GENERATION OF RANDOM SEQUENCES BY HUMAN SUBJECTS: A CRITICAL SURVEY OF LITERATURE

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The subjective concept of randomness is used in many areas of psychological research to explain a variety of experimental results. One method to study randomness is to have subjects generate random series. Unfortunately, few results of the experiments that used this method lend themselves to comparison and synthesis because the investigators employed such a variety of experimental conditions and definitions of mathematical randomness. Some suggestions for future research are made.

In many different fields of psychological research, the concepts of "subjective chance" and "subjective randomness" have been used almost exclusively to account for unexpected results. Characteristic of subjective chance is that it is not equal to mathematical chance; subjects seem to expect dependencies between successive events in spite of the fact that they know that the events occur independently of each other. Early in this century, psychophysics became interested in this phenomenon or the fact that successive responses of a subject are mutually dependent. In the psychophysical setting, the usual procedure is that a binary choice is made. Particularly experienced subjects are well aware that they are supposed to choose the alternatives in a random order. Even so, the subjective chance phenomenon persists. Hence, one possible explanation of interdependency of responses is that the subject has his own idea of what a random sequence looks like.

More recent research on subjective probability, probability learning, and gambling behavior also revealed that successive responses of a subject were mutually dependent in experimental settings where independent responses were expected. Once again the subjective concept of randomness was mentioned as an explanation.

In experiments on telepathy the concept was used to account for too many correct predictions of serial events. Clinical psychologists have used the subjective concept of chance for the diagnosis of neurotics. Finally, randomization tasks were employed as a secondary task in mental load measurements. Tune (1964b) presented a review on the interdependency of successive responses in various fields of psychological research.

In spite of the wide use of concepts like subjective chance or randomness, the question of whether such a thing really exists has never been settled. There is even less unanimity with respect to the nature and degree of dissimilarity between objective and subjective randomness. This lack of information, and the fundamental interest in how people form expectations in situations where chance is involved, induced a fair amount of research during the past 15 years. A score of experimental methods was designed to discover what subjects expect to happen by chance.

Reichenbach (1949) was the first to claim that humans are unable to produce a random series of responses, even when instructed and duly motivated to do so. Subsequent publications generally supported Reichenbach's proposition, but, with respect to the details, much confusion was introduced. Four randomization experiments and other relevant publications were reviewed by Tune (1964a). As the number of publications dealing with the randomization experiment has increased to at least 15, a new investigation of the status of the art seems justified. The present survey is confined to experiments in which subjects were instructed to produce a random series of events.

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Definition of the Randomization Experiment

The 15 experiments discussed in the present review are characterized by several requirements:

1. The subject is *explicitly* instructed to produce a random series of events. Often the instruction refers to random processes like coin tossing or throwing dice.

2. The series are long enough to prevent complete memorization.

3. No stimulus or feedback is given to the subject during the experiment, except for an eventual pacing signal.

4. The subjects are normal adults.

Comparison of Experimental Procedures and Conditions

Experimental evidence on randomization is highly contradictory. One reason may be the striking divergence of experimental procedures used by the various experimenters. Some relevant factors, contributing to the disagreement among experimental results, are presented in Table 1.

The number of alternative choices ranged from 2 to 26. It is likely that this difference in range is at least one of the reasons for the different experimental findings since Baddeley (1966 [1962]²), Rath (1966), and Warren and Morin (1965) found that nonrandomness increases with the number of alternatives.

Some authors (Baddeley, 1966; Chapanis, 1953; Lincoln & Alexander, 1955; Rath, 1966; Teraoka, 1963) reported that part of nonrandomness was caused by a tendency to arrange the alternatives in a natural ordering. Other experimenters (Mittenecker, 1958; Teraoka, 1963; Zwaan, 1964) used alternatives that had no natural ordering. Hence, the nature of the alternatives can be considered another factor responsible for the disagreement among experimental results.

The number of generated elements per series varied from 20 to 2,520, while some experimenters used several series in one experimental condition. Since boredom may be a

² A. D. Baddeley. Some factors influencing the generation of random letter sequences. (Tech. Rep. No. 422/62) Cambridge, England: Applied Psychology Research Unit, 1962. factor that increases nonrandomness (Weiss, 1964), it is likely that also sequence length influenced the results to some extent.

The experimental situation seldom included a visually displayed choice set. When the choice set was only defined by instruction, subjects first had to activate their internal representation of the set and, next, make a random selection. In case of small choice sets, the difference between an internally or externally represented choice set may be negligible, but it is at least doubtful whether in Baddelev's experiment the 26 letters of the alphabet were equally available to the subjects during the whole session. It is plausible that subjects used one small subset at a time, that they tried to make random selections only within the subset, and changed subsets occasionally. In that case, the series should have contained many digrams with elements in their natural ordering as, indeed, was reported frequently (see Table 3). Visual display of the set of alternatives, as used by Lincoln and Alexander (1955), Mittenecker (1958), Ross (1955), and Weiss (1964), may be one way to overcome this difficulty.

Still another factor that should be taken into account is the mode of production. Only Weiss (1964) reported automatic registration of responses by means of push buttons. Most of the other experimenters had their subjects call out or write down the series. These two modes of production differ with respect to the availability of previous responses: the spoken items can only be remembered, responses that are written down on a sheet of paper remain present until the page is turned. Only Wolitzky and Spence (1968) used an apparatus by which all (written) responses but one were covered. Since Tune (1964a) attributed nonrandomness to the limited span of short-term memory, the number of previous responses visible for the subject is a variable that should not be overlooked.

The rate of production was reported to be an important factor by Baddeley (1966), Teraoka (1963), and Warren and Morin (1965). Among the 15 experiments under discussion, response rate varied from .25 to 4 seconds per response, whereas production could be paced or unpaced. Although there is no agreement about the effect of an increas-

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No. c previo respon visibl for S	
Paced (P) or un- paced (U)	ͷϥϫϫϫϫϫͻͻͻͻͻϫϫϲͻϫϫϫϫϫϫϫϫ
No. of secs. per response	$\begin{array}{c} \vdots, 1, 2, 4\\ \vdots, 1, 2, 4\\ \vdots, 2, 1, 2, 4\\ \vdots, 1, 2, 4\\ \vdots, 1, 2, 4\\ \vdots, 1, 2, 4\\ \vdots, 1, 2\\ \vdots,$
Mode of production	calling out calling out calling out calling out calling out calling out calling out writing down writing down
Inter- nal (I) or exter- nal (E) repre- sented choice set	
No. of responses per series	$\begin{array}{c} 100\\ 360\\ 360\\ 360\\ 360\\ 360\\ 360\\ 360\\ 3$
No. of series per S per con-	0»0.5www0-44000
Nature of the alternatives	letters letters letters letters letters letters letters letters letters digits digits digits digits digits digits digits heads-tails digits digits digits letters digits letters digits letters digits letters digits digits letters digits digits letters digits digits digits letters digits digits letters digits digits digits digits digits digits letters digits digits digits digits digits letters digits
No. of alternatives in the choice set	26 26 26 26 27 26 26 26 26 26 26 26 26 26 26 26 26 26
Author (s) and year	Baddeley (1962) Baddeley (1966) Bakan (1960) Chapanis (1953) Lincoln & Alexander (1955) Mittenecker (1953) Mittenecker (1953) Mittenecker (1953) Rath (1966) Ross & Levy (1958) Ross & Levy (1958) Teraoka (1964) Warren & Morin (1965) Weiss (1964) Weiss (1964) Weiss (1964) Wolitzky & Spence (1968) Zwaan (1964)

TABLE 1 Experimental Conditions of 15 Experiments on Subjective Randomization

^a Production as fast as possible.

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ing rate of production since both increases and decreases of nonrandomness have been found, this factor evidently complicates the randomization experiment.

Finally, the number of subjects varied from 2 to 124. Individual differences were sometimes rather large, which means that results based on small numbers of subjects cannot always be generalized.

In general, it can be stated that no two experiments of our sample differ only in one of the factors mentioned above. Therefore, comparisons are questionable, to say the least.

DEFINITION OF MATHEMATICAL RANDOMNESS

With respect to the definition of mathematical or objective randomness, little standardization is evident concerning the criterion for calling a series random or nonrandom. Here a methodological problem arises, as randomness is easier disproved than proved. For disproving randomness it is sufficient to show one type of systematic trend in the series, whereas for the establishment of real randomness it is required to prove that not a single serial regularity of the many possible ones is present. An endless repetition of the alphabet, for instance, is perfectly random regarding single-letter frequencies, but extremely nonrandom with respect to frequencies of pairs. A similar difficulty occurs when an experimenter is interested in the increase or decrease of randomness: one series can be more random than another according to one criterion and, at the same time, less random in another respect. Recognition of this problem is crucial for the interpretation and comparison of experimental results. The measures of nonrandomness most frequently used are presented in Table 2. If only frequencies of single responses are taken into account, analyses are said to be of zero order.³ In zeroorder analyses, no dependencies among responses can be established. For first-order analyses, frequencies of digrams (pairs) are used, for second-order analyses frequencies of trigrams, etc. The general rule is that analyses of order n, which require a count of (n + n)

⁸ In information theory, the zero order of dependency is usually called the first order of redundancy. 1)-grams, can yield dependencies between responses that are maximally n places apart. As shown in Table 2, few experimenters use analyses higher than second order. The mathematical origin of the measures is also rather diverse: Witness the third column in Table 2.

One class of measures bears relation to occurrence of runs, which are strings of identical responses. The total number of runs, used by Bakan (1960) and Zwaan (1964), is essentially a first-order measure, since it equals the number of digrams with unequal elements. The frequency distribution of runs with length *i*, as used by Ross and Levy (1958) and Teraoka (1963), is a measure with all orders mixed in a mathematically complex way. Distance of repetition curves (Mittenecker, 1953, 1958; Zwaan, 1954) gives the frequency distribution of gaps with length *i* between two identical responses, which are actually runs of nonoccurrence of that alternative. This again is a measure with all orders mixed.

A second class contains measures from information theory, like information per response (Baddeley, see Footnote 2) and relative redundancy in the series (Baddeley, 1966; see Footnote 2; Lincoln & Alexander, 1955; Mittenecker, 1958; Warren & Morin, 1965). Measures of this type require very long series for higher-order analyses. Baddeley (1966) mentioned 4,000 responses for a firstorder analysis of 26-alternative sequences. Hence, in practice, the analysis is limited to the third order.

Finally, for analyses above Order 4, often autocorrelation curves are used, which have again the disadvantage that estimates of dependencies are not given separately for each order (Chapanis, 1953; Mittenecker, 1958). A series with an endless repetition of the digram 0-1 will yield an endless autocorrelation function with values +1, -1, +1, -1, etc. Yet the simplest description of the dependencies is a first-order alternation model. One way to overcome this difficulty is to calculate a power spectrum on the basis of the autocorrelation (Pöppel, 1967). For the computation of a power spectrum with six terms, however, at least 72 autocorrelations are needed, whereas the computation will be

TABLE 2

MEASURES OF NONRANDOMNESS USED BY VARIOUS EXPERIMENTERS

Author(s) and year	Order of analysis Description of the measure for nonrandor		
Baddeley (1962)	1	repetition of digrams	
	0	redundancy	
	1	stereotyped responses	
	1	information per response	
Baddeley (1966)	0	redundancy	
	1	stereotyped and repeated digrams	
Bakan (1960)	1 number of runs		
	2	alternation and symmetry in trigrams	
	0	frequency of alternatives	
Chapanis (1953)	0	frequency of alternatives	
	1,2	frequency of digrams and trigrams	
	1-?	autocorrelation function	
Lincoln & Alexander (1955)	0-2	redundancy	
	0	frequency of alternatives	
	1	spatial distance between two alternatives in the	
		digrams	
	2	frequency of trigrams	
Mittenecker (1953)	mixed	distance of repetition	
	0	frequency of alternatives	
Mittenecker (1958)	0,1	redundancy	
	111	autocorrelation function	
Rath (1966)	0	frequency of alternatives	
	1	frequency of digrams corrected for frequency of alternatives	
	2	frequency of trigrams corrected for frequency of digrams	
	1	frequency of digrams as a function of the distance between the two elements in the natural ordering	
Ross (1955)	1	number of alternations	
	0	frequency of alternatives	
Ross & Levy (1958)	1	number of alternations	
, , ,	1-?	occurrence of runs	
Teraoka (1963)	0	frequency of alternatives	
	1	conditional probabilities	
	1	frequency of digrams as a function of the distance between the two elements in the natural ordering	
	1-4	occurrence of runs	
Warren & Morin (1965)	0-3	redundancy	
Weiss (1964)	1-9	frequency of (n)-grams corrected for lower-order dependencies	
	2	frequency of trigrams	
Wolitzky & Spence (1968)	0	frequency of alternatives	
Zwaan (1964)	1	number of runs	
	mixed	distance of repetition	

successful only if the autocorrelation function is fairly periodic over this interval. Unfortunately, this is not a priori true for attempted random sequences.

In general, it can be concluded that most measures of nonrandomness are neither powerful enough for disproving all serial regularities nor adequate for establishing increases and decreases of nonrandomness.

RESULTS AND THEORIES

Considering the divergence of experimental procedure and method in measurement, it is not surprising that results are quite contradictory. Actually, there is no way of combining details of the results of the 15 publications discussed into one coherent theory. Some major outcomes are presented in Table 3.

TABLE 3

SOME RESULTS OF EXPERIMENTS ON SUBJECTIVE RANDOMIZATION

Author(s) and year	Are sub- jects good random- izers?	Positive (Pos.) or negative (Neg.) recency	Other systematic deviations from randomness	Factors increasing nonrandomness	
Baddeley (1962, 1966)	no	 ?	unbalanced 1- and 2-gram fre- quencies, stereotyped di- grams	increase of rate of production and number of alternatives, introduction of secondary task	
Bakan (1960)	no	Neg.	avoidance of symmetric re-		
Chapanis (1953)	no	Neg.	unbalanced 1-, 2-, 3-gram fre- quencies, preference to de- creasing series, avoidance of increasing series	naivete of Ss	
Lincoln & Alexander (1955)	no	Neg.	preference to the easy motor responses, to alternatives with a large spatial distance to the previous alternative, and to clockwise or counter- clockwise ordered sequences	giving verbal response instead of motor response	
Mittenecker (1953, 1958)	no	Neg.	balancing of frequencies with- in small samples	neuroticism	
Rath (1966)	no	Neg.	preference to symbols adja- cent in the natural sequence	increase of number of alterna- tives	
Ross (1955)	yes	Pos.			
Ross & Levy (1958)	no	Pos. and Neg.ª	overuse of run length with ex- pected frequency of at least 1	naivete with respect to the ex- pected frequency of runs	
Teraoka (1963)	no	Neg.	response chaining related to the natural order of the al- ternatives, dependencies over at least 5 places, periodicity with period of 5 responses	presence of a natural order of alternatives, decrease of rate of production	
Warren & Morin (1965)	no	Ş		increase of rate of production and of number of alterna- tives	
Weiss (1964)	no	Pos.?	preference for symmetric tri- grams	boredom	
Wolitzky & Spence (1968)	no	5	G	increase of the informational load of a secondary task	
Zwaan (1964)	по	Neg.			

* Negative after briefing about expected frequency of runs.

First, almost all experimenters found systematic deviations from randomness. Only Ross (1955) claimed that his subjects were good randomizers. Second, most experimenters yielded negative recency, which means too many alternations or too many runs. Some authors did not mention the direction of nonrandomness because their measures could not distinguish between negative and positive recency. Positive recency was reported only for first-order dependencies. Weiss's (1964) data seemed to point to second-order positive recency, providing that his relative frequencies of trigrams were corrected to add up to 100%. Although Ross's (1955) experiments yielded real randomness, some objections can be raised. His subjects were requested to stamp symbols (X or O) on cards. This procedure may have favored repetition (going on with the same stamp) over alternation (taking the other stamp), for instance, because the subjects were bored by the experiment, and hence took the easygoing way. Thus, the frequently observed tendency toward alternation may have been balanced out by this unintentional facilitation of repetition.

Third, several other systematic deviations from randomness were found, such as preference to the natural order of the alternatives and preference as well as avoidance of symmetric patterns. In general, these systematic trends are related to the nature of the stimuli.

Finally, Table 3 presents some factors that are supposed to increase nonrandomness, but, in view of the difficulties in defining such an effect mathematically, these outcomes should be evaluated with caution.

As far as the different theories are concerned, there is one point of view that attributes nonrandomness to the limitations of short-term memory. Tune (1964a) argued that subjects who can tally frequencies of all *n*-grams may be random up to order (n-1). Baddeley (1966) claimed that the very use of memory was responsible for serial dependencies and proposed a theory based on a limited capacity for generating information. According to this theory, information generated per time unit should be constant. The increasing rate of production did make the series more nonrandom, but, as shown before, the results of this experiment might have been contingent on mental representation of large sets or parts thereof, rather than on random selection. Teraoka (1963) found a decrease of nonrandomness with an increase of speed. whereas Warren and Morin (1965) found the opposite. The latter authors stated, however, that the amount of information generated per time unit also increased with rate of production. An interesting theory proposed by Mittenecker (1953) and extended by Zwaan (1964) suggests that subjects try to balance the frequencies of alternatives within small samples. Weiss (1965) supported a theory which states that both attention for being random and distraction from previous responses are necessary conditions for being random. This theory cannot be discredited since any effect can be explained, either in terms of decreased attention or in terms of decreased distraction. Other theories deal with boredom, experience with ordered sequences in normal, daily life, etc. Thus far, there is no reason to favor one theory over another,

since no reliably decisive experiments have been published.

THE RELEVANCE EXPERIMENT

Some theories mentioned above attribute nonrandom behavior in the randomization experiment to functional factors like memory, attention, and boredom. This implies that the randomization paradigm involves two factors at a time: subjective concept of randomness and some functional limitations of serial randomization. A necessary control experiment can be made by presenting sets of random and nonrandom series to subjects with the instruction to select the "true" random ones. If nonrandomness is indeed attributable to a subjective concept, subjects should not be able to discriminate between random and nonrandom sequences even in this situation. This experiment determines the relevance of the notion of "subjective randomness" in the randomization experiment.

Few authors report such a control experiment. Baddeley (1966) mentioned, without presenting any data, that subjects could select the correct series, suggesting that their concept of randomness was perfectly alright. Cook (1967) arrived at the same conclusion. He used nonrandom series that were so obviously nonrandom that the data cannot be taken as decisive. Mittenecker (1953) and Zwaan (1964) both reported that subjects were unable to make the correct identification. The error was in the direction of negative recency. Wagenaar (1970b) found that subjects were generally not able to indicate the true random series, the bias being in the direction of negative recency.

RECOMMENDATIONS FOR FUTURE RESEARCH

The first problem to be solved is the problem of measurement. A method is needed for measuring higher-order nonrandomness in short sequences. A very original approach was made by Vitz and Todd (1969), but this author feels that their method is not developed well enough to allow for comparison of series with unequal length or number of alternatives. The present author is involved in another attempt to define nonrandomness in short sequences up to high orders of dependency (Wagenaar & Truijens).⁴ Some promising results were obtained with this method (Wagenaar, 1970a, 1971), but more experimentation is needed to establish whether the method is powerful enough.

The second need is to develop the proposed theories mathematically to check more thoroughly on the phenomenon that different theories predict identical results.

The third step is to design decisive experiments that single out all factors responsible for nonrandomness. Especially the discrimination between subjective concepts and functional factors, like memory and attention, deserves more experimentation.

CONCLUSION

Thus far, randomization experiments have not led to conclusive results. Further research in this field will yield useful information only if the experimental conditions are better controlled, if mathematical randomness is defined in a uniform way, and if the problems are so stated as to permit more critical experiments.

⁴ W. A. Wagenaar & C. L. Truijens. Measurements of high-order sequential dependency in short sequences. (Tech. Rep. No. IZF 1970-19) The Netherlands: Institute for Perception.

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(Received July 7, 1970)

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