



Detecting saccades with jerk

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Abstract

In studies of smooth eye movements, saccades are often detected and removed from eye movement records during analysis. A simple and effective method for saccade detection is described; the method uses jerk (the third derivative of eye position with respect to time). © 1998 Elsevier Science Ltd. All rights reserved.

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In studies of smooth eye movements (pursuit, vestibulo-ocular reflex (VOR), etc) it is common practice to detect and remove saccades from records of eye velocity, often replacing the saccades with a straight-line segment [1]. This permits groups of velocity records, obtained under the same experimental conditions, to be averaged. The critical steps in saccade removal are: (i) detection of saccadic onset; (ii) determination of saccade termination; and (iii) replacement of the data segment so determined. Various techniques employing eye position, velocity and/or acceleration have been used for saccade detection; for example: ‘a position reset criterion’ [2]; simple velocity thresholds [3,4]; acceleration threshold [5]; velocity and acceleration thresholds [6]; ‘deviations [of velocity] from the current smooth velocity’—essentially acceleration [7]; maximum angle between linear segments of position record [8]—amounting to change in slope, or acceleration, [9]; and a Kalman filter of the velocity [10]. This laboratory has for some time employed a simple detection scheme based on ‘jerk’—the third derivative of eye position with respect to time.

Figs. 1 and 2 show sample eye movement records for two subjects. Eye position signals were obtained from an infrared scleral reflection system (for details, see [11]). The eye position signal was filtered (analog passive low-pass filter, single-pole, $f_c = 80$ Hz), and sampled at 167 s^{-1} . For analysis, the position data were digitally smoothed with a cutoff at 50 Hz and differentiated to

give eye velocity ($d\theta/dt$), acceleration ($d^2\theta/dt^2$), and jerk ($d^3\theta/dt^3$). These operations were performed using ASYST (Keithley/Metrabyte); the smoothing filter in ASYST is derived from an inverse Fourier transform of a Blackman window; for a cutoff at 50 Hz (50% at approximately 20 Hz) the weights for 6 ms bins are: [0.0037, 0.0399, 0.1178, 0.2103, 0.2527, 0.2103, 0.1178, 0.0399, 0.0037]. The differentiation utility in ASYST interpolates polynomials through the data and differentiates the polynomials. The polynomials used were one order higher than the degree of the derivative, and were fit over the minimum number of points centered on the point in question; e.g. for a 3rd derivative, a 4th order polynomial was used, which was fit through the five points centered on the point being evaluated. (An n th order polynomial used n or $n + 1$ points to provide an odd number of points.)

The subject of Fig. 1 was a nystagmat looking straight ahead at a display of sequentially presented single words (‘RSVP text’, e.g. [12]). The subject of Fig. 2 (the author) was making voluntary head movements about the vertical axis (roughly sinusoidal and approximately 10 deg peak-to-peak at approximately 3 Hz) while fixating one of three targets at 35 cm. The three targets were approx 1 deg apart, and the subject shifted fixation voluntarily between them during the head movement, in order to create a record of small saccades superimposed on substantial smooth eye movements. Calibrations were obtained by fixation of calibration targets. (For the nystagmat, an estimate of calibration was used.)

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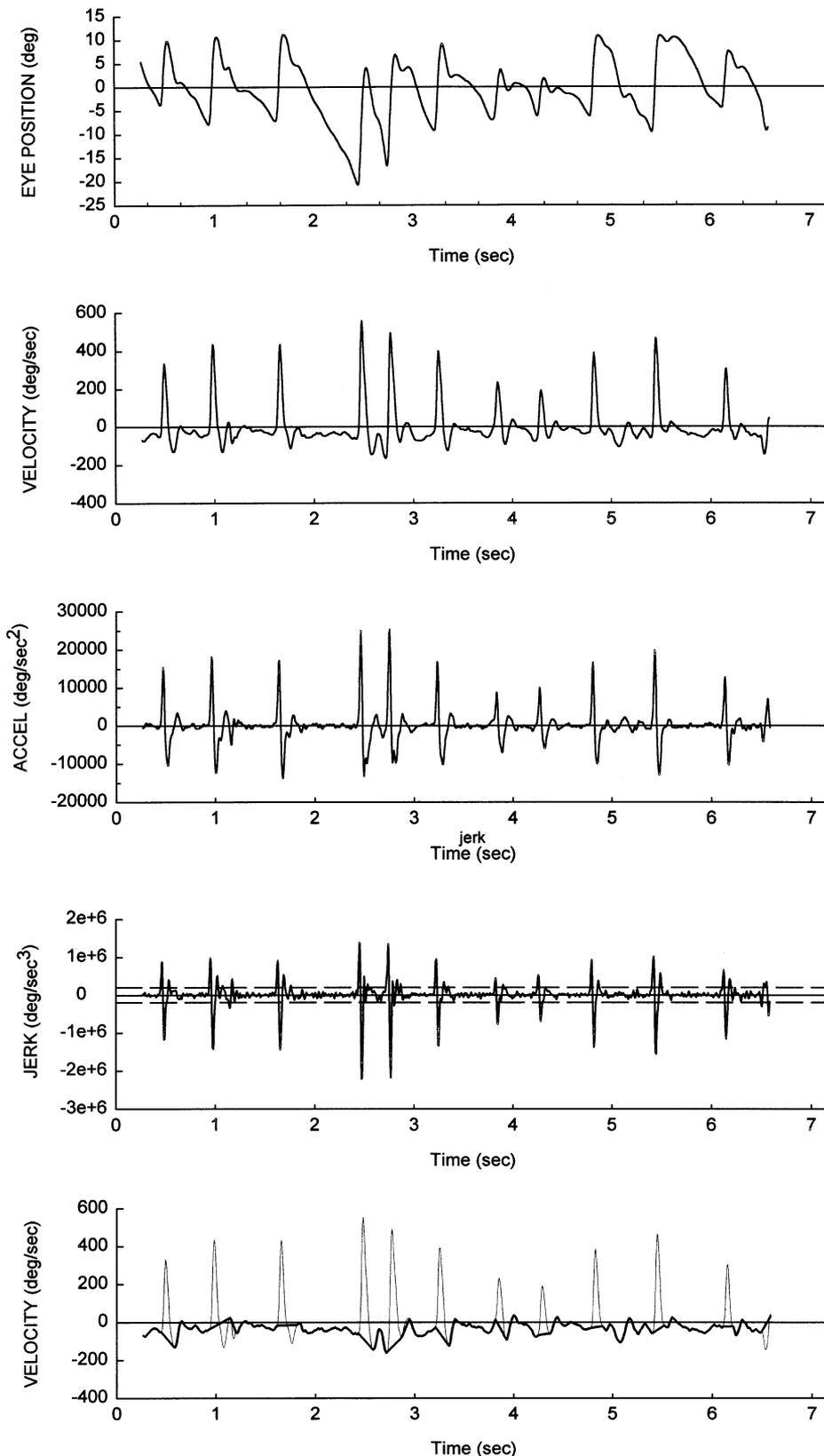


Fig. 1. Eye movement records of a subject who was a nystagmat. Subject was looking straight ahead at serially presented single words of text ('RSVP text'). From top to bottom, records are: eye position, velocity, acceleration, jerk, and the effect of saccade removal on the velocity record. The threshold values of jerk used for detecting the onset of saccades are shown as dashed lines.

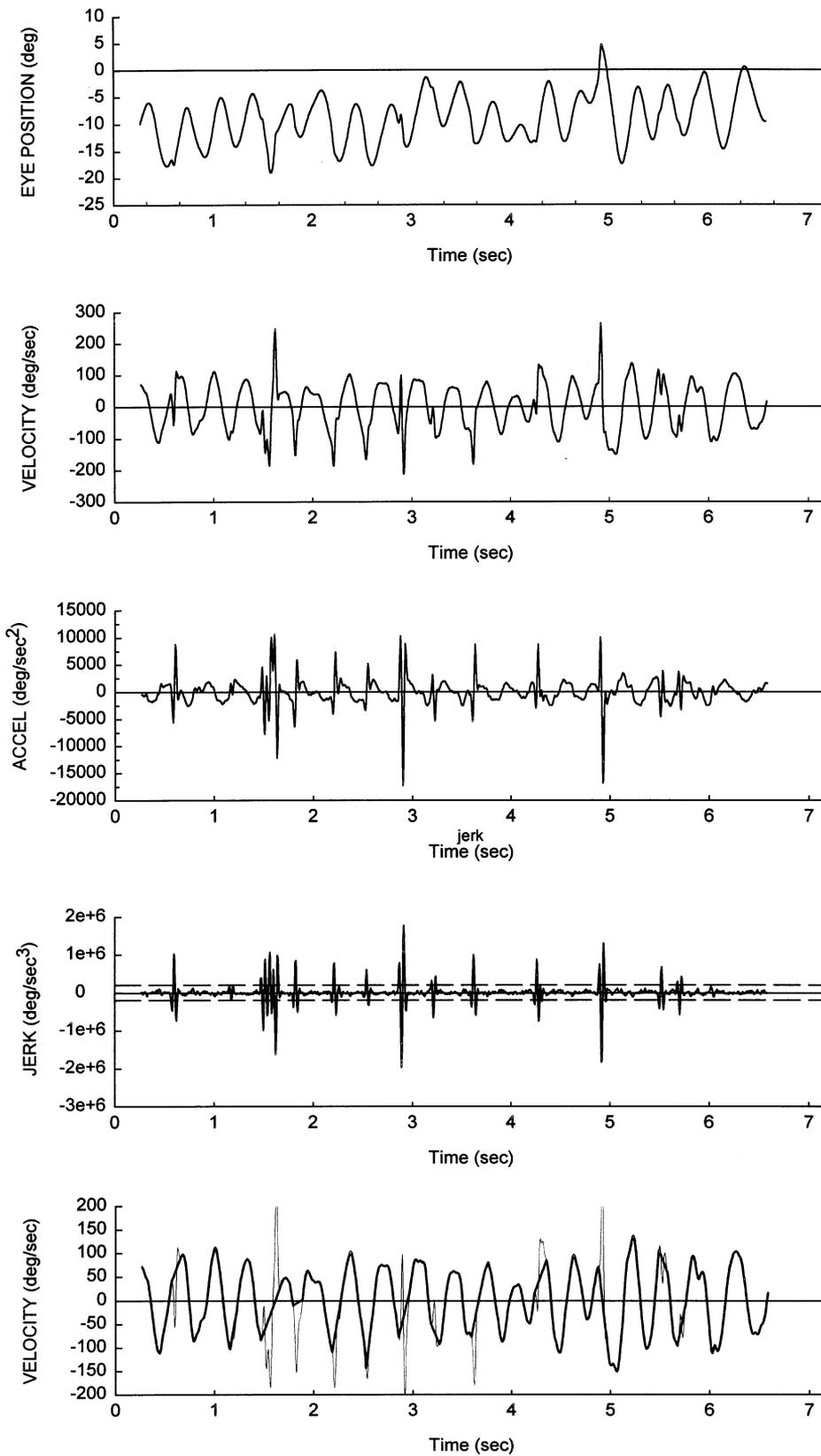


Fig. 2. Eye movement records of the author, who was making voluntarily head rotations about the vertical axis, and simultaneously shifting fixation between three targets separated by 1 deg. Conventions as in Fig. 1.

Each figure shows, from top to bottom: eye position, velocity, acceleration, jerk, and the effect of saccade removal on the velocity record. (In the last, the thin line shows the original velocity record and the heavy line shows the record following saccade removal.) While the onset of a saccade is an abrupt event (see below), the termination of a saccade is often less clear-cut, since small, slower movements such as ‘glissades’ may occur at termination ([13]). Thus, jerk is particularly useful for determination of onset; the same simple jerk threshold ($2 \times 10^5 \text{ deg s}^{-3}$) was used for onset detection in both figures. Following onset and prior to termination of a given saccade, eye acceleration was required to change sign at least once. Given its less abrupt nature, identifying saccade termination is less exact than is the case for onset, and various criteria work reasonably well. (It is worth noting that the primary task in saccade detection, particularly for saccade removal, is the detection *per se*—the finding that a saccade occurs.) As examples of termination criteria, in Fig. 1, the criterion for termination used an acceleration window—acceleration had to fall inside the window ($\pm 1200 \text{ deg s}^{-2}$) for two consecutive 6 ms samples. In Fig. 2, the criterion for termination used a jerk window—jerk had to fall inside the window ($\pm 10^5 \text{ deg s}^{-3}$) for four consecutive samples. Both of these techniques for determining saccade termination worked satisfactorily for the data of Fig. 1; the jerk technique was superior for the data of Fig. 2. For a given subject, some tuning of the criteria for onset and termination are sometimes necessary; however, jerk has been found easy to work with. Comparison of the acceleration and jerk records of Figs. 1 and 2 shows that, although the saccades of Fig. 1 could be handled with an algorithm based on either acceleration or jerk, the data of Fig. 2 are more demanding, and an algorithm based on acceleration could easily miss some of the smaller saccades detected with jerk. The demands made of the detection algorithm obviously depend on the nature of the eye movements; a saccade following a period of fixating a stationary target can be located by any reasonable algorithm.

If one looks at a record of eye velocity such as that of Fig. 2, the nature of a jerk criterion for detecting saccade onset can be appreciated at an intuitive level: small saccades stand out not because of their velocity, nor necessarily because of the slope of their velocity (their acceleration); it is the abrupt change of slope (jerk), appearing as a ‘zig’ in the trace, that marks saccades. (In fact, the name ‘saccade’ comes from the French for a jerk in the movement sense¹). An approx-

imate treatment of the dynamics of small saccades in comparison to representative (vigorous) smooth eye movements is given in Appendix A; this analysis suggests that saccade discriminability improves substantially in going from velocity detection to acceleration detection to jerk detection.

A question worth considering is whether still higher derivatives might provide still greater benefits. In fact, the 4th derivative does work well, and successively higher derivatives do have successively higher ratios of values for saccades compared to smooth movements (see Appendix A). Some drawbacks to climbing further up this ladder are: (a) The contribution of 60 Hz noise will increase by 120π times with each differentiation (about six times faster than the saccadic derivatives). This provides a constraint which depends on the amount of noise present and the amount of filtering which can be performed without affecting frequencies of interest. (b) Each differentiation increases the number of peaks in the signal generated by a single saccade (see the figures). (c) Each differentiation employs additional samples surrounding the sample at the time for which the derivative is evaluated. Taking (b) and (c) together means that the precision with which saccade onset is determined will be progressively reduced for higher derivatives, which could be a problem for some applications.

The particular techniques used here for smoothing and differentiation are not presented as ideal; the choices were made to provide specific examples. The digital smoothing was used largely to reduce 60 Hz noise contributions. Since fine details of saccades involve components with frequencies up to approx 100 Hz [17,18], some saccadic details were buried in the noise and/or lost in the smoothing; in particular, the transient peaks of velocity, acceleration and jerk were reduced. However, the focus here was on saccade removal for the study of smooth movements. If eye position were measured with a device such as a search coil or Purkinje image eye-tracker, and the signal sampled at $500\text{--}1000 \text{ s}^{-1}$ and not smoothed below 100 Hz, these details could be preserved, and jerk could then provide finer discrimination of saccade onset. Even with the limited bandwidth employed here to study smooth eye movements (which typically have power spectra extending no higher than about 10 Hz ([17]), it may be seen that the technique is quite effective and works within a conceptually simple framework.

After preparing this paper, it has been learned that there has apparently been previous publication of the concept of jerk-based saccade detection ([19], referenced

¹ *Dictionnaire de L'Académie Française* (5th ed., 1798) gives several definitions, the most applicable referring to a sharp (movement) given to the reins of a horse, which would usually be translated as a jerk on the reins. (This dictionary is available at a website maintained by the

ARTFL project at the University of Chicago.) Westheimer (1973) [14] noted that Landolt (1891) [15] used the term in connection with saccadic eye movements. Harrap's New Standard French and English Dictionary, Mansion [16] defines a saccade as a jerk, start, shake, jolt, or violent pull.

in [10]). The author has not so far been able to obtain a copy of this publication, and the concept appears to merit further presentation.

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Appendix A. Approximate derivation of jerk values for small saccades

The oculomotor plant can be approximately represented (neglecting a pure-delay element) by a second-order transfer function with time constants 0.200 and 0.016 s ([20]) or 0.150 and 0.012 s ([21]). Developing the equations along similar lines to Van Opstal et al. (1985) [21], the response of this overdamped system to a step of innervation is

$$\Theta(t) = \left(\frac{P}{k}\right) \left[1 - \left(\frac{T_1}{T_1 - T_2}\right) e^{-t/T_1} + \left(\frac{T_2}{T_1 - T_2}\right) e^{-t/T_2} \right] \quad (1)$$

where P (spikes s^{-1}) is the firing rate of the step of innervation, and k (spikes $s^{-1} \text{ deg}^{-1}$) relates eye position to firing rate. Differentiating Eq. (1) gives explicit equations for velocity, acceleration, and jerk.:

$$\begin{aligned} \frac{d\Theta}{dt} &= \frac{P}{k} \left(\frac{1}{T_1 - T_2} \right) (e^{-t/T_1} - e^{-t/T_2}) \\ \frac{d^2\Theta}{dt^2} &= \frac{P}{k} \left(\frac{1}{T_1 - T_2} \right) \\ &\quad \times \left(\frac{1}{(-T_1)^{n-1}} e^{-t/T_1} - \frac{1}{(-T_2)^{n-1}} e^{-t/T_2} \right) \end{aligned} \quad (2)$$

For $n > 1$, these derivatives have zeros:

$$\begin{aligned} \left. \frac{d^n\Theta}{dt^n} \right|_{\max} &= 0 \quad \text{at} \\ t = (n-1)t_{\text{peak}} &= (n-1) \left[\frac{\ln(T_1/T_2)}{1/T_2 - 1/T_1} \right] \end{aligned} \quad (3)$$

The innervation for a real saccade has a form often called a ‘pulse-step’—a brief high-frequency burst of innervation (the ‘pulse’) followed by a steady firing rate (the ‘step’) which holds the eye at the new position [22]. In what follows, an estimate of pulse height as a function of saccade size is obtained and then used in Eq. (2) for acceleration and jerk ($n = 2, 3$). The estimate of pulse height is obtained by evaluating Eq. (2) for velocity at the time of maximum velocity. (Since the focus is on saccade onset, only the pulse is considered;

references in what follows to ‘step responses’ are made in the mathematical sense of system response to a step-function input which starts abruptly and then remains constant.)

For a pulse input of height (P/k) , the response *during* the pulse is given by Eq. (1); i.e. the response is the system ‘step response,’ truncated at the end of the pulse. As the pulse becomes progressively shorter, the *overall* response changes from the step response to a function with the same time course as the impulse response (but smaller amplitude). (The impulse response is position as a function of time in response to an input $\delta(t)$. For linear systems, the system impulse response is the same as the velocity of the system step response, since the derivative of the response is the same as the response to the derivative of the input, and $\delta(t)$ is the derivative of a unit step function.) The impulse response has maximum position at t_{peak} given by Eq. (3); for $T_1, T_2 = 0.200, 0.016$ s, respectively and $t_{\text{peak}} = 0.044$ s. (This is the same time at which the system step response has maximum velocity.) The velocity of the impulse response (which has the same time course as the acceleration of the system step response) has a local extremum at $2t_{\text{peak}}$, but has its maximum value at $t = 0$. Short saccades often have innervation pulses of intermediate length—too long to be considered impulses, but shorter than t_{peak} . For such pulses, maximum velocity occurs at the end of the pulse, i.e. at the time when the step response is truncated. (This is the same approach used by Van Opstal et al. (1985) [21].)

Although the height of the innervation pulse is constant for large saccades, for small saccades, it depends on saccade size ([23,24]). Various approaches to evaluating P are possible; for the present purposes, an estimate is derived as follows by using empirical estimates of peak velocity and pulse length, and then obtaining (P/k) from Eq. (2) for velocity. Maximum saccade velocity for small saccades is given empirically by:

$$v_{\text{max}} \approx C\Theta_0 \quad (4)$$

where C is approx $75\text{--}80 \text{ s}^{-1}$ and Θ_0 is the saccade amplitude in deg ([25]; data collected in [26]). Saccade duration is approximately given by:

$$D = \frac{21 + 2.2\Theta_0}{1000} \quad (5)$$

where D is in s and Θ_0 is in deg ([27]). Duration of the innervation pulse is taken as half of saccade duration for the small saccades under consideration².

² For large saccades, pulse duration approximately equals saccade duration [28], but for small saccades the relationship is a little more complicated. For a simple, positive pulse, the response time course cannot be briefer than the impulse response; however, Van Opstal et al. (1985) [21] found that some small saccades *were* briefer, which

Eq. 2 for velocity is now assumed to give maximum velocity at the end of the pulse; velocity and time as functions of saccade amplitude are obtained from Eqs. (4) and (5) and substituted. Rearranging, (P/k) is found as a function of Θ_0 :

$$\frac{P(\Theta_0)}{k} = C\Theta_0 \left[\left(\frac{1}{T_1 - T_2} \right) \left(\exp - \left(\frac{21 + 2.2\Theta_0}{2000T_1} \right) - \exp - \left(\frac{21 + 2.2\Theta_0}{2000T_2} \right) \right) \right]^{-1} \quad (6)$$

(The '2000' replaces the '1000' of Eq. (5) because pulse duration was taken as half of saccade duration, as described above.) This $P(\Theta_0)$ is nearly linear for small angles. Using 0.200 and 0.016 for T_1 and T_2 , respectively, in the case of small Θ_0 Eq. (6) reduces approximately to:

$$\begin{aligned} \frac{P(\Theta_0)}{k} &\approx 0.428C(\Theta_0 - 0.0708\Theta_0^2) \\ &= 0.428C\Theta_0(1 - 0.0708\Theta_0) \end{aligned} \quad (7)$$

Substituting Eq. (7) into Eq. (2) for acceleration and jerk at time zero, using $C = 75 \text{ s}^{-1}$, and retaining only the term in Θ_0 :

$$\frac{d^2\Theta}{dt^2_{\max}} \approx 1.00 \times 10^4 \Theta_0 \quad \frac{d^3\Theta}{dt^3_{\max}} \approx 6.77 \times 10^5 \Theta_0 \quad (8)$$

For purposes of comparison, consider sinusoidal motion with frequency 2 Hz and amplitude Θ_{\sin} ; the maximum velocity, acceleration, and jerk for this motion are given by:

$$\frac{d^n\Theta}{dt^n_{\max}} = (4\pi)^n \Theta_{\sin} \quad (9)$$

As discriminability measures, the ratios of maximum (saccade to sine) velocity, acceleration, and jerk are obtained from Eqs. (4), (8) and (9):

$$\begin{aligned} \text{vel. ratio} &\approx 5.97 \left(\frac{\Theta_0}{\Theta_{\sin}} \right), \quad \text{acc. ratio} \approx 63.3 \left(\frac{\Theta_0}{\Theta_{\sin}} \right), \\ \text{jrk. ratio} &\approx 341 \left(\frac{\Theta_0}{\Theta_{\sin}} \right) \end{aligned} \quad (10)$$

The dominant term in taking higher derivatives is the T_2^{-n} term in Eq. (2), giving a 60 times increase with each differentiation compared to the increase of 4π for the sinusoid.

In regard to possible effects of filtering on detectability, the initial behavior of acceleration and jerk may be treated approximately by assuming that the innervation pulse is long enough so that the simple double exponential functions obtained in Eq. (2) describe early acceleration and jerk. In that case, it is easy to show that acceleration has its first zero at

t_{peak} of Eq. (3), and jerk has its first zero at $2t_{\text{peak}}$. Therefore, since bandwidth of a pulse is approximately inversely related to the pulse duration, the bandwidth of the first peak of acceleration considered in isolation would be at least as wide as that of the first peak of jerk. (So the effect of filtering on these initial peaks should be no greater for jerk than for acceleration.)

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