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Miscellanea

Rayleigh Test for Randomness of Circular Data

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Critical values of the Rayleigh test for testing whether the population of circular data from which a sample is drawn differs from randomness are sometimes presented in an inconvenient form (Batschelet, 1981) which makes it necessary to interpolate to obtain a value at a chosen probability level. Even when the values are presented at fixed probability levels (Mardia, 1972) it can be useful for computer programming and other purposes to have a simple expression that describes the tables.

According to Mardia (1972) a good approximation, based on the work of Pearson (1906) and Greenwood and Durand (1955) for $P = P(n\bar{R}^2 \geqslant K)$, where $\bar{R}$ is the mean resultant vector of n unit vectors, is given by

$$P = e^{-K}\,[1 + (2K - K^2)/4n - (24K - 132K^2 + 76K^3 - 9K^4)/288n^2] \qquad (1)$$

A study of the tabulated critical values of $\bar{R}$ reveals that K is quite well represented by a linear function of $\frac{1}{n}$ over a wide range of n for each probability level P (Fig. 1). Substituting $K = A + B/n$ in (1) yields

$$\ln P = -A + (-4B + 2A - A^2)/4n + O(n^{-2}),$$

so we choose

$$A = -\ln P \text{ and } B = (2A - A^2)/4.$$

Thus approximate critical values of $n\bar{R}^2$ are given by

$$K = -\ln P - \frac{2\ln P + (\ln P)^2}{4n} \qquad (2)$$

The intercepts and slopes are given in Fig. 1 for 5 probability levels.

It can be seen that there is very good agreement, except at low P and n, between the equations and the values calculated from Mardia's table. The tabulated values for $n = 100$ are given to two significant figures only and are clearly in error.

The fit for $P = 0.001$ can be improved by adding a quadratic term, determined empirically—such a curve is shown in Fig. 1.

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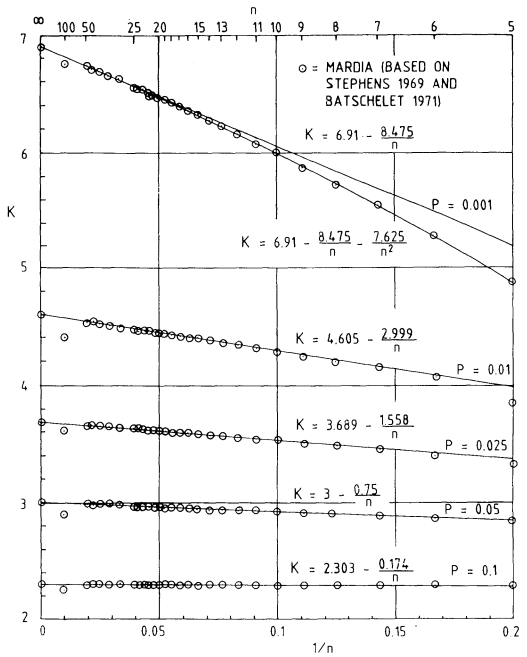

Fig. 1. Rayleigh test for circular data – Critical values K of $n\bar{R}^2$ as a function of $1/n$

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