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Recovering Reflectance and Illumination in a World of Painted Polyhedra

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To be immune to variations in illumination, a vision system needs to be able to decompose images into their illumination and surface reflectance components. Although this problem is greatly underconstrained, the human visual system is able to solve it in diverse situations. Most computational studies thus far have been concerned with strategies for solving the problem in the restricted domain of 2-D Mondrians. This domain has the simplifying characteristic of permitting discontinuities only in the reflectance distribution while the illumination distribution is constrained to vary smoothly. Such approaches prove inadequate in a 3-D world of painted polyhedra which allows for the existence of discontinuities in both the reflectance and illumination distributions. We propose a two-stage computational strategy for interpreting images acquired in such a domain.

1. Introduction

Figure 1 illustrates a classic problem for vision systems: one of the circled edges is due to a change in surface color (a reflectance edge) while the other is due to a change in surface orientation leading to a change in illumination (an illuminance edge). In the image, though, both these transitions have identical luminance profiles. How might one distinguish between the two situations? Making this distinction, which humans do effortlessly, is important for most vision systems since it enables them to factor out from the image the varying effects of illumination from the reflectance distribution intrinsic to the object [3]. The recovery of such intrinsic properties is important for many visual tasks.

While the importance of such computations has long been recognized in the machine vision community, the approaches developed so far are capable of functioning only in highly restricted domains. We wish to develop techniques that will be somewhat more general, and to that end we introduce the world of 'painted polyhedra' in which to try out our ideas. We use human perception as a guide in our investigations, but make no claims of the biological plausibility of the individual computational steps involved.

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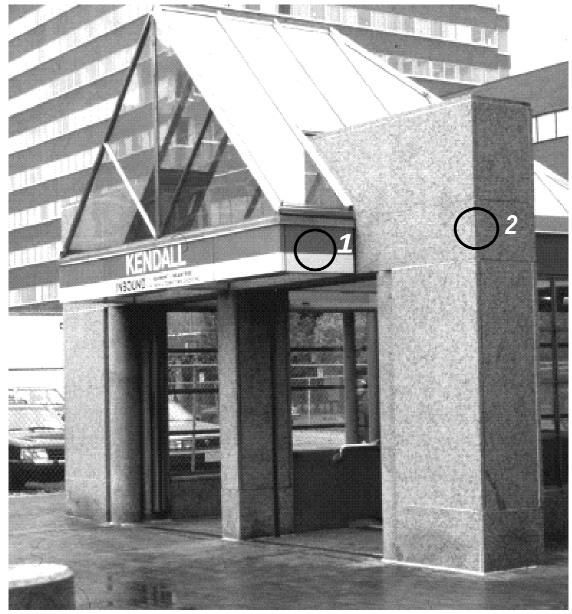

Figure 1. A typical real world scene. Changes in surface reflectance or orientation lead to variations in image luminance that may have identical profiles locally.

2. Previous work:

The predominant paradigm for analysing images in terms of their illumination and reflectance components is Retinex [16, 17, 12, 19, 5], motivated by Land's lightness constancy experiments with 'Mondrian' stimuli (the term 'lightness' denotes a perceptual estimate of true reflectance). Mondrian patterns consist of patches of differently colored paper pasted on a planar background. The illumination across the patterns is constrained to vary smoothly. In such a setup, discontinuities in image luminance coincide exclusively with reflectance discontinuities in the scene. Retinex exploits this characteristic to recover the lightness distribution from a given image. The image is differentiated and then thresholded to get rid of slow intensity variations due to illumination. Subsequent reconstitution of the image through integration leads to the recovery of the underlying reflectance distribution upto an unknown offset.

While Retinex performs well in the Mondrian domain, its assumption of a smoothly varying illumination gradient limits its usefulness in a 3-D world containing painted polyhedra. The sharp intensity transitions exhibited by images in this domain can arise not only from surface reflectance variations but also from changes in illumination across differently oriented faces of the polyhedra. Having no notion of three-dimensionality, Retinex classifies all such transitions as being due to reflectance changes - an interpretation which is often perceptually and physically incorrect, as illustrated in figure 2 (cf. [1, 11, 15]).

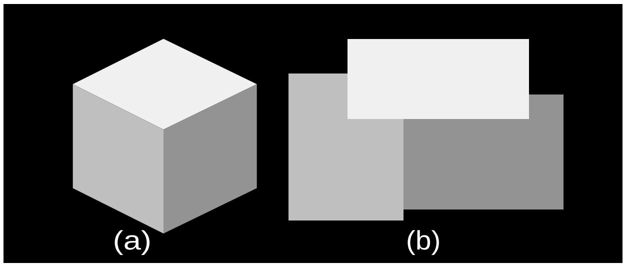

Figure 2. Retinex treats both the patterns shown above identically; as far as it is concerned, the pattern on the left is just another flat Mondrian with three differently colored patches. The more natural interpretation of pattern (a) would be as a uniform albedo 3-D cube illuminated in a particular fashion.

Adelson and Pentland [2] proposed a cost based 'workshop metaphor' for interpreting images of simple painted polyhedra. A set of specialists deal with 3-D shape, illumination, and reflectance, and seek a least cost model for the image. The model was able to deal with some simple images but did not readily generalize to other situations.

In the rest of the paper, we describe a new approach that seems better suited to interpreting images of objects belonging to the world of painted polyhedra.

3. A new direction:

In the present investigation, our domain consists of painted polyhedral/origami objects such as those shown in figure 3. We assume an absence of inter- and intra-object occlusions and cast shadows. The surfaces shall be assumed to be qualitatively matte without necessarily being precisely Lambertian. The scene shall be assumed to be illuminated with a diffuse ambient and a single distant light source (see [20] for justification).

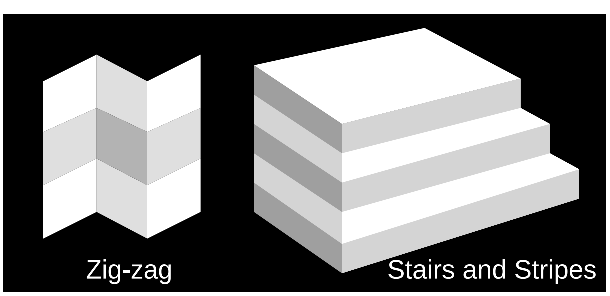

Figure 3. Two sample objects from our domain of interest. Notice that the figure on the right represents an object whose physical realization is impossible.

Our strategy has two stages. The first stage attempts to use simple local gray-level junction analysis to classify the observed image edges into the illumination or reflectance categories. Subsequent processing verifies the global consistency of these local inferences while also reasoning about the 3-D structure of the object and the illumination source direction. This work draws on a host of ideas each of which merits detailed description. For reasons of space, we shall lay emphasis on how the various ideas work together in the overall computational framework for lightness recovery without describing any individual one in detail. Further details may be found in [21].

3.1 Gray-level junction analysis:

Figures 1 and 3 demonstrate that edges with identical intensity profiles can have very different perceptual/ physical interpretations. Local analysis of such profiles, then, seems unlikely to be capable of classifying edges differently. Can any other source of local information aid in distinguishing between reflectance and illumination edges?

In a non-accidental image, an edge representing the line of join of two differently oriented surfaces necessarily exhibits either an arrow or Y junction at both its end-points. A reflectance edge that is not deliberately made to coincide with the corners of the underlying polygonal surface, on the other hand, exhibits T-junctions at its end-points (see figure 4). This mapping from scene to images may be inverted and the existence of particular types of junctions in the image may be used as evidence for the presence of particular physical characteristics in the underlying scene. Following this line of reasoning, we may construct a simple junction catalogue that can be used for classifying scene edges. Figure 5 shows the junction catalogue used in our implementation. This catalogue is not exhaustive for all possible polyhedral junction types; our intent here is only to indicate the basic idea behind the approach. Besides the afore-mentioned arrow and Y junctions, it also includes the psi-junction along with constraints on the gray-levels that need to be satisfied before equating an image junction with the reference junction pattern.

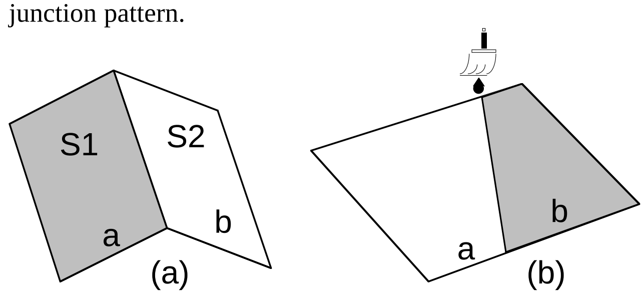

Figure 4. Since an illumination edge represents a line of join of two differently oriented surfaces S1 and S2, lines 'a' and 'b' must be non-collinear in the scene and consequently in a non-accidental image. However, for a reflectance edge arising simply out of a change in albedo over different regions of the same surface (as in (b)), line segments 'a' and 'b' are collinear both in the scene and the image.

Due to the 1-to-1 relationship between junction-types and interpretations, no constraint propagation is required during the labelling process which simply involves determining an edge's label from the junction-type it exhibits at its end-points. An iterative brightness equalization across illumination edges leads, eventually, to the lightness distribution [21]. For most objects, the labels derived from both end-points are identical. However, impossible objects (of the type shown in figure 3) may exhibit edges whose interpretation changes along their length. To handle such objects, the 'strength' of the inferred label is made to decrease monotonically (linearly, in the current implementation) from the label inducing end to the other end. Figures 6 and 7 show some examples of applying the gray-level junction analysis ideas to images of synthetic polyhedral/origami objects. All occluding edges are left unclassified.

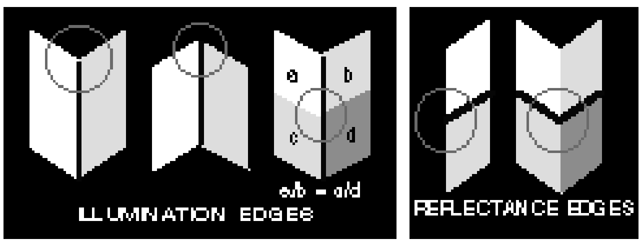

Figure 5. The junction catalogue we used. 'a', 'b', 'c' and 'd' represent brightness values.

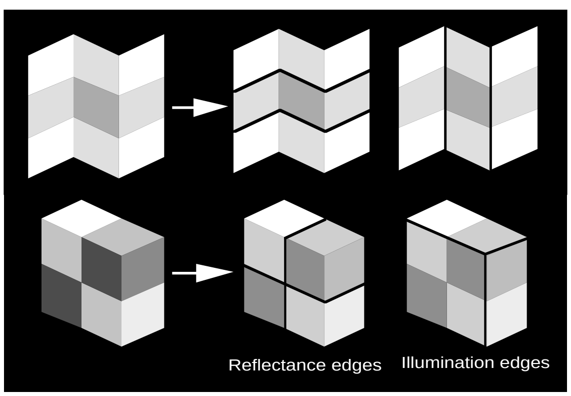

Figure 6.Two images interpreted by the junction analysis program.

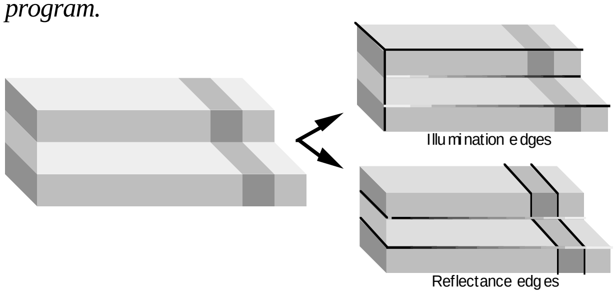

Figure 7. Interpreting the image of an 'impossible' object using the junction analysis program. The change in shade from black to white along the labelled edges indicates the decreasing 'strength' of the label.

The gray-level junction analysis approach predicts that in the absence of any junctions corresponding to illumination edges (and therefore to 3-D shapes), an image should appear to represent a flat painted scene with no 3-D attributes. That this prediction holds true is demonstrated in figure 8. By maintaining all other attributes of an image constant and varying only the geometric structure slightly to include or abolish particular junction types, the appearance of the scene can be changed dramatically. This compelling demonstration highlights the importance of simple gray-level junction analysis in image interpretation.

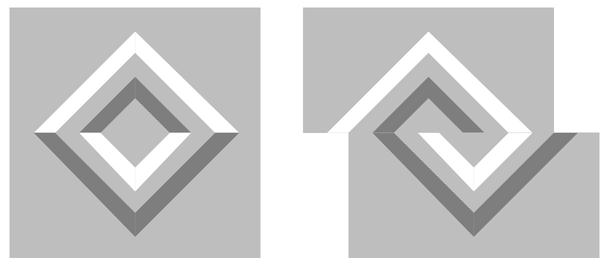

Figure 8. By including or excluding junction types that signal illumination edges in images, a figure may be made to appear flat or three dimensional. Notice that the top and bottom halves of both these figures are identical.

3.2 The need for global analysis:

The apparent success of the simple junction analysis scheme in interpreting the examples so far might induce one to accept it as the complete solution to the problem we set out to solve. However, it is simple to show that this scheme has some fundamental limitations due primarily to its local nature. Figure 9 is a case in point. Perceptually, the image on the left appears to depict a properly shaded uniform albedo 3-D truncated hexagonal pyramid; all the edges are perceived as arising out of illumination changes. The image on the right is seen as a flat painted pattern with all the edges arising out of reflectance changes. Our junction analysis scheme, however, labels all edges in *both* images as illumination edges since all the observed junctions (arrows and Y's) are associated with illumination edges in the junction catalogue. The cause of this problem, of course, is that the inferences suggested by the catalogue are justified *locally*, but the *global* structure of the image in figure 9(b) renders them incorrect. Evidently, a process capable of reasoning about the global structure of the image is required. Such a global scope would also enable it to verify whether the image gray-levels constitute a globally 'consistent' shading pattern. But exactly what do we mean by 'consistency'?

A pattern of gray-levels shall be considered 'consistent' if it can be produced by illuminating with a single distant light source a uniform albedo 3-D structure consistent with the geometric structure of the pattern. While there are clearly infinitely many 3-D structures consistent with the pattern's geometric structure, we restrict our attention to those that are perceptually likely. Also, for reasons described later, two gray-level patterns shall be considered 'equivalent' if their ordinal structures (relative ordering of the different patches based on their associated gray-levels) are identical; precise gray-level matches are not required.

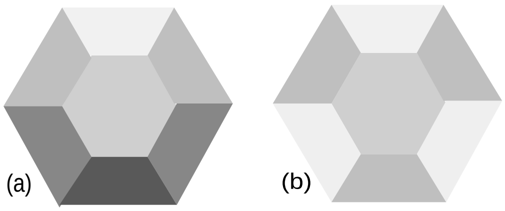

Figure 9. Two figures with identical geometric structures but very different interpretations. Figure (a) is perceived to be a shaded 3-D truncated hexagonal prism while figure (b) appears to be a flat pattern of paint.

From the definition of gray-level pattern consistency given above, it is apparent that a method for verifying such consistency needs to perform two conceptually different tasks:

1. it needs to recover a set (possibly singleton) of perceptually likely 3-D structures consistent with the geometric structure of the input pattern, and
2. it needs to verify whether (any of) the recovered 3-D structures can be illuminated with a single distant light source so as to produce a gray-level pattern equivalent to the input pattern.

Figure 10 shows this two part strategy. The second part of the strategy, in essence, attempts to determine whether the variations in gray-levels across different faces are due to shading or changes in intrinsic surface reflectance. We next describe briefly computational schemes to accomplish each of these tasks.

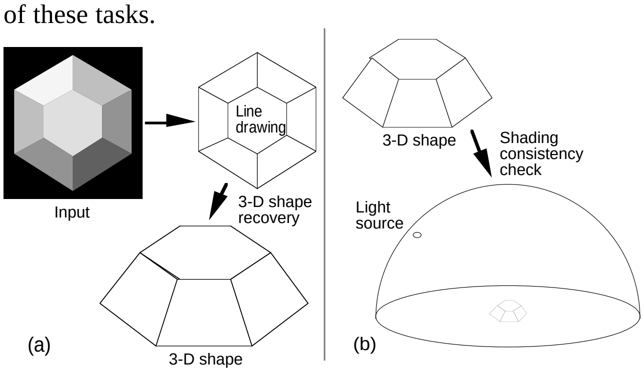

Figure 10. Our two-part strategy comprises of (a) deriving the likely 3-D structures corresponding to the geometric configuration of the image, and (b) verifying the structure's consistency with the image shading pattern.

3.3 Deriving 3-D shapes from 2-D drawings:

As stated earlier, our aim here is to interpret 2-D line-drawings extracted from the input gray-level patterns in terms of their perceptually/physically likely 3-D structures. The difficulty of this task, well documented by sevral reserachers [14, 22], arises from its highly underconstrained nature; any planar line-drawing is geometrically consistent with infinitely many 3-D structures, as shown in figure 11. In light of this observation, two questions that need to be addressed are: 1.what distinguishes the 'correct' 3-D structure from the rest?, and 2.how might we search for the 'correct' structure in the infinite space of all possible 3-D structures consistent with the given line-drawing?

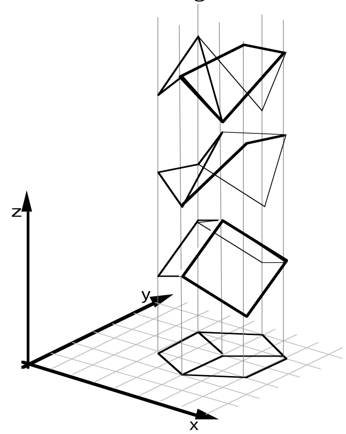

Figure 11. Any planar line-drawing is geometrically consistent with infinitely many 3-D structures.

It has long been suggested that the distinguishing characteristic of a perceptually favored 3-D interpretation is its low 'complexity'. The variance of the included angles has been proposed as a measure of complexity [4, 18]; minimizing this metric leads to perceptually correct interpretations for many line drawings. However, we find that using this metric alone results in unexpected and bizarre interpretations for certain figures (see figure 12). We propose that to properly characterise the perceptually 'correct' interpretations, three types of measures are required: angle variance, planarity of faces and overall compactness; we wish to obtain that 3-D configuration which, while having planar faces is maximally regular and compact so as not to require excessive foreshortening of any line segment to relate it to the input 2-D drawing. A similar suggestion was made in [9].

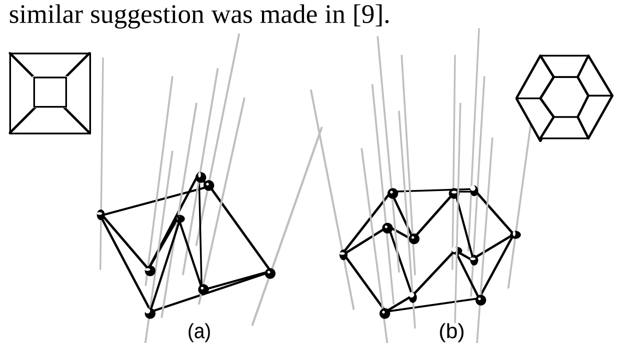

Figure 12. Perceptually incorrect shapes recovered from the input line-drawings by Marill's algorithm. The shapes are shown as states of a 'beads-on-wires' model. The 'wires' are aligned to the line-of-sight and the positions of the beads represent the depth values associated with each vertex.

The question of how to search for the desired configuration in the infinite search-space of all possible configurations is a tricky one. Traditional approaches usually involve formulating and then optimizing a composite cost function (such as a weighted sum of the relevant metrics [9]). This approach suffers not only from the need to make *ad hoc* choices for the relative weights but also has the same drawbacks that regularizing techniques have, viz., the constructed cost-function might not represent the original problem. Also, the parameter values that might be appropriate for one problem instance might be inappropriate for another.

Our search strategy belongs to the class of 'greedy' optimization algorithms [6]. Here we only give a conceptual description of the strategy and refer the reader to [21] for a formal treatment of the same. Imagine that one is given a 2-D line drawing that one wishes to derive the maximally regular planar 3-D shape of. What kinds of intermediate stages should one expect to pass through on way to the final configuration? One natural way of getting to the desired shape is to incrementally modify the originally planar configuration so that at every intermediate step the most regular planar faceted configuration is obtained. This can be thought of as doing gradient descent in regularity space where the points considered in the space correspond to the different planar-faceted 3-D configurations. The first local minima reached in this fashion is reported as the recovered 3-D shape. This strategy does not require the construction of one composite cost function from different metrics. Besides obviating the need for *ad hoc* choices of parameters, this also has the desirable result of having the same algorithm work unchanged on all problem instances. Figure 13 shows two sample results.

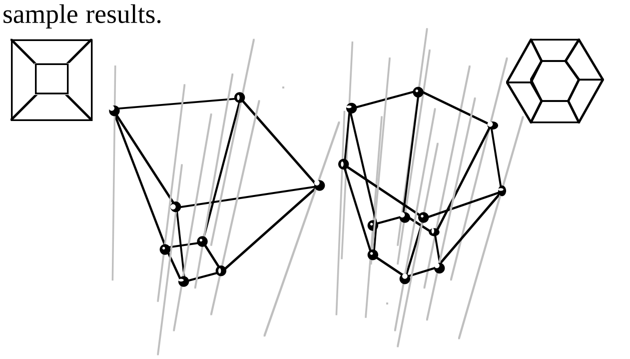

Figure 13. Two examples of 3-D shape recovery using constraints of symmetry, planarity and compactness.

3.4 Verifying the 'consistency' of shading:

With a method for recovering 3-D structures from 2-D line-drawings in hand, we need now to determine whether the image shading pattern is 'consistent' with the recovered structure. In other words, we wish to find out whether there exist any source directions that would completely account for all gray-level variations in the image without having to invoke the hypothesis of surface reflectance changes.

Given a 3-D structure and the gray-levels associated with each of its faces, the problem of determining the source direction under the assumption of a precisely specified reflectance function is not too difficult and closed-form solutions for this task have already been described [13, 20, 23]. These methods, however, have some fundamental limitations. Firstly, because of their dependence on precise measurements of gray-level values, minor alterations in image gray-levels can radically alter the computed solution, even though perceptually they may be of no consequence at all. Figure 14 provides an illustration of this problem.

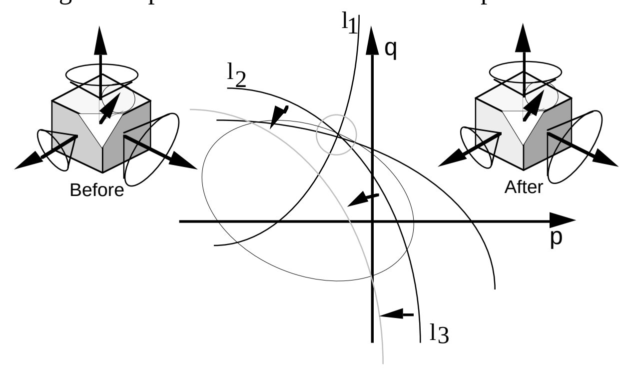

Figure 14. The various faces of a polyhedral object define curves of valid light directions in gradient space. The common point of intersection of all these curves corresponds to the light direction that would account for the brightnesses of all faces simultaneously. A slight alteration in the gray-level of one of the faces shifts the corresponding locus of valid light directions and the four curves (corresponding to the valid source directions for the four visible faces in this example) no longer have a common point of intersection implying that the sliced-cube on the right is no longer consistently shaded. Perceptually, however, the minor gray-level alteration is inconsequential.

Secondly, such methods rely critically on a precise specification of the surface reflectance function. Minor alterations in this function profoundly influence the computed solution. This is a serious drawback considering that in most situations, the choice of the reflectance function is at best an educated guess. What we seek to have is a method that would be gracefully tolerant of changes in the image gray-levels and the specification of surface reflectance function. Instead of adopting an ad-hoc patch-up like the use of least-square error minimization, we propose a qualitatively different paradigm.

One of the key motivating observations behind our approach is that our perceptual apparatus is far more sensitive to detecting relations like 'brighter than'/'darker than' between pairs of adjacent surfaces than to estimating their absolute brightnesses. Perceptual interpretations of images are quite stable over alterations in image gray-levels that leave the binary relations between adjacent pairs of surfaces unaltered (in a sense, these relations define perceptual equivalence classes for images). In our approach, we use only such binary relations extracted from the underlying images. The other key idea is to use these relations to constrain the source direction in a manner that least commits us to a particular reflectance function. Consider figure 15. If *S1* and *S2* are two surfaces with normals *n1* and *n2* (remember that the 3-D shape of the object has already been recovered in phase-1 of the strategy) and *S1* appears darker than *S2* in the image, then the valid light directions can be represented on a Gaussian sphere by one of the two hemispheres formed by a plane passing through the origin that perpendicularly bisects the vector joining the tips of the normals *n1* and *n2* on the sphere surface. This set of light directions is valid for any reflectance function that results in a monotonically decreasing relationship between image luminance and angle of incidence. We may further constrain the light directions to lie above the horizontal plane. A light direction chosen from this set will maintain the ordinal relationship between the brightnesses of surfaces *S1* and *S2*. Other pairs of surfaces will similarly define hemispheres of valid light directions. The complete shading pattern can be considered consistent if the intersection of all the hemispheres corresponding to different adjacent surface pairs yields a finite set. The problem of checking for the consistency of the observed shading pattern is thus rendered equivalent to determining whether a set of hemispheres have a non-null intersection. The non-null intersection set, if obtained, represents the valid set of light directions. Since (as shown below) each hemisphere is equivalent to a linear constraint on the possible positions of the source vector, this approach lends itself naturally to a linear programming solution method such as the Fourier-Motzkin elimination technique [10, 7]. Interestingly, the Perceptron Learning algorithm [8] is also perfectly suited to solving this problem. This approach also has the desired properties of not being critically dependent on precise measurements of the absolute surface brightness values and not having to assume a precisely specified formal reflectance model.

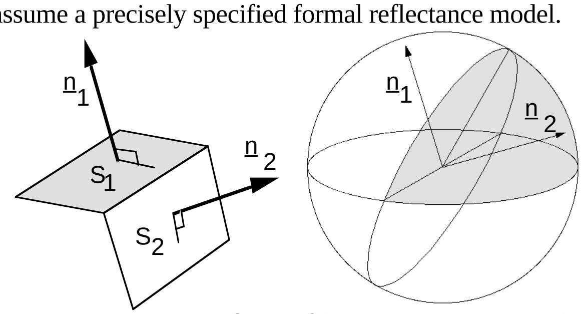

Figure 15. Two surfaces of known orientation and their relative brightnesses constrain the light source to lie in a particular sector of the Gaussian sphere.

3.4.1. Using linear-programming to determine light directions from shaded images of polyhedral objects:

Consider the pair of surfaces S1 and S2 with normals $\bar{n}_1$ and $\bar{n}_2$ respectively. Compute a vector $\bar{s}$ such that

$\bar{s} \cdot ((\bar{n}_1 + \bar{n}_2)/2) = 0$; $\bar{s} \times (\bar{n}_1 \times \bar{n}_2) = 0$;

$\bar{s} \cdot \bar{n}_1 > 0$ and $\bar{s} \cdot \bar{n}_2 < 0$ if S1 is brighter than S2

$\bar{s} \cdot \bar{n}_1 < 0$ and $\bar{s} \cdot \bar{n}_2 > 0$ if S2 is brighter than S1

The hemisphere of valid directions defined by the surface pair S1 and S2 then is precisely the set of vectors t satisfying the inequality $s \cdot t > 0$. To constrain the valid directions to lie above the horizontal plane, we may wish to enforce the additional inequality $z \cdot t \geq 0$ (assuming without loss of generality that the ground plane is the X-Y plane). For each adjacent pair of surfaces Si and Sj, we get one such linear inequality , viz. $s_{ij} \cdot t > 0$. We wish to find a vector t (if it exists) that satisfies *all* these inequalities. This is a simple linear programming problem. There are 'e' linear inequalities for a polyhedral object with 'e' internal edges. Since we are interested only in the direction of t, there are only two degrees of freedom to be solved for. As no objective function is being extremized, there will exist an infinite number of solutions if there are any solutions at all. All of these solutions will lie in a convex polygon on the unit sphere. The sides of this polygon are portions of great circles corresponding to constraints imposed by some surface pairs (see figure 16).

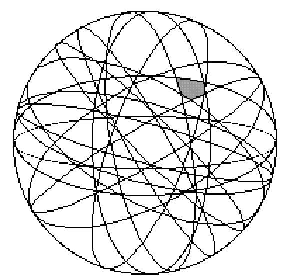

Figure 16. The solutions to the system of constraints set up by the various surface pairs lie on a convex polygon on the unit sphere. The sides of this polygon are portions of great circles corresponding to constraints imposed by some surface pairs.

3.4.2 Determining the illuminant direction - examples:

We now present some examples illustrating the use of the aforementioned ideas for checking the consistency of the observed shading pattern in the image and for recovering the illuminant direction. The first two examples comprise of a cube illuminated from two different directions. The graphical solutions (figure 17) show that the recovered sets of valid light directions are consistent with human perception.

The next two examples are more interesting. They are the figures that we used to motivate the need for a global analysis (figure 9). The graphical solutions, (figures 18 and 19) suggest that while the shading pattern in figure 9(a) is consistent with the shape recovered by the module responsible for 3-D shape recovery from 2-D line drawings, that of figure 9(b) is not. In other words, while a distant light source can be positioned to illuminate a truncated hexagonal pyramid shape to make it look similar to figure 9(a), there is no way that it may be made to look like figure 9(b) by solely manipulating the light direction. The consistency or inconsistency of the 2-D shading pattern in the image determines whether the percept obtained is one of either a solid 3-D shape illuminated in a particular manner or simply a 2-D pattern of paint.

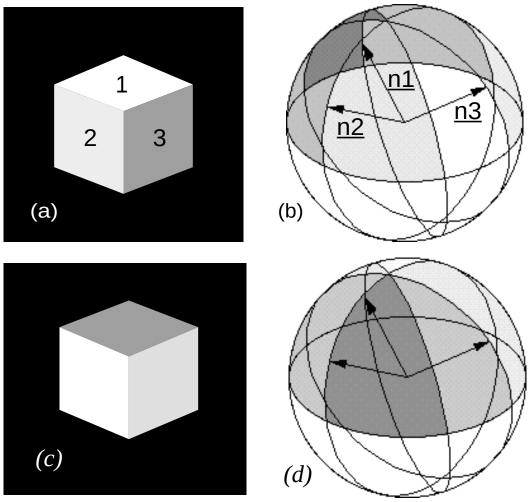

Figure 17. Computing the set of valid light directions for the cube image, (a). The darkest sector of the Gaussian sphere in (b) represents the solution set. The ni's are the surface normals for the three visible faces of the cube. (c) and (d): Varying the ordinal relationships between the gray-levels of the faces of the cube changes the computed solution set for illuminant directions in a manner consistent with human perception.

We may mention in passing that situations where a majority, but not all, of the inequalities are simultaneously satisfiable have interesting perceptual/physical correlates. The inequalities left unsatisfied correspond to 'compound edges' - image brightness transitions that are perceived as being due simultaneously to changes in illumination and surface reflectance [21].

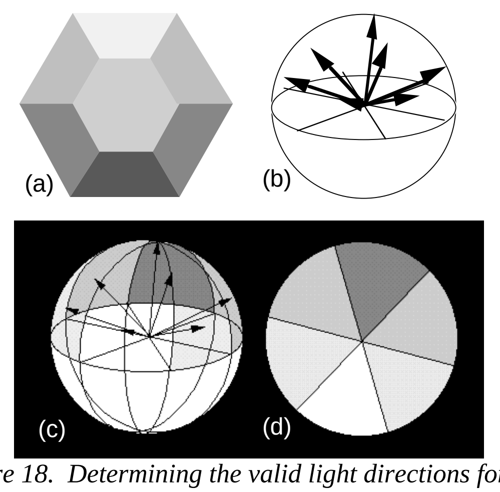

Figure 18. Determining the valid light directions for figure (a) with associated 3-D structure shown in figure (b) using the Gaussian sphere/surface normal representation. The darkest sectors in (c) and (d) represent the computed solution set. Figure (d) shows the Gaussian sphere in (c) viewed with the line of sight aligned to the polar axis.

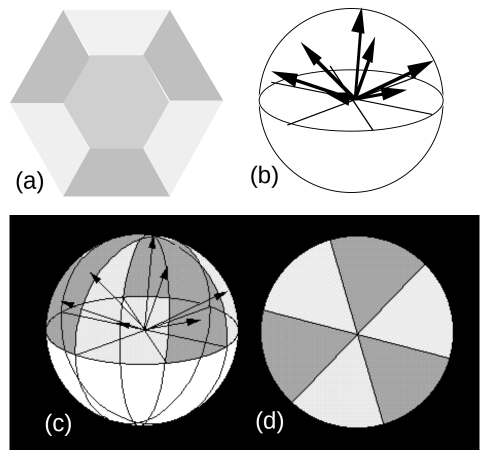

Figure 19. The system of constraints set up by the surface pairs of figure (a) do not admit a solution. No region of the Gaussian sphere satisfies all the constraints. Each of the three dark sectors of the Gaussian sphere satisfies only four of the six constraints simultaneously.

3.5. Putting it all together - a summary of the complete interpretation process:

The process begins with a local junction analysis of the image. An asymmetry exists in the confidence levels in classifying edges as being due to reflectance or illumination variations. Under the assumption of a generic view-point, the classification of reflectance edges must necessarily be correct. An edge initially classified as an illumination edge, however, might turn out to be a reflectance edge upon global analysis (as seen in an earlier example). Thus, while we can be completely confident about the reflectance labels of the local junction analysis module, the illumination labels can, at best, be taken to be tentative, to be verified by subsequent global analysis.
The next step in the interpretation involves a global analysis of the shading components. This, as described above, comprises of two basic steps: 3-D shape recovery from the geometric structure of the image and subsequently checking the consistency of the image gray-levels and the recovered 3-D structure. The solutions, besides verifying the simultaneous satisfiability of all the constraints also provide information about the valid light directions and the existence of 'compound' edges. This completes the interpretation process. A summary of all the information recovered from two simple images is shown in figure 20.

We acknowledge the fact that our model is quite certainly just a small part of the complete answer to the question of how we interpret 2-D images in terms of their shading and reflectance components. Several issues, like how to deal with occlusions and cast shadows, remain to be explored. What we hope to have accomplished in this paper is to have provided a beginning of the computational investigation of the lightness recovery problem in a 3-D domain.

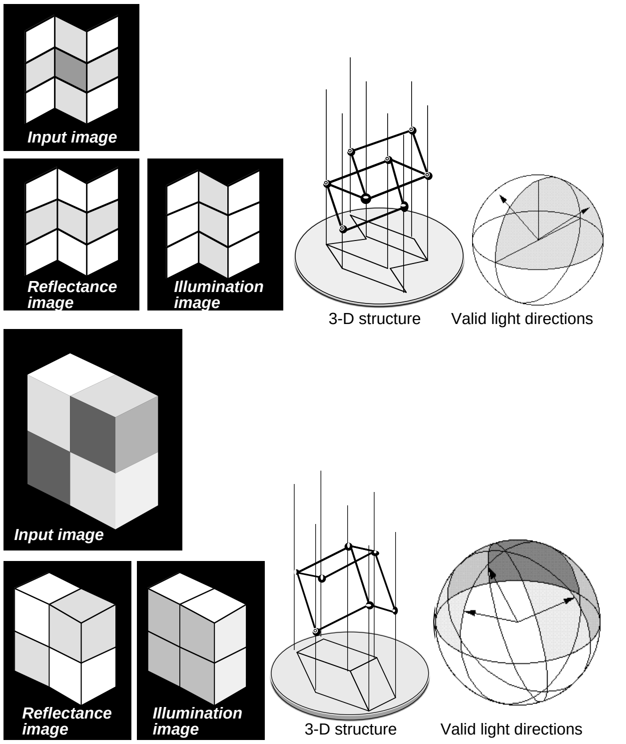

Figure 20. A summary of all the information recovered from two input images.

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