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External Representations in Complex Information Processing Tasks

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External representations are involved in many complex information processing tasks, such as multiplication with paper and pencil, grocery shopping with a written list, web surfing, library information retrieval, geometrical problem solving, graph understanding, diagrammatic reasoning, chess playing, most computer-based tasks, and so on. Few would deny that external representations play certain roles in these tasks. However, in comparison with internal representations, relatively little research in mainstream cognitive science has been directed towards the nature of external representations in complex information processing tasks. Recently, there has been a growing awareness of the much more important roles of external representations than previously thought, and there have been a growing number of studies on external representations. The purpose of this article is to review and summarize the recent studies on external representations and discuss their implications in cognitive and information sciences.

Background: Internal and External Representations

Knowledge representation is a fundamental issue in cognitive science. It is impossible to imagine a cognitive system in which representations do not play a central role. Due to such importance, there has been a large body of research in cognitive science over the past few decades that has greatly enhanced our understanding of the nature of representations. Most of these studies are concerned with internal representations—the knowledge structures in people's head. Rumelhart & Norman (1988) summarized the major achievements in the study of internal representations in terms of four types of representations: propositional, analogical, procedural, and parallel and distributed representations. Propositional representations are knowledge structures that are based on a set of discrete symbols or propositions so that knowledge and concepts in the world are represented by formal statements. Examples include symbolic logic and predicate calculus, semantic networks, and schemas and frames. Analogical representations are knowledge structures that have direct correspondence between the represented world and the representing world in a continuous manner, such as a mental map of the United States. Procedural representations are knowledge structures in the form of procedures and processes such as the procedural skills of riding a bike or producing a speech sound. Parallel and distributed representations are knowledge structures that are not represented at any discrete place in memory but instead is distributed over a large set of representing units with each unit representing a piece of a large

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amount of knowledge. Parallel and distributed representations are also called connectionist representations and neural network representations.

In comparison with internal representations, systematical studies of external representations did not begin until recently (see Zhang, 1997, for a review). This late start might be due to the belief that very little knowledge about the internal mind can be gained by studying external representations, or due to the view that external representations are nothing but inputs and stimuli to the internal mind, or simply due to the lack of a suitable methodology for studying external representations. External representations are the knowledge and structures in the environment, as physical symbols, objects, or dimensions (e.g., written symbols, beads of abacuses, dimensions of a graph, etc.), and as external rules, constraints, or relations embedded in physical configurations (e.g., spatial relations of written digits, visual and spatial layouts of diagrams, physical constraints in abacuses, etc.). The information in external representations can be picked up, analyzed, and processed by perceptual systems alone, although the top-down participation of conceptual knowledge from internal representations can sometimes facilitate or inhibit the perceptual processes. External representations are just as important as internal representations. Much can be learned about the internal mind by studying external representations because much of the structure of the internal mind is a reflection of the structure of the external environment (e.g., Anderson, 1993; Kirlik, Plamondon, Lytton, Jagacinski, 1993a, 1993b; Shepard, 1984; Simon, 1981). External representations are not simply inputs and stimuli to the internal mind; rather, they are so intrinsic for many cognitive tasks that they guide, constrain, and even determine cognitive behavior.

Theory: Distributed Representations

A wide variety of complex information processing tasks are distributed cognitive tasks—tasks that require the processing of information distributed across internal and the external representations. It is the interwoven processing of internal and external information that generates much of a person's intelligent (Hutchins, 1995a, 1995b; Norman, 1988, 1993; Zhang & Norman, 1994). Let us consider multiplying 735 by 278 using paper and pencil (Figure 1). The internal representations are the meanings of individual symbols (e.g., the numerical value of the arbitrary symbol "7" is seven), the addition and multiplication tables, arithmetic procedures, etc., which have to be retrieved from memory. The external representations are the shapes and positions of the symbols, the spatial relations of partial products, etc., which can be perceptually inspected from the environment (see Zhang & Norman, 1995). To perform this task, people need to process the information perceived from external representations and the information retrieved from internal representations in an interwoven, integrative, and dynamic manner.

External representations can be transformed into internal representations by memorization. But this internalization is not necessary if external representations are always available, and not possible if external representations are too complex. Internal representations can also be transformed into external representations by externalization. Externalization can be beneficial if the benefit of using external representations can offset the cost associated with the externalization process.

```
         7  3  5
      ×  2  7  8
      ----------
      5  8  8  0
   5  1  4  5
1  4  7  0
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2  0  4  3  3  0
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Figure 1. This is an example of internal and external representations. In this multi-digit multiplication task using paper and pencil, the internal representations are the meaning of individual symbols, addition and multiplication tables, and arithmetic procedures, and the external representations are the shapes of the symbols, positions of the symbols, and spatial relations of partial products

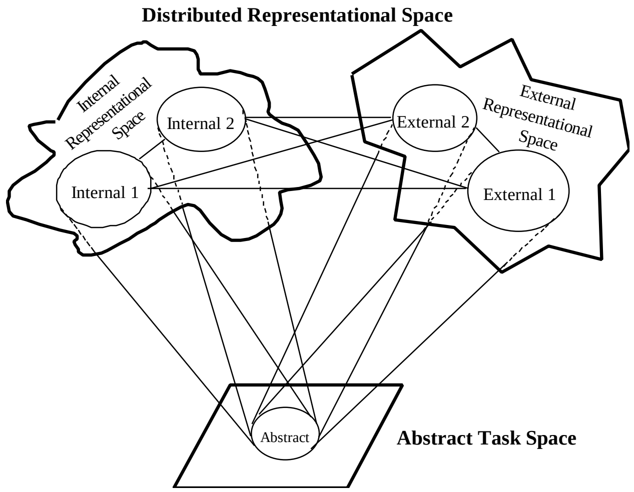


Figure 2. The theory of distributed representations developed by Zhang & Norman (1994). The internal representations form an internal representational space, and the external representations form an external representational space. The internal and external representational spaces together form a distributed representational space, which is the representation of the abstract task space.

Zhang & Norman (1994) developed a theory of distributed representations to account for the behavior in distributed cognitive tasks. The theory is sketched in Figure 2, which shows the representational system of a task with two internal and two external representations. Each internal representation resides in a person's mind, and each external representation resides in an external medium. The internal representations form an internal representational space, and the external representations form an external representational space. These two spaces together form a distributed representational space, which is the representation of the abstract task space that describes the abstract structures and properties of the task. In this theory, the representation of a distributed cognitive task is neither solely internal nor solely external, but distributed as a system of distributed representations with internal and external representations as two indispensable parts. The theory of distributed representations is consistent with the theory of situated cognition, which argues that people's activities in concrete situations are guided, constrained, and to some extent, determined by the physical and social context in which they are situated (e.g., Barwise & Perry, 1983; Clancey 1993; Greeno, 1989; Greeno & Moore 1993; Lave, 1988; Lewis, 1991; Suchman, 1987). In the views of distributed representations and situated cognition, it is not necessary to construct an internal model of the environment to mediate actions: people can directly access the situational information in their environment and act upon it in an adaptive manner.

Properties: External Representations

External representations are more than inputs and stimuli to the internal mind. They have many non-trivial properties. The most obvious one is that they can serve as memory aids: extend working memory, form permanent archives, allow memory to be shared, etc. However, the properties that truly make external representations crucial are not memory aids. For many tasks, external representations are intrinsic components, without which the tasks either cease to exist or completely change in nature. The following review describes examples of such non-memory-aid properties of external representations.

Diagrams, graphs, and pictures are a few typical types of external representations. They are used in many cognitive tasks such as problem solving, reasoning, and decision making. In the studies of the relationship between mental images and external pictures, Chambers & Reisberg (1985; Reisberg, 1987) showed that external pictures can give people access to knowledge and skills that are unavailable from internal representations. In the studies of diagrammatic problem solving, Larkin & Simon (1987; Larkin, 1989), for example, argue that diagrammatic representations support operators that can recognize features easily and make inferences directly. In the studies of logical reasoning with diagrams, Stenning & Oberlander (1995) argue that diagrammatic representations such as Euler circles limit abstraction and thereby aid processibility, that is, graphical representations can make some information interpretable and transparent in a specialized form at the expense of limiting abstraction in general forms. The representation, perception, and comprehension of graphs have been extensively studied since last century (for a few integrative studies, see Bertin, 1983; Cleveland, 1985; Schmid, 1983; Tufte, 1990). It is well known that different forms of graphic displays have different representational efficiencies for different tasks and can cause different cognitive behaviors. For example, Kleinmuntz & Schkade (1993) showed that different representations (graphs, tables, and lists) of the same information can dramatically change decision making strategies. Zhang (1996) suggested that all graphs could be systematically studied under a representational taxonomy based on the properties of external representations.

The studies on literacy also show the important functions of external representations. The classical view on writing, originally developed by Aristotle (1938) and restated in our own time by Bloomfield (1993) and Saussure (1959), is that writing merely transcribes or re-represents speech from one external representation in auditory form to another external representation in visual form. For some people, however, it is not a simple transcription because writing supports reflective thought (Norman, 1993) without which the logical, analytic, rational, and scientific modes of modern thought are impossible (e.g., Goody, 1977; Ong, 1982). For example, Goody argues that the shifts from the so-called prelogical to more and more rational mode of thought resulted from the shifts from orality to various stages of literacy, that is, writing systems are not only the products of the mind but also part of the determining features of the mind. Without writing, the human mind was so occupied by the participation in dynamic utterance of speech that it could not organize and elaborate logical relations in the analytic form of linear sequences. Rational mode of thought was possible only because certain procedures were made available by the technology of writing. Ong also argues that writing has reconstructed cognition: writing systems are not mere external aids but also internal transformations of cognition. In a recent paper, Olson (1996) has made a convincing argument that writing does not merely transcribe but rather brings structural properties of speech into consciousness, that is, the development of writing was also the discovery of the representable structures of speech. From an evolutionary perspective, Donald (1991) also illustrated the important roles of external representations in the emergence of the modern mind. According to Donald, the changes in cognitive architecture mediated by external representations were no less fundamental than those mediated by biological changes in the brain: the external symbolic system, especially writing, is the most important representational system responsible for much of the virtually unlimited cognitive capacity of the modern mind.

The above brief review clearly demonstrates that external representations are not simply inputs and stimuli to the internal mind, and they are much more than memory aids. For many tasks, external representations are so intrinsic to the tasks that they guide, constrain, and even determine the pattern of cognitive behavior and the way the mind functions.

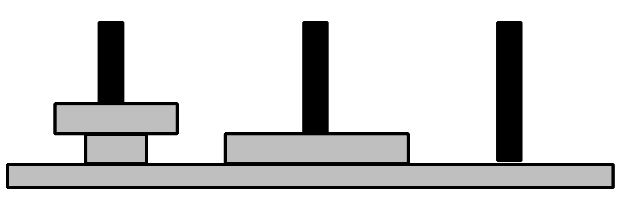
Phenomenon: The Representational Effect

The representational effect is a ubiquitous phenomenon that exists in nearly all cognitive tasks. It refers to the phenomenon that different representations of a common abstract structure can generate dramatically different representational efficiencies, task complexities, and behavioral outcomes (for a review, see Zhang & Norman, 1994). Different representations of a common abstract structure are usually called isomorphic representations. One simple example of the representational effect is the representation of numbers: Arabic numerals are more efficient than Roman numerals for multiplication (e.g., 73×27 is easier than $\text{LXXIII} \times \text{XXVII}$) even though both types of numerals represent the same entities—numbers. The most dramatic case is probably the Copernican revolution, where the change from the geocentric representation of the solar system (Ptolemaic system) to the heliocentric representation (Copernican system) laid the foundation of modern science and fundamentally changed people's conception of the universe. Between these two extremes, the representational effect is usually in the form of a good or bad display, an easy or difficult task, a direct or indirect engagement, a short or long sequence of actions, and so on.

The focus of this article is to show the representational effect in several task domains under the theoretical framework of distributed representations. In terms of distributed representations, isomorphic representations are those that have the same abstract task space but different distributions of information across internal and external representations (see Figure 2). The domains to be considered are problem solving, relational information displays, and numeric tasks.

Domain 1: Problem Solving

Zhang & Norman (1994) studied the effects of different distributions of information across internal and external representations on problem solving behavior in the Tower of Hanoi task (see Figure 3). Each of the three rules of the Tower of Hanoi can be implemented either in internal or external representations. Figure 4 shows three isomorphic representations of the Tower of Hanoi with different distributions of internal and external rules. In the Orange version, three plastic balls were used. Rules 1, 2, and 3 were all internal because they all had to be memorized. In the Donut version (the standard Tower of Hanoi), plastic rings were used. Rules 1 and 2 were internal and Rule 3 was external. Rule 3 was external because the physical constraints guaranteed that it was followed, i.e., a smaller donut could be moved out without the larger one on the top being moved first. In the Coffee version, all cups were filled with fresh coffee. Rule 1 was internal, and Rules 2 and 3 were external. A smaller cup could not be placed on the top of a larger cup (external Rule 2), as this would cause the coffee to spill. Rule 3 was external because a cup could not be moved if there was another cup on its top.



Rule 1: only one disk can be transferred at a time.

Rule 2: a disk can only be transferred to a pole on which it will be the largest.

Rule 3: only the largest disk on a pole can be transferred to another pole.

Figure 3. The Tower of Hanoi problem. The task is to move the three disks from one configuration to another, following the three rules.

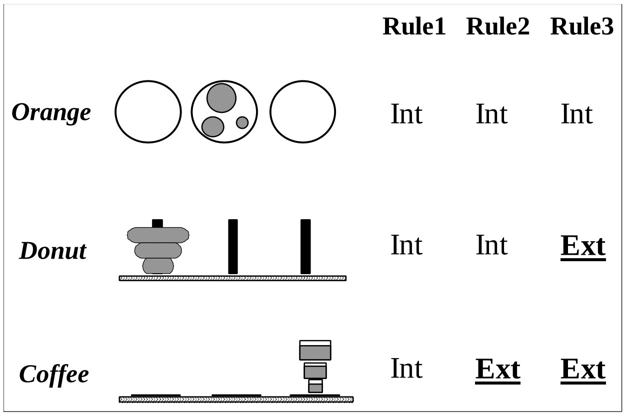


Figure 4. Three isomorphs of the Tower of Hanoi problem. See text for explanations.

Thus, in terms of the amount of information in external representations, the order is Coffee > Donut > Orange. An experiment was carried out for these three isomorphs and solution times, solution steps, and errors were measured. The experimental task was defined as a set of problems with an initial and an ending state that could be solved in seven steps. The experimental results showed that the more information in external representations, the easier the task. The solution times for Coffee, Donut, and Orange versions were 131.0, 83.0, 53.9 seconds, respectively, and the solution steps were 19.7, 14.0, and 11.4, respectively. The optimum number of solution steps was seven. The results also showed that the externalization of information could also reduce error rates. The error rates for Coffee, Donut, and Orange versions were 1.4, 0.61, and 0.22.

Based on these empirical results, Zhang & Norman (1994) made the following statements about external representations. First, external representations provide information that can be directly perceived and used without being interpreted and formulated explicitly. Second, they can anchor cognitive behavior. That is, the physical structures in external representations constrain the range of possible cognitive actions in the sense that some actions are allowed and others prohibited. Third, they change the nature of tasks: tasks with and without external representations are completely different tasks from a task performer' point of view, even if the abstract structures of the tasks are the same.

Domain 2: Relational Information Displays

There are a large number of tabular and graphic displays, such as line graphs, bar charts, pie charts, scatter plots, matrices, tables, networks, maps, and many others. Despite of the great variety, they are all relational information displays—displays that represent relations between dimensions. Zhang (1996) applied the theory of distributed representations (Zhang & Norman, 1994) to relational information displays and developed a taxonomy that can classify all types of tabular and graphic displays. The following two subsections describes the representation of dimensions, which are the basic structure of relational information displays, and a taxonomy of relational information displays.

The Representation of Dimensions

A dimension, as defined by Garner (1978), is a component property of a stimulus that has alternative, mutually exclusive levels. For example, hue, brightness, shape, length, and orientation are all examples of dimensions. Every dimension is on a certain type of scale, which is the abstract measurement property of the dimension. Stevens (1946) identified four major types of psychological scales: ratio, interval, ordinal, and nominal. Each type has one or more of the following properties: category, magnitude, equal interval, and absolute zero (Table 1). Category refers to the property that the instances on a scale can be distinguished from each another. Magnitude refers to the property that one instance on a scale can be judged greater than, less than, or equal to another instance on the same scale. Equal interval refers to the property that the magnitude of an instance represented by a unit on the scale is the same regardless of where on the scale the unit falls. An absolute zero is a value that indicates that nothing at all of the property being represented exists.

Nominal scales only have one formal property: category. Names of people are an example of nominal scales: they only discriminate different entities but have no information about magnitudes, intervals, and ratios. Ordinal scales have two formal properties: category and magnitude. The ranking of movie quality is an example of ordinal scales: a movie ranked “1” is better than a movie ranked “2” (magnitude) and the quality of a movie ranked “5” is different from that of a movie ranked “7” (category). However, the rankings themselves tell us nothing about the differences and ratios between the rankings. Interval scales have three formal properties: category, magnitude, and equal interval. Time is an example of interval scales: 02:00 is different from 22:00 (category), 14:00 is later than 09:00 (magnitude), and the difference between 15:00 and 14:00 is the same as between 09:00 and 08:00 (equal interval). However, time does not have an absolute zero. Thus, we cannot say that 10:00 is twice as late as 05:00. Ratio scales have all of the four formal properties: category, magnitude, equal interval, and absolute zero. Length is an example of ratio scales: 1 inch is different from 3 inches (category), 10 inches are longer than 5 inches (magnitude), the difference between 10 and 11 inches is the same as the difference between 100 and 101 inches (equal interval), and 0 inch means the nonexistence of length (absolute zero). For length, we can say that 10 inches are twice as long as 5 inches.

Table 1. Properties of Psychological Scales

Formal Properties	Scale Types			
	ratio	interval	ordinal	nominal
category	yes	yes	yes	yes
magnitude	yes	yes	yes	no
equal interval	yes	yes	no	no
absolute zero	yes	no	no	no
Example	length	time	ranking	names

Table 1 shows that the four types of scales have an order of representational power: ratio > interval > ordinal > nominal. A higher scale (e.g., ratio) possesses more information (more formal properties) than a lower scale (e.g., nominal). In general, the scale information of a dimension is distributed across internal and external representations as a distributed representation. More specifically, the scale information of a dimension is the set of formal properties of the dimension, which constitute the abstract representational space of the dimension. A distributed representation of the dimension means that some of its formal properties are represented internally and some externally.

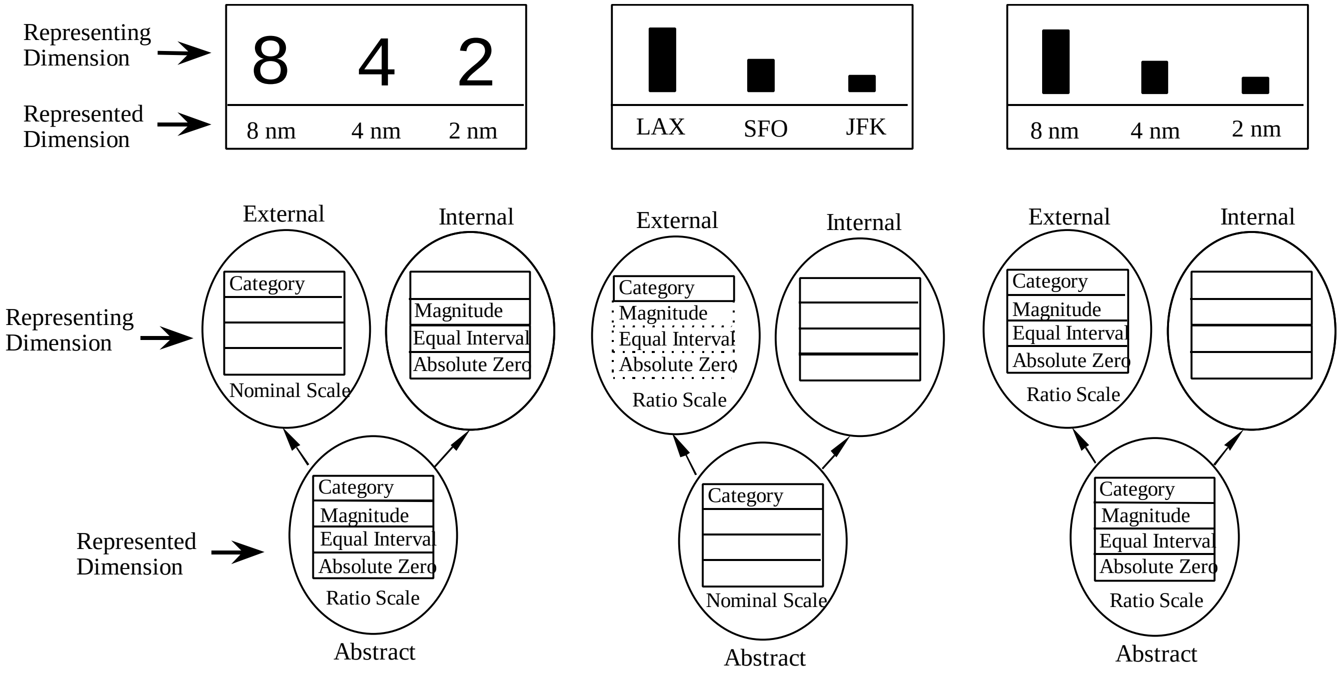
Figure 5 shows three variations of distributed representations of dimensions. In Figure 5A, a higher dimension (distance in nautical miles, ratio scale) is represented by a lower dimension (shape of digit, nominal scale). Because shape is on a nominal scale, it can only represent the category property of distance in the external representation. The other three properties of distance (magnitude, equal interval, and absolute zero) are represented internally because they are not embedded in the physical properties of the shapes of digits. In Figure 5B, a lower dimension (names of airports, nominal scale) is represented by a higher dimension (length of bar, ratio scale). In this case, all the scale information of the lower dimension is represented externally by the higher dimension because the formal properties of the lower dimension is a subset of those of the higher dimension. However, the extra information in the higher dimension may cause misperception on the lower dimension. In Figure 5B, what we really need to represent is the category property of names of airports, that is, LAX, SFO, and JFK are different airports. Because length is a ratio dimension, the extra information it has (magnitude, equal interval, absolute zero) may cause misperception on the represented dimension (names of airports). For example, we may get the misperception that LAX is twice as large as SFO. In Figure 5C, the scale type of the represented dimension (distance, ratio scale) matches the scale type of the representing dimension (length of bar, ratio scale). In this case, all the scale information of the represented dimension is represented externally by the representing dimension. This is a direct, efficient, and accurate representation.

A Taxonomy of Relational Information Displays

Relational information displays can be analyzed at three levels: dimensionality, scale types, and dimensional representations (Figure 6). At the level of dimensionality, different displays can have different numbers of dimensions, e.g., 2-D, 3-D, 4-D, etc. At the level of scale types, the dimensions of a display can have different scale types: ratio (R), interval (I), ordinal (O), and nominal (N) scales. For example, the two dimensions of a 2-D display can be both on ratio and ratio scales, one on ratio and one on nominal scale, and so on. At the level of dimensional representations, each scale type can be implemented by different physical dimensions. For example, ratio scale can be represented by length, distance, and angle; interval scale by position and orientation; ordinal scale by cell position; and nominal scale by shape, direction, texture, and position. With these physical dimensions, the scale combination R-R can be represented by length-length (Rectangle, Cross), length-angle (Coxcomb, Polar Plot), distance-distance (Line Graph, Cartesian Plot), and so on. The scale combination R-I can be represented by length-position (Histogram), length-orientation (Glyph, Polygon), distance-position, and so on. The scale combination R-N can be represented by length-position (segmented and vertical bar charts), length-direction, angle-direction (Pie Chart), and so on. The scale combinations O-O-N can be represented by CellPosition-CellPosition-shape (Table, Matrix), position-position-texture (Network), and so on.

The hierarchical structure in Figure 6 is a representational taxonomy that can classify all relational information displays, including most graphs, charts, tabular displays, maps, networks, etc. For example, among the displays in Figure 6, the pie chart and vertical bar chart are in the same category at the level of dimensional representations because they are different representations of the same scale types; the line graph and the pie chart are in the same category at the level of scale types because they have different scale types with the same dimensionality; and all the displays in Figure 6 are in the same category at the level of dimensionality because they are all relational information displays.

With this taxonomy, we can get an estimate of the similarity between any two relational information displays. The lower the level at which two displays are in the same category, the more similar they are. For example, the pie chart and the vertical bar chart are more similar to each other than the pie chart and the line graph, because the former two are in the same group at the level of dimensional representations whereas the latter two are at the level of scale types.



(A) Shape® Distance **(B) Length® Name** **(C) Length® Distance**

Figure 5. The distributed representation of scale information. (A) A nominal dimension (shape) represents a ratio dimension (distance in nautical miles, nm). The extra information of the ratio dimension either has to be represented in the internal representation or not represented at all. (B) A ratio dimension (length) represents a nominal dimension (names of airports). The extra information of the ratio dimension may cause misperception on the nominal dimension. (C) A ratio dimension (length) represents a ratio dimension (distance). This is an accurate and efficient representation.

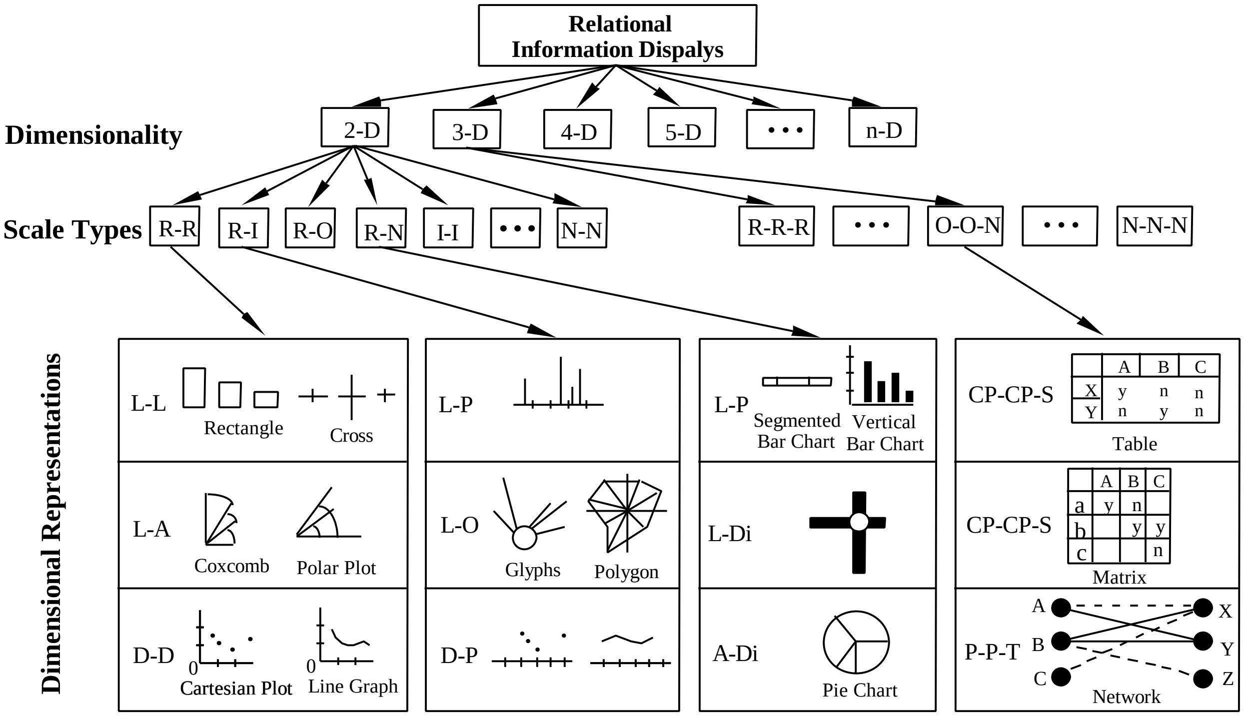


Figure 6. A taxonomy of relational information displays. A = Angle, CP = Cell Position, D = Distance, Di = Direction, L = Length, O = Orientation, P = Position, S = Shape, T = Texture. See text for details.

Domain 3: Numeric Tasks

Numbers can be represented in many different ways, such as Arabic and Roman numerals. Over the past two or three thousand years, hundreds of numeration systems have been invented across the world. Zhang & Norman (1995) applied the theory of distributed representations (Zhang & Norman, 1994) to study the representational systems of numbers. Similar to relational information displays, the basic structure of numeration systems is also dimensions. For example, Arabic system has two dimensions: base dimension represented by the shapes of the ten digits and power dimension represented by the positions of the digits. The next two subsections describe a cognitive taxonomy of numeration systems and an efficiency analysis of multiplication tasks for different numeration systems.

A Taxonomy of Numeration Systems

Numeration systems can be analyzed at four levels: dimensionality, dimensional representation, bases, and symbol representation. Each level has an abstract structure that can be implemented in different ways. The different representations at each level are isomorphic to each other in the sense that they all have the same abstract structure at that particular level (Figure 7).

At the level of dimensionality, different numeration systems can have different dimensionalities: 1D, 1×1D, (1×1)×1D, and others. However, they are all isomorphic to each other at this level in the sense that they all represent the same entities—numbers. This level mainly affects the efficiency of information encoding. 1D systems are linear, while 1×1 D and (1×1)×1 D systems are polynomial. Polynomial systems encode information more efficiently than linear systems: the number of symbols needed to encode a number in a polynomial system is proportional to the logarithm of the number of symbols needed to encode the same number in a linear system.

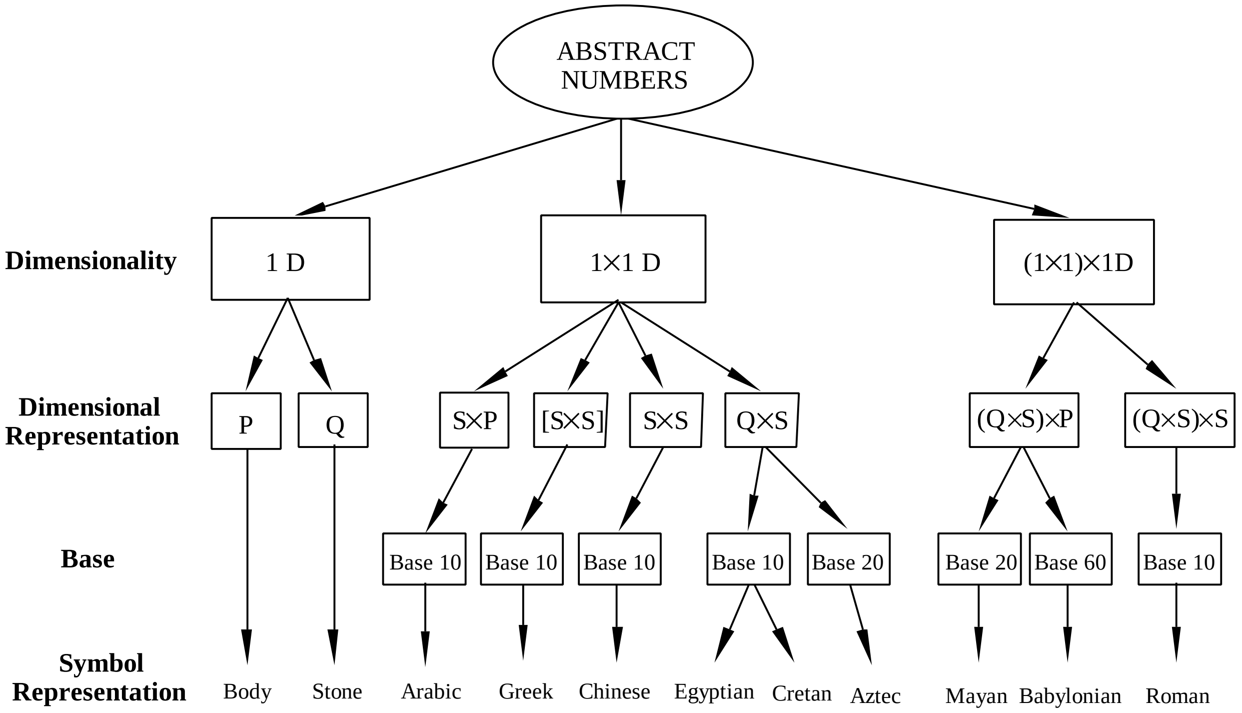


Figure 7. A cognitive taxonomy of numeration systems. At the level of dimensionality, different systems have different dimensionalities. At the level of dimensional representations, the dimensions of different systems are represented by different physical properties. P = Position, Q = Quantity, S = Shape. At the level of bases, different systems may have different bases. At the level of symbol representations, different systems use different symbols.

At the level of dimensional representations, isomorphic numeration systems have the same dimensionality but different dimensional representations. The physical properties used to represent the dimensions of numeration systems are usually quantity (Q), position (P), and shape (S). For example, the base and power dimensions of 1×1D systems can be represented by shape and position (S×P, Arabic system), shape and shape (S×S, Chinese system), quantity and shape (Q×S, Egyptian system), etc. This level is crucial for the representational effect of numeration systems.

At the level of bases, isomorphic numeration systems have the same dimensionality, same dimensional representations, but different bases. For example, both the Egyptian and the Aztec systems are 1×1D systems, and the base and power dimensions of both systems are represented by quantity and shape. However, the base of the Egyptian system is ten while that of the Aztec system is twenty. This level is important for tasks involving addition and manipulation tables: the larger a base is, the larger the addition and multiplication tables are and the harder they can be memorized and retrieved.

At the level of symbol representations, isomorphic numeration systems have the same abstract structures at the previous three levels. However, different symbols are used. For example, both the Egyptian and the Cretan systems are 1×1D systems, the two dimensions of both systems are represented by quantity and shape, and both systems have the base ten. However, in the Egyptian system, the symbols for 10^0, 10^1, and 10^2 are |, ↔, and 𓍢, whereas in the Cretan system, the corresponding symbols are ʻ, ◑, and ∕. This level mainly affects the reading and writing of individual symbols.

The hierarchical structure of numeration systems shown in Figure 7 is a cognitive taxonomy of numeration systems. For example, the Egyptian and Cretan systems are in the same group at the level of symbol representations; the Mayan and Babylonian systems are in the same group at the level of bases; the Arabic, Greek, Chinese, Egyptian, Cretan, and Aztec systems are in the same group at the level of dimensional representations; and all the systems in Figure 7 are in the same group at the level of dimensionality. Under this taxonomy, the lower the level at which two systems are in the same group, the more similar they are. For example, the Egyptian and the Cretan systems are more similar to each other than the Arabic and the Babylonian systems, because the former two are in the same group at the level of symbol representations whereas the latter two at the level of dimensionality. This taxonomy can classify nearly all numeration systems that have been invented across the world. In addition to numeration systems of written numerals, this taxonomy can also classify numeration systems of object numerals. The following are a few examples (see Ifrah, 1987). The Peruvian knotted string system is a P×Q (base 10) system; the Chimpu (knotted strings used by the Indians of Peru and Bolivia) is a Q×Q (base 10) system; the knotted string system used by the German millers is a S×S (base 10) system; the Roman counting board, the Chinese abacus, and the Japanese Soroban are (Q×P) ×P (main base 10 and sub-base 5) systems, and the Russian abacus is a Q×P (base 10) system.

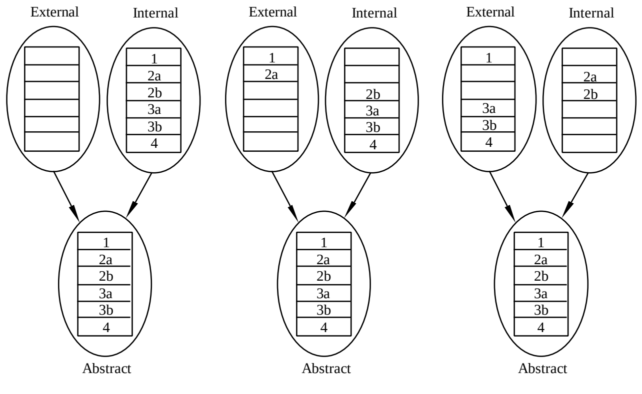
Internal and External Representations in Multiplication

For 1×1D system, a numeral is represented as a polynomial: ? $a_i x^i$. Multiplication by the polynomial method is performed by multiplying every term of the multiplicand with every term of the multiplier and then adding the partial products together, regardless of which particular algorithm is used. Thus, the two basic components of polynomial multiplication are the multiplication of individual terms and the addition of partial products.

For all 1×1D systems, term multiplication ($a_i x^i \times b_j y^j$) has the same set of six basic steps:

1. Separate power and base dimensions
2a. Get base values of $a_i x^i$ & $b_j x^j$
2b. Multiply base values
3a. Get power values of $a_i x^i$ & $b_j x^j$
3b. Add power values
4. Attach power values

The information needed to carry out each of the six steps can be either in internal or external representations. Figure 8 shows how the information for the six steps is distributed across internal and external representations for Arabic, Greek, Egyptian systems. The detailed analyses can be found in Zhang & Norman (1995). If we assume that with all other conditions identical, the more information needs to be retrieved from internal representations, the harder the task (e.g., due to working memory load), then for term multiplication, the Greek system (six internal steps) is harder than the Egyptian system (four internal steps), which in turn is harder than the Arabic system (two internal steps).



(A) Greek System (B) Egyptian System (C) Arabic System

Figure 8. The distributed representation of the information needed for the six basic steps of term multiplication under Greek, Egyptian, and Arabic systems.

Conclusion

This article reviewed and summarized recent studies on external representations in complex information processing tasks. In comparison to internal representations, systematic studies of external representations did not start until recently. External representations were previously considered as mere inputs and stimuli to the internal mind. The current review shows that external representations are neither mere inputs and stimuli to nor mere memory aids to the internal mind. They are intrinsic components of many cognitive tasks; they guide, constrain, and even determine cognitive behavior. For complex tasks requiring interactions with the environment, the complexity of the environment and the limitations of the mind suggest that cognitive behavior is much like constraint satisfaction through the execution of the operations directly activated by external and internal representations and the processing of the information directly available from external and internal representations. External representations have different properties from those of internal ones. They need to be studied on their own right, not as something peripheral to internal representations.

In the descriptions of detailed studies about external representations, this article focused on a specific line of research: the theory of distributed representations and its applications in three task domains. The basic idea of the theory of distributed representations is that a distributed cognitive task can be considered as a distributed representational system with internal and external representations as two indispensable parts. Given the same information or structure, the more information is distributed in external representations, the easier and less error-prone the task. The enhancement of performance by external representations is due to many factors, including efficient perceptual processing, reduction of working-memory load, more efficient processing routines, visibility of actions and feedbacks, structuring and anchoring of actions, different knowledge and skill bases, and so on. With the explosive growth of computer-based information systems (e.g., digital library, electronic medical records, etc.), we are interacting more and more with computer-generated information displays. To make these displays effectively and accurately generate the information that people need for specific tasks in specific places at specific times, we need a good design of these displays. Systematic studies of external representations will provide more theory-based design principles to the field of human-computer interaction and interface design and will improve quality of complex information processing tasks.

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