A Representational Analysis of Relational Information Displays*

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Abstract

Graphic and tabular displays are analyzed under a common, unified form—relational information displays (RIDs), which are displays that represent relations between dimensions. First, the mapping between the representing and the represented dimensions of RIDs is analysed from the perspective of distributed representations (Zhang & Norman, 1994). Second, the structures of RIDs are analyzed at three levels: dimensionality, scale types, and dimensional representations. From this analysis, a representational taxonomy is developed that not only can classify all RIDs but also can serve as a framework for systematic studies of RIDs. Third, a task taxonomy of RIDs is developed, which can classify the majority of dimension-based display tasks. Finally, the relation between representations of displays and structures of tasks is analyzed in terms of a mapping principle: the information perceivable from a RID should exactly match the information required for the task. Thus, although there are no best displays that are efficient for all types of tasks, there is a correct or incorrect mapping between the representation of a display and the structure of a task.

1. Introduction

There are a wide variety of graphic and tabular displays, such as line graphs, bar charts, pie charts, scatter plots, matrices, tables, networks, maps, and many others. Despite of the great diversity, these displays all can be unified under a common form—*relational information displays* (henceforth, RIDs), which are displays that represent relations between dimensions. The objective of this article is to understand how the different displays all work under a single framework.

1.1. The Formal Structure of RIDs

According to the mathematical theory of relations, a relation is a structure on a set of dimensions. If $\mathbf{D_1}$, $\mathbf{D_2}$, ..., $\mathbf{D_n}$ are N dimensions and each dimension $\mathbf{D_i}$ is a set with n_i elements (d_{i1}, d_{i2}, d_{i3}, ..., d_{in}), then a relation **R** on these N dimensions is a subset of the Cartesian product $\mathbf{C} = \mathbf{D_1} \times \mathbf{D_2} \times ... \times \mathbf{D_n}$. Each element in the relation set **R** is a n-tuple. Let us consider an example. Figure 1 shows a simplified directory display on a Macintosh computer. It has five dimensions:

* This article will appear in *International Journal of Human-Computer Studies*. I am very grateful to Andrew Monk, whose valuable comments and suggestions reshaped the conceptual organization of this article. I'd also like to thank three anonymous reviewers for their valuable comments and constructive criticisms. Correspondence and requests for reprints should be sent to Jiajie Zhang, Department of Psychology, The Ohio State University, 1827 Neil Avenue, Columbus, OH 43210, USA. Email: zhang.52@osu.edu.

Name = (data, final, record, work)
Size = (120K, 100K, 70K, 55K)
Type = (MacDraw, Excel, Word)
Label = (hot, warm, cold)
Time = (2:40pm, 3:35pm, 4:30pm, 6:20pm)

The Cartesian product **Name**×**Size**×**Type**×**Label**×**Time** has a total of 576 (4×4×3×3×4) 5-tuples. **Directory** is a relation on the five dimensions. It has four 5-tuples, which are a subset of the Cartesian product **Name**×**Size**×**Type**×**Label**×**Time**.

Directory = ((final, 120K, Word, hot, 2:40pm),
(work, 70K, Word, warm, 6:20pm),
(record, 55K, Draw, warm, 3:35pm),
(data, 100K, Excel, cold, 4:30pm))

In addition to the tabular display shown in Figure 1, other types of tabular displays and a variety of graphic displays can represent the same relational information **Directory**. Figure 2 shows just one example of graphic display that represents the same information as the tabular display in Figure 1. It is common experience that some displays can be better than others in conveying the same information. This representational effect, that different representations of a common abstract structure can have different representational efficiencies and produce different cognitive behaviors, has long been the focus of studies on graphic displays (for a few reviews and integrative studies, see Bertin, 1983; Carswell & Wickens, 1988; Cleveland, 1985; Schmid, 1983; Tufte, 1983, 1990).

1.2. Outline

This article, by unifying diverse graphic and tabular displays under a common form—RIDs, develops a general theoretical framework that not only can describe the properties and structures but also can account for the representational effects of all RIDs. This theoretical framework has four components: dimensional representations, a representational taxonomy, a task taxonomy, and a mapping principle for the relation between representations and tasks. These four components are developed in four steps. First, dimensions, the basic structures of RIDs, are analyzed from the perspective of distributed representations (Zhang & Norman, 1994, 1995). The basic idea is that the representations of dimensions in RIDs are distributed representations with internal and external representations as two indispensable components. Second, the structures of RIDs are analyzed at three levels: dimensionality, scale types, and dimensional representations. From this analysis, a representational taxonomy of RIDs is developed, which not only can classify all RIDs but also can specify the important structures and factors for systematic studies of RIDs. Third, a task taxonomy of RIDs is developed, which can classify most dimension-based display tasks. Fourth, the relation between representations of displays and structures of tasks is analyzed in terms of a mapping principle: the information that can be perceived from a RID should exactly match the information required for the task. Thus, although there are no best displays that are efficient for all types of tasks, there is a correct or incorrect mapping between the representation of a display and the structure of a task.

Name	Size	Type	Label	Time
final	120K	Word	hot	2:40pm
work	70K	Word	warm	6:20pm
record	55K	MacDraw	warm	3:35pm
data	100K	Excel	cold	4:30pm

Figure 1. A simplified directory (folder) display on an Apple Macintosh computer. It has five dimensions: Name, Size, Type, Label, and Time. Label is a user-defined property of files, which is defined as an ordinal dimension representing the relative activity levels of files in the present study. Time indicates the time when a file is last modified.

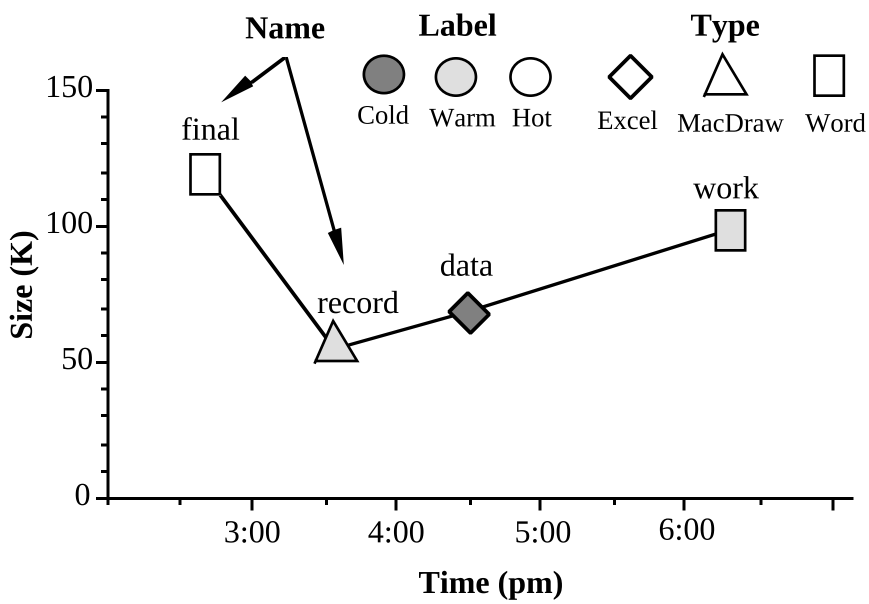

Figure 2. A 5-dimensional graphic display that represents the same information as the tabular display in Figure 1.

2. Representations of Dimensions

Dimensions are the basic structures of RIDs. Thus, the properties of RIDs are mainly determined by the properties of their dimensions. This section (a) introduces the formal properties of dimensions as scales, (b) describes the representations of scales from the perspective of distributed representations, (c) analyzes the mapping between represented and representing dimensions, and (d) shows the similarities and differences between graphic and tabular displays.

2.1. Formal Properties of Dimensions—Scales

Every dimension, whether it is a physical dimension such as the length of a bar or a more abstract dimension such as the amount of money, is on a certain type of scale. The scale of a dimension is the abstract measurement property of the dimension (see Krantz, Luce, Suppes, & Tversky, 1971; Narens, 1981; Stevens, 1946). Stevens identified four major types of scales: ratio, interval, ordinal, and nominal. Each type has one or more of the following formal properties: category, magnitude, equal interval, and absolute zero (see Table 1). *Category* refers to the property that the instances on a scale can be distinguished from each another. *Magnitude* refers to the property that one instance on a scale can be judged greater than, less than, or equal to another instance on the same scale. *Equal interval* refers to the property that the magnitude of an instance represented by a unit on the scale is the same regardless of where on the scale the unit falls. An *absolute zero* is a value which indicates that nothing at all of the property being represented exists.

Let us use the dimensions in Figure 1 to illustrate the four types of scales. Nominal scales only have one formal property: category. Names of computer files are an example of nominal scales: they only discriminate between different entities but have no information about magnitudes, intervals, and ratios. Ordinal scales have two formal properties: category and magnitude. The activity levels of computer files are an example of ordinal scales: the activity level of a "hot" file is different from that of a "cold" file (category) and a "hot" file is more active than a "cold" file (magnitude). However, the activity levels themselves tell us nothing about the interval differences and ratios between the activity levels. Interval scales have three formal properties: category, magnitude, and equal interval. Time is an example of interval scales: 02:00 is different from 22:00 (category), 14:00 is later than 09:00 (magnitude), and the difference between 15:00 and 14:00 is the same as that between 09:00 and 08:00 (equal interval). However, time does not have an absolute zero. Thus, we cannot say that 10:00 is twice as late as 05:00. Ratio scales have all of the four formal properties: category, magnitude, equal interval, and absolute zero. The sizes of computer files are an example of ratio scales: 1K is different from 2K (category), 10K are larger than 5K (magnitude), the difference between 10K and 11K is the same as the difference between 100K and 101K (equal interval), and 0K means the nonexistence of size (absolute zero). For file sizes, we can say that 10K are twice as large as 5K.

Table 1. The Formal Properties of Scales

Formal Properties	Scale Types			
	ratio	interval	ordinal	nominal
category	yes	yes	yes	yes
magnitude	yes	yes	yes	no
equal internal	yes	yes	no	no
absolute zero	yes	no	no	no
Example	file size	time	activity level	file name

2.2. Distributed Representation of Scale Information

RIDs have two types of dimensions: represented and representing dimensions. The represented dimensions of a RID are the dimensions of an original domain that are to be represented by the physical dimensions of the RID. The representing dimensions of a RID are the physical dimensions of the RID that are used to represent the dimensions of the original domain. For example, in Figure 2, the represented dimensions are name, size, type, label, and time, which are the dimensions of computer files (the original domain); the corresponding representing dimensions are text (for name), distance (for size), shape (for type), density (for label), and position (for time), all of which are the physical dimensions of the display.

In order for a RID to be accurate and efficient, its physical dimensions must represent the dimensions of the original domain accurately and efficiently. Because the information on a dimension that is perceived, processed, and manipulated is always the scale information of the dimension, the representation of dimensions is the representation of its scale information.

In general, the representation of the scale information of a represented dimension of an original domain by a representing dimension of a RID is a distributed representation with internal and external representations as two indispensable parts. Specifically, the scale information of a represented dimension is the set of the formal properties of the dimension. These formal properties constitute the abstract representational space of the represented dimension. A distributed representation of the dimension means that some of its formal properties are in the external representation and some in the internal representation (see Figure 3 and the explanations below). General treatments of distributed representations can be found in Zhang & Norman (1994, 1995).

From Table 1 we can see that the four types of scales have an order of representational power: ratio > interval > ordinal > nominal. A higher scale (e.g., ratio) possesses more information (more formal properties) than a lower scale (e.g., nominal). Figure 3 shows three possible forms of distributed representations of dimensions. In Figure 3A, a higher dimension (file size, ratio scale) is represented by a lower dimension (shape of digit, nominal scale). Because shape is on a nominal scale, it can only represent the category property of file size in the external representation. The other three properties of file size (magnitude, equal interval, absolute zero) are represented internally because they are not embedded in the physical properties of the shapes of digits. Generally speaking, when a higher dimension is represented by a lower dimension, the extra information of the higher dimension either has to be represented internally (as in Figure 3A) or is not represented at all.

In Figure 3B, a lower dimension (file type, nominal scale) is represented by a higher dimension (length of bar, ratio scale). In this case, all the scale information of the lower dimension is represented externally by the higher dimension because the formal properties of the lower dimension is a subset of those of the higher dimension. However, the extra information in the higher dimension may cause misperceptions on the lower dimension (see Mackinlay, 1986; Norman, 1993). In Figure 3B, what we really need to represent is the category property of file types, that is, Word, Excel, and MacDraw indicate different types of files. Because length is a ratio dimension, the extra information it has (magnitude, equal interval, absolute zero) may cause misperceptions on the represented dimension (file type). For example, we may get the misperception that Word is twice as large as Excel, or Excel is twice as expensive as MacDraw, or other misperceptions.

In Figure 3C, the scale type of the represented dimension (file size, ratio scale) matches the scale type of the representing dimension (length, ratio scale). In this case, all the scale information of the represented dimension is represented externally by the representing dimension.

Among the three forms of representations in Figure 3, Figure 3C is the best one for two reasons. First, Figure 3C is more efficient than Figure 3A because all the information in Figure 3C is external and can be processed by perceptual mechanisms. Second, Figure 3C is more accurate than Figure 2B because misperception can be produced by Figure 3B, but not by Figure 3C.

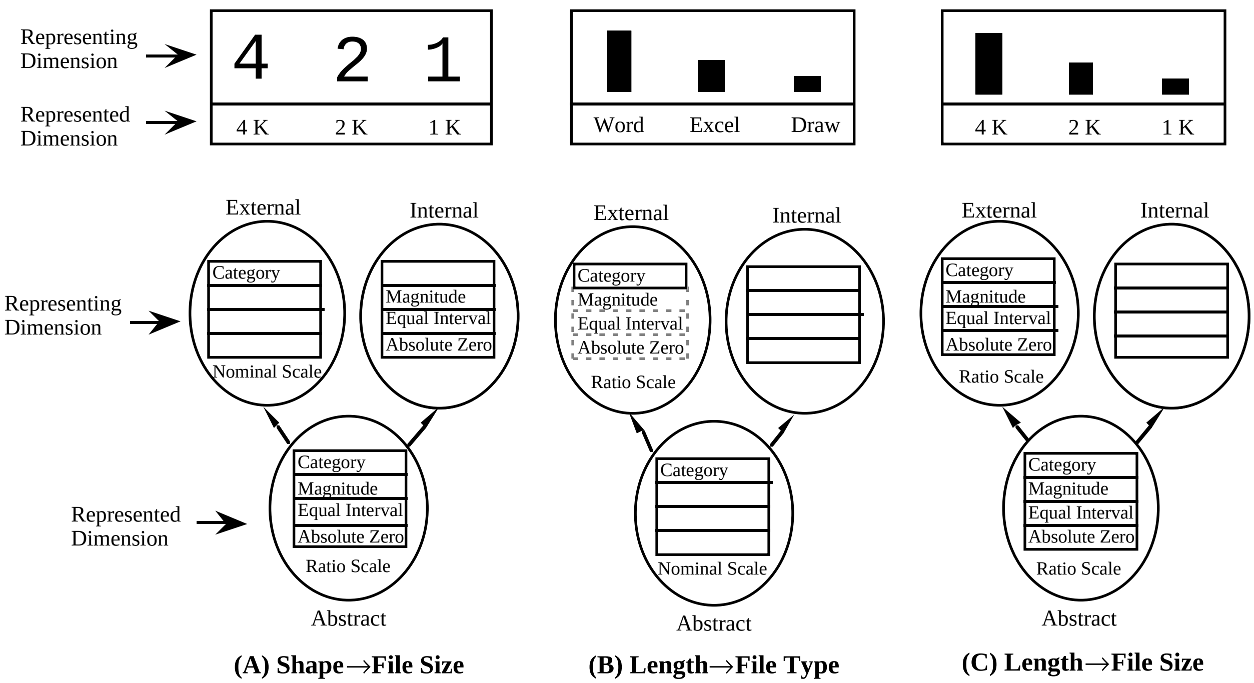

Figure 3. The distributed representation of scale information. The scale information of a dimension is in the abstract space as a set of formal properties, which are distributed across an internal and an external representation. (A) A nominal dimension (shape of digit) represents a ratio dimension (file size). The extra information of the ratio dimension either has to be represented in the internal representation or is not represented at all. (B) A ratio dimension (length) represents a nominal dimension (file type). The extra information of the ratio dimension may cause misperceptions on the nominal dimension. (C) A ratio dimension (length) represents a ratio dimension (file size). This is an efficient and accurate representation.

2.3. The Mapping between Represented and Representing Dimensions

Last section shows that in order for a representation to be efficient and accurate, the represented and representing dimensions should match in scale types. This section shows examples of different types of mappings between represented and representing dimensions in RIDs.

In Figure 4, the represented dimensions are the dimensions of the relation **Directory** (see Figure 1): size (ratio), time (interval), label (ordinal), and type (nominal). The representing dimensions are the physical dimensions used to represent the represented dimensions: length (ratio), orientation (interval), density (ordinal[1]), and shape (nominal). In the four displays on the diagonal in Figure 4 (A, F, K, P), the scale types of the represented dimensions match the scale types of the representing dimensions. In these displays, the scale information of the represented dimensions is represented efficiently and accurately. In the six displays

[1] According to Stevens' Law (Stevens, 1957), $\psi = \mathbf{kS^n}$, where ψ is the ratio judgment by psychological measurement, **S** is the ratio judgment by physical measurement, **k** is a constant, and **n** is the power index determined by the properties of the stimulus and the measurement process. For density, $n \approx 1.3$. Thus, although density is a ratio dimension by physical measurement, the perception of the radio (and interval) information of density is distorted and difficult. Therefore, in Figure 4 density is used as an ordinal dimension.

above the diagonal, the representing dimensions have more information than the represented dimensions. The extra information of the representing information may cause misperceptions on the represented dimensions. For example, in Figure 4C, a "warm" file may be perceived as twice as active as a "cold" file. This is a misperception, because "hot", "warm", and "cold" only indicate the relative activity levels of the files: they have no ratio and interval information. In the six displays below the diagonal, the representing dimensions have less information than the represented dimensions. In these displays, the extra information of the represented dimensions either has to be represented internally or is not represented at all. For example, in Figure 4M, the shapes possess no ratio, interval, and ordinal information about the sizes of the four files.

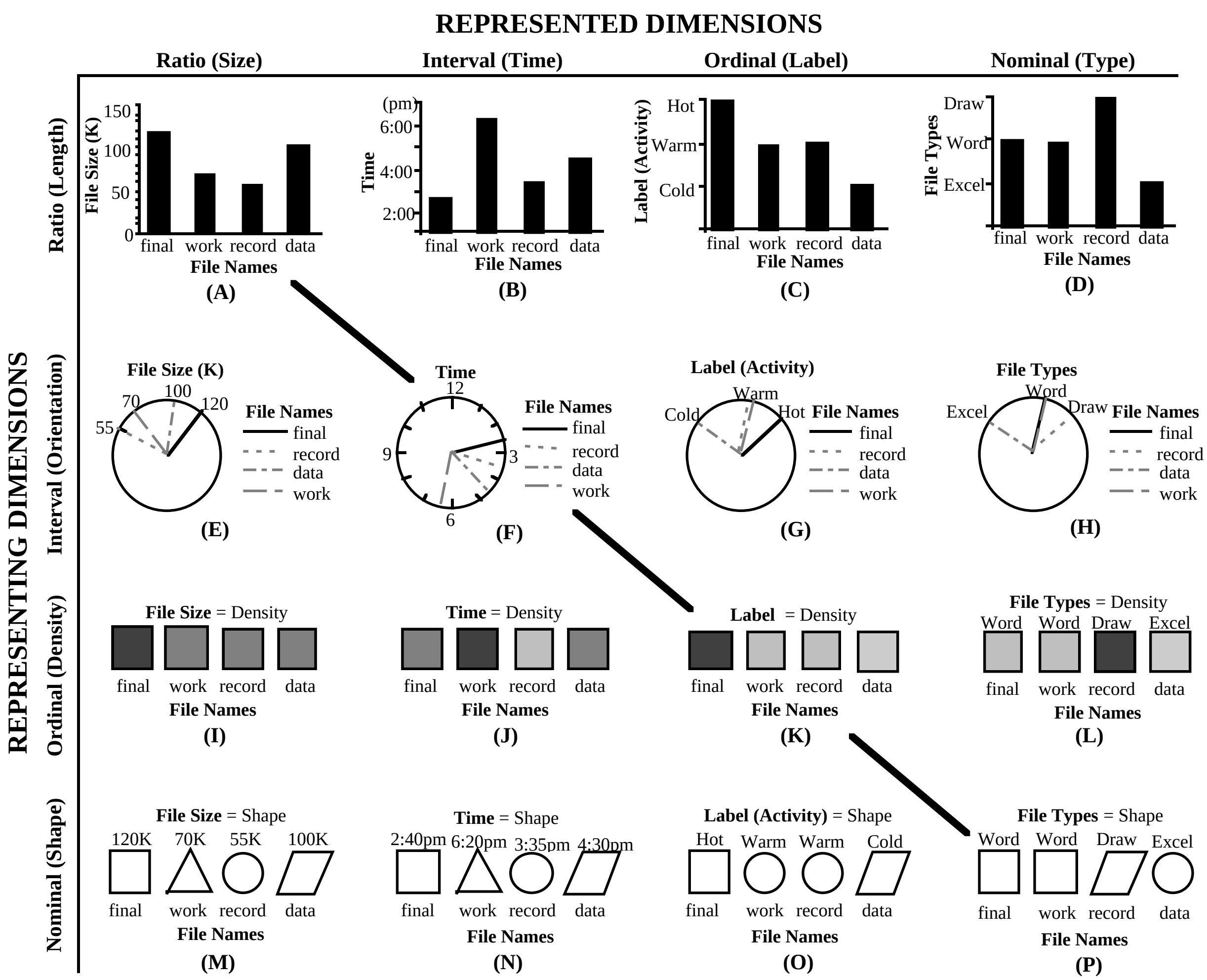

Figure 4. The mapping between represented and representing dimensions. The representing dimensions in the four displays on the diagonal have the same amount information as the represented dimensions. The representing dimensions in the six displays above the diagonal have too much information. The representing dimensions in the six displays below the diagonal have insufficient information.

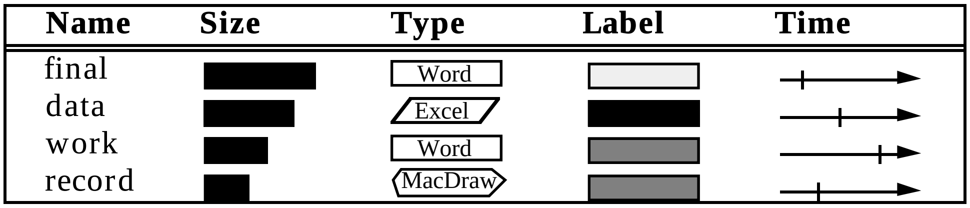

Figure 5. An example of graphical tabular display.

2.4. Graphic vs. Tabular Displays

Based on the analysis of dimensional representations in Section 2.2., we can analyze the similarities and differences between graphic and tabular displays. In alphanumeric tabular displays, the dimensions of relational information are represented by alphanumeric symbols and positions of table cells. Alphanumeric symbols are nominal dimensions, which can only represent nominal information externally. Cell positions are ordinal dimensions, which can only represent nominal and ordinal information externally. Thus, neither alphanumeric symbols nor cell positions can represent interval and ratio information externally. In contrast, in graphic displays, not only nominal and ordinal information but also interval and ratio information can be represented externally (e.g., by length, distance, etc.). This is the main reason why graphic displays are better than alphanumeric tabular displays when interval and ratio information needs to be represented. However, for relational information that only has nominal and ordinal information, graphic and tabular displays do not differ much in their representational efficiencies.

Although alphanumeric tabular displays can not represent interval and ratio dimensions externally, graphic tabular displays can do so. This is because in graphical tabular displays, alphanumeric symbols can be replaced by other physical dimensions such as length, position, etc., which are ratio and interval dimensions. Figure 5 is an example of graphic tabular display. It represents the same information as the alphanumeric tabular display in Figure 1.

3. A Representational Taxonomy of RIDs

RIDs can be analyzed at three levels: dimensionality, scale types, and dimensional representations (see Figure 6 and the explanations below). This hierarchical structure of RIDs can serve as a representational taxonomy of RIDs.

3.1. The Hierarchical Structure

At the level of dimensionality, different RIDs can have different numbers of dimensions, e.g., 2-D, 3-D, 4-D, etc. For example, all the displays in Figure 4 have two dimensions and the display in Figure 2 has five dimensions.

At the level of scale types, the dimensions of a RID can have different scale types: ratio (R), interval (I), ordinal (O), and nominal (N) scales. For example, the RID in Figure 2 has one ratio (distance), one interval (position), one ordinal (density), and two nominal scales (shape and text). In general, a n-dimensional display can have (n+3)(n+2)(n+1)/6 combinations of scale types[2] . For example, a

[2] To calculate the total number of combinations of scale types for a n-dimensional display, let us consider the sequence: R••...•I••...•O••...•N••...•, where the numbers of dots after R, I, O, and N are the num-

2-D display can have (2+3)(2+2)(2+1)/6 = 10 combinations: R-R, R-I, R-O, R-N, I-I, I-O, I-N, O-O, O-N, N-N.

At the level of dimensional representations, each scale type can be implemented by different physical dimensions. In Figure 6, for example, ratio scale is represented by length, distance, and angle; interval scale by position and orientation; ordinal scale by cell position; and nominal scale by shape, direction, texture, and position. With these physical dimensions, the scale combination R-R can be represented by length-length (Rectangle, Cross), length-angle (Coxcomb, Polar Plot), distance-distance (Line Graph, Cartesian Plot), and so on. The scale combination R-I can be represented by length-position (histogram), length-orientation (glyph, polygon), distance-position, and so on. The scale combination R-N can be represented by length-position (segmented and vertical bar charts), length-direction, angle-direction (pie chart), and so on. The scale combinations O-O-N can be represented by CellPosition-CellPosition-shape (table, matrix), position-position-texture (network), and so on.

3.2. The Representational Taxonomy

The hierarchical structure in Figure 6 can serve as a representational taxonomy of RIDs. This taxonomy can classify all RIDs, including most graphs, charts, tabular displays, maps, networks, etc. For example, among the displays in Figure 6, the pie chart and vertical bar chart are in the same category at the level of dimensional representations because they are different representations of the same scale types; the line graph and the pie chart are in the same category at the level of scale types because they have different scale types with the same dimensionality; and all the displays in Figure 6 are in the same category at the level of dimensionality because they are all RIDs.

With this taxonomy, we can get a rough estimate of the similarity between any two RIDs. The lower the level at which two RIDs are in the same category, the more similar they are. For example, the pie chart and the vertical bar chart are more similar to each other than the pie chart and the line graph, because the former two are in the same group at the level of dimensional representations whereas the latter two are at the level of scale types.

In addition to the classification of all RIDs, this taxonomy can also specify the structures important for any systematic studies of RIDs, whether they are empirical studies or practical applications. These structures are the three levels of the hierarchical structure. As an example of empirical studies, we can compare the behavioral outcomes of different dimensional representations of the same scale type in a variety of display tasks. As an example of practical applications, the taxonomy tells us that in order to decide whether a RID is appropriate for a task, we need to analyze how many dimensions the RID has, what scale types the dimensions have, and how the scale types are represented.

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bers of dimensions on ratio, interval, ordinal, and nominal scales, respectively. Thus, the total number of dots in the sequence is n—the number of the dimensions of the display. The number of permutations of this sequence with R fixed at the beginning is (n+ 3)!. Because the n dots and I, O, and N are interchangeable, their permutations (n! and 3!, respectively) should be excluded. Thus, the total number of possible scale types of a n-D display is (n + 3)!/n!3! = (n+3)(n+2)(n+1)/6.

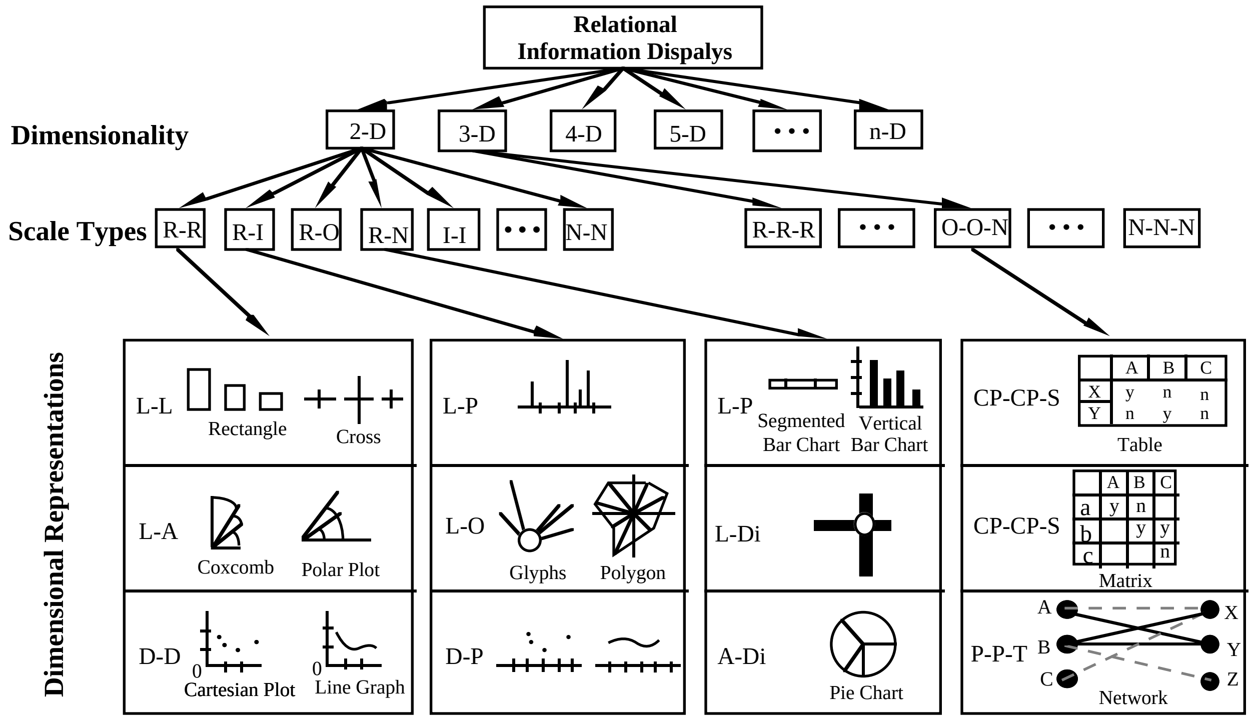

Figure 6. The hierarchical structure of RIDs. It is also a representational taxonomy of RIDs. A = Angle, CP = Cell Position, D = Distance, Di = Direction, L = Length, O = Orientation, P = Position, S = Shape, T = Texture. See text for details.

4. A Task Taxonomy of RIDs

The same RID can support different tasks and the same task can be performed on different RIDs. Last section developed a representational taxonomy of RIDs. The purpose of this section is to develop a task taxonomy of RIDs. Next section will analyze the relations between representations and tasks.

The task taxonomy developed here is similar to the one described by Carswell and Wickens (1988). However, the current task taxonomy is based on the principles of dimensional representations developed in the previous sections of the present study. According to this task taxonomy, there are three major types of display tasks: information retrieval, comparison, and information integration.

4.1 Information Retrieval Tasks

An information retrieval task is to search for specific information in a RID. The dimensions of a RID can be divided into a base dimension on which the search is carried out, and a set of target dimensions on which the specific information is identified. Thus, search on the base dimension and identification on the target dimensions are the two components of an information retrieval task.

To start an information retrieval task, one base dimension and one or more values on the base dimension are specified. Section 1.1 shows that a relation on N dimensions is a set of n-tuples, which are a subset of the Cartesian product of the N dimensions. In the language of tuples, the search task is to find all the tuples that contain the specified values on the base dimension. The identification task is to identify the values on the target dimensions of all matching tuples. Let us consider a task for the display in Figure 2. The task is to examine all the properties of all *word* files. In this case, the base dimension is "type", the specified value on this base dimension is *word*, and the target dimensions are "name", "size", "label", and "time". The search on the base dimension “type” brings out two tuples: (*final*, *120K*, *word*, *hot*, *2:40pm*) and (*work*, *70K*, *word*, *warm*, *6:20pm*), which both contain the specified value *word*. The values on the four target dimensions of these two matching tuples are then identified as (*final*, *120K*, *hot*, *2:40pm*) and (*work*, *70K*, *warm*, *6:20pm*), which indicate that the first *word* file called *final* is a *120K hot* file modified at *2:40pm* and the second *word* file called *work* is a *70K warm* file modified at *6:20pm*. If the task is to examine only the names of all *word* files, then "name" is the only target dimension. In this case, the resulting tuples are (*final*, *word*) and (*work*, *word*), which indicates that the first *word* file is called *final* and the second *word* file is called *work*.

The search task is affected by the scale type of the base dimension. In other words, the scale type of a base dimension determines which search tasks can be performed on the base dimension. Let us use the display in Figure 2 as an example to describe the allowable search tasks on dimensions with different scale types. On nominal dimensions, there is only one search task: finding one or more specified values, such as finding *word* on the "type" dimension. On ordinal dimensions, there are two search tasks: the one for nominal dimensions and a second one about magnitude information such as searching for the hottest file on the "label" dimension. On interval dimensions, there are three search tasks: the two search tasks for ordinal dimensions and a third one about interval information such as searching for the two times that are closest to each other. On ratio dimensions, there are four search tasks: the three for interval dimensions and a fourth one about ratio information such as finding all the sizes that have a 1/2 ratio relative to each other.

The identification task on the target dimensions is affected by the arrangement of the positions of the dimensions. In a tuple format tabular display such as Figure 1, the identification task is easier because the dimensions are arranged as parallel columns and the values to be identified on the target dimensions are in the same row as a tuple. In contrast, in a graphical display such as Figure 2, the identification task is harder because the dimensions are scattered in different places.

4.2 Comparison Tasks

There are two types of comparison tasks: within-dimension comparisons and between-dimension comparisons. Similar to information retrieval tasks, both types of comparison tasks have a base dimension and a set of target dimensions.

A within-dimension comparison task is to compare the values on the same target dimension. Given two or more specified values on the base dimension, there are two or more tuples that contain the specified values on the base dimension. The comparison is between the corresponding values on any of the target dimensions. Let us consider a task for the display in Figure 2. The base dimension is "name", the two specified values on this base dimension are *final* and *record*, and the target dimensions are "size", "label", "type", and "time". We can compare the sizes, labels, types, or times of the two files named *final* and *record*. For example, we can compare *final* and *record* on the “size” dimension: whether *final* is larger in size than *work*.

A between-dimension comparison task is to compare the values across different dimensions. The dimensions to be compared must have the same scale type.

Given one specified value on the base dimension, there are one or more tuples that contain the specified value on the base dimension. The comparison is between the corresponding values of two or more target dimensions of one of the matching tuples. Let us consider a between-dimension comparison task for the display in Figure 7, which has three dimensions: names (nominal), lengths (ratio), and breadths (ratio) of lakes. The task is to compare the length and breadth of Lake Erie. In this task, the base dimension is the "names" of the lakes, the specified value on this base dimension is *Erie*, and the two target dimensions to be compared are "lengths" and "breadths". These two target dimensions can be compared because they are both on ratio scales.

For both within-dimension and between-dimension comparison tasks, the scale types of dimensions determine what properties can be compared. For ratio dimensions, the comparison can be about ratios, intervals, magnitudes, and categories; for interval dimensions, about intervals, magnitudes, and categories; for ordinal dimensions, about magnitudes and categories; and for nominal dimensions, about categories only.

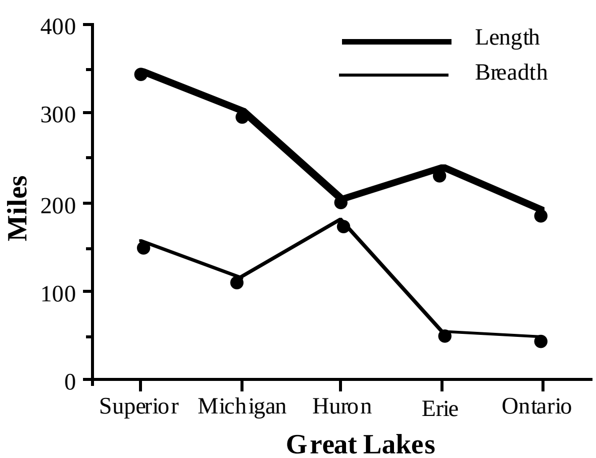

Figure 7. An example of between-dimension comparison.

4.3. Integration Tasks

Although the focus of this article is on the representation of relational information, not on the integration of information, integration tasks are briefly described here because they are part of the task taxonomy of RIDs.

The dimensions of RIDs can interact with each other in various ways to produce different effects on the perception of the dimensional structures of displays. Garner (1974) identified two major types of dimensional interactions: separable and integral. Separable dimensions are those whose component dimensions can be directly and automatically separated and perceived. The size and shape of an object are separable dimensions: the levels on size and shape are perceived as isolated, unrelated entities. Integral dimensions can only be perceived in a holistic fashion: they can not be separated without a secondary process that is not automatically executed. The width and height of a rectangle are integral dimensions (Dykes, 1979): the perception of width interacts with the perception of height.

In general, if the function of a display is to represent relational information, the dimensions should be separable. However, there are tasks that deliberately make use of the integrality of dimensions. In these integration tasks, integral dimensions are better than separable dimensions (e.g., Carswell & Wickens, 1988) because the nature of the task is the integration of information. Trend analysis and information integration are two examples of such integration tasks.

Trend analysis is usually carried out for two dimensions. The task is to determine the trend of the target dimension in relation to the base dimension. The two dimensions for trend analysis have to be on ratio or interval scales.

Trend is primarily based on the slopes of all adjacent pairs of data points in a display. Each slope is an emergent property which is the ratio of the two intervals on the two dimensions. Integral dimensions are generally better than separable dimensions for the perception of slopes (e.g., Schutz, 1961).

An information integration task is to integrate the information from several dimensions to make decisions. Generally, the more integral the dimensions, the better the integration and the easier the task. For example, Jacob, Egeth, & Bevan (1976) showed that polygons (integral) are better than glyphs (separable) for clustering tasks (see Figure 6). More studies on information integration tasks can be found in Carswell & Wickens (1988).

5. The Mapping Between Representations and Tasks

There are a large body of empirical studies on the relative representational efficiencies of graphic displays (for a review, see Carswell and Wickens, 1988). However, there has been little consensus on what makes a graph good or poor. The only consistent finding, contrary to initial expectations, is that there does not exist a universally best display which is efficient for all types of tasks. In other words, whether a display is effective for a task depends on not just the representational properties of the display but also the structure of the task.

Although there are no general principles for designing a best representation which is efficient for all types of tasks, the representational taxonomy and the task taxonomy of RIDs described in the last two sections suggest that there does exist a general principle that can identify correct or incorrect mappings between representations and tasks. This mapping principle is that the information perceivable from a RID should exactly match the information required for the task, no more and no less. In other words, the tasks assigned to a display should be the tasks afforded by the external representations of the display and the displays assigned to a task should be the displays whose external representations support the task.

If a RID does not have sufficient information for a task, then either the task can not be performed or the extra information needed for the task has to be compensated by internalized information in memory. For example, 1-digit Arabic numerals are not a good representation for magnitude comparison tasks because the shapes of Arabic numerals are on a nominal scale that does not have magnitude information (see Zhang & Norman, 1995). The magnitude information has to come from memorized knowledge acquired over years of learning.

If a RID has more information than a task requires, the extra information may cause misperceptions or misunderstandings. For example, length is not a good representation for magnitude comparison tasks because length not only has magnitude information but also has extra information about intervals and ratios. If we use length to represent the activity levels of computer files, we may get misperceptions about the differences and ratios between different activity levels, which are in fact meaningless because activity level is on an ordinal scale.

When a RID has necessary and sufficient information for a task, the mapping between the display and the task is perfect. For example, length is a good representation for ratio comparison tasks because length is on a ratio scale and ratio comparison tasks require ratio scales.

5. Conclusion

5.1. Related Research

Graphic and tabular displays have long been considered having a variety of advantages over textual and alphanumeric representations: visually appealing; bringing out hidden information and relations; stimulating diagrammatic thinking; making information transparent; and so on. Early experimental studies (for a review, see Carswell & Wickens, 1988) focused on the relative efficiencies of different display formats. Bertin (1983) was among the first to take an extensive and systematic approach to graphics. Through a detailed semiotic analysis, Bertin developed a taxonomy of graphic displays and a set of principles for graphics design. His work has many great insights and is very useful in practice. However, it does not have a psychological foundation, which is important because the comprehension of graphs is essentially a psychological activity. Likewise, many other studies on graphics, including Schmid (1983) and Tufte (1990), did not address the psychological issues, either.

Recently, there has been a revival of interest in the psychological foundation of graphics. Most of the psychological studies are primarily concerned with the difficulty factors and perceptual and cognitive processes involved in graph comprehension. Cleveland and his colleagues (Cleveland, 1985; Cleveland & McGill, 1985) studied the comparisons of quantitative information and the relative goodness of a set of physical dimensions for ratio judgments. Carswell & Wickens (1988), based on the psychological studies of integral and separable dimensions, proposed the proximity compatibility hypothesis to account for a variety of graphics problems, especially those that involve information integration. Another line of research has focused on the perceptual and cognitive processes involved in graph and diagram comprehension (e.g., Casner & Larkin, 1989; Larkin & Simon, 1987; Pinker, 1990). In these studies, the perceptual and cognitive processes involved in graph comprehension are analyzed and modeled by computational systems such as production systems and semantic networks. Recently, Norman (1993) analyzed the representational properties of information displays and the role of psychological scales in graphics design. On the basis of psychological studies, several researchers have developed computer systems that can automate the design of graphics (e.g., Casner, 1990; Machinlay, 1986).

5.2. Significance of the Current Research

Although the current approach shares the same interest with other approaches in the properties, structures, and representational effects of graphic and tabular displays, it differs in several aspects. First, it unified a great diversity of graphic and tabular displays under a common form—relational information displays. Second, it offered a new conceptual framework: RIDs are distributed representations that have internal and external representations as two indispensable parts. In comparison with internal representations, the important roles of external representations in RIDs have not been fully recognized. The present study made a contribution by emphasizing and exploring the important properties of external representations in RIDs. Third, it developed a representational taxonomy of RIDs that not only can classify all RIDs but also can serve as a framework to guide empirical and practical studies of RIDs in a systematic way. Fourth, it developed a task taxonomy of RIDs that can classify most dimension-based display tasks. Finally, it pointed out that although there are no general principles for designing the best displays that are efficient for all types of tasks, there is a general mapping principle that can identify whether the mapping between a representation and a task is correct or incorrect. The practical implication of this mapping principle is that when we design RIDs we should always consider the relation between displays and tasks and should not waste time to find the best display or the easiest task in isolation.

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