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THE USE OF CONTROL SYSTEMS ANALYSIS IN THE NEUROPHYSIOLOGY OF EYE MOVEMENTS

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INTRODUCTION

When we ask how the brain works, the question is perceived quite differently by people working at the many levels of the nervous system. Certainly it is necessary to know how the hardware of the nervous system works how transmitters affect membranes, how myelin is formed, how action potentials are conducted—but we all recognize that the solutions to these problems are not an end in themselves. They are the means that will enable us to understand how the brain processes information, which, we assume, is the substrate for behavior. If our knowledge of the connections between nerve cells and the signals that they carry were adequate, many theoreticians would be at work using the techniques of information theory, signal theory, and automatic control theory to explain the bases of such things as perception, recognition, and memory because these mathematical techniques are the tools of those who must deal with information-processing systems, of which the brain is by far and away the most complex and powerful. The fact that such theoreticians are remarkable for their scarcity simply reminds us that our knowledge of signals and connections in neural networks is, in most cases, so fragmented that such progress is impossible.

Our knowledge of signals and connections in the oculomotor system is still at a stage where much more information is needed, but, thanks to recent advances in tracing nerve cell processes and recording from cells in alert animals, coupled with certain simplifying features of the eye movement

control system, enough data are available that one can at least begin to use quantitative, analytic methods for organizing these data into circuits and systems to explain how oculomotor control is effected. Thus, one begins to see in the literature increasingly complex models of oculomotor organization and more sophisticated methods of analyzing their responses and comparing them to experimental results. In short, control systems analysis is being used more and more in the study of the oculomotor system. This review attempts to examine this rather new phenomenon and illustrate how the use of analytical techniques is employed in interpreting the wealth of new data coming from laboratories, and in guiding the course and nature of future research.

There are a number of levels at which systems analysis can be applied in the oculomotor system. The device being controlled-the eyeball and its muscles-must be described in a format suitable for systems analysis. Next, the oculomotor signals that flow about in the brainstem and cerebellum can be described with emphasis on their quantification. At higher levels of organization one may consider simple neural circuits that do rather basic signal conditioning before sending commands on to the motoneurons, and more complex circuits that go beyond the data and require additional hypothetical connections. Such proposals challenge the experimenter by providing possible explanations for a good deal of observed behavior and indicating further experiments to test the hypotheses. Finally, at the highest levels, models have been proposed to explain the overall behavior of one or several entire oculomotor subsystems. These models, however, are often filled with black boxes that are usually of little interest to the neurophysiologist since they do not suggest methods of experimental testing at the level of neural circuits. Before reviewing these levels of signal processing, however, it is interesting to examine those features of the oculomotor system that have permitted such extraordinary progress in recent years.

UNIQUE FEATURES OF THE OCULOMOTOR SYSTEM

One of the features of eye movement control that facilitates its study is the simplicity of the organization of the eyeball and its muscles (Robinson 1978). Because of the following simplifying features, it is easy to relate the discharge rate of motoneurons and other, more central, neurons directly to the motion of the eye:

- 1. The eyeball may be considered to rotate around a fixed point so there is only one "joint" in the system.
- 2. There are only two muscles to rotate the eye in any one plane.

- 3. The muscles are straight with parallel fibers so that the force of each fiber is applied directly to the globe.
- 4. The tendons wrap around the globe so that the moment arm of the muscles does not depend on eye position.
- 5. The muscles are reciprocally innervated and usually do not cocontract.
- 6. Most importantly, the eyeball is not used to apply forces to external loads, as are most other muscles; so much of the circuitry, such as the stretch reflex, required by other motor systems to deal with a wide variety of changing loads is absent in the oculomotor system.

Conceptually the oculomotor system is simple because we are able to understand what it does and why it does it. In afoveate, lateral-eyed animals there are three major oculomotor subsystems: (a) the vestibulo-ocular reflex, (b) the optokinetic system, and (c) the saccadic (or quick-phase) system. The purpose of the first two is to prevent images from moving on the retina when an animal's head (or body) turns. The vestibulo-ocular reflex senses head velocity by means of the semicircular canals and causes the eyes to move in the opposite direction, at the same speed, so that the line of sight remains constant in the visual environment. This reflex, described in detail below, enables animals to move and see at the same time. So, it is not surprising to find that the reflex is common to all vertebrates, in essentially the same form, and is even found, with modifications, in invertebrates. Because of the dynamic properties of the canals, this reflex works best at intermediate and high frequencies (0.1 to 7.0 Hz) but not at low frequencies (below 0.01 Hz). To supplement this reflex at low frequencies, the optokinetic system evolved. This system, also described in detail below, uses image slip on the retina as an error signal in a negative feedback scheme to move the eyes so as to lessen the motion of images on the retina. It is designed, not to duplicate the vestibulo-ocular reflex at high frequencies, but to complement it at the low frequency range where the canals do not operate correctly. Together these two systems allow an animal to turn slowly or rapidly, in a transient or sustained manner, while maintaining clear, stable vision by rotating the eyes in such a way that the images of the visual environment remain relatively stable on the retina.

The purpose of the saccadic system, on the other hand, is to reorient the eyes quickly in space. Since vision during saccades is poor, this system has specialized in making such eye movements very rapid to minimize the time during which vision is lost. In afoveate animals, such as rabbits and goldfish, the rapid eye movements occur as part of a coupled, programmed, eye-head reorientation. Frontal-eyed, foveate animals, such as cats, monkeys, and humans, have extended the saccadic system so that the rapid movement can also put the image of a specific target of interest onto the fovea and they

are able to make these eye movements without an associated head movement. These animals have also developed a vergence system designed to put the images of targets at various distances on the fovea of each eye simultaneously for binocular vision, and, especially in primates, they have developed a smooth pursuit system to track a moving target with smooth eye movements and keep its image relatively stationary on the fovea.

The objectives of these five major subsystems (pursuit, vergence, saccadic, optokinetic, and the vestibulo-ocular reflex) seem fairly obvious and the manner in which they achieve these objectives is so stereotyped that the function of each system can be specified mathematically. Thus, what the neural networks do is known; one is therefore free to concentrate on how they do it. This is not true for most other motor control systems, where one usually does not even know what a neural circuit is trying to accomplish, let alone how it might achieve it. Understanding what a neural system is trying to achieve is a powerful advantage in any sort of neurophysiology, one that is often not appreciated, and without which the application of systems analysis is impossible.

Experimental methods also play an important part in the recent increase in our knowledge of eye movement control. Through the work of Evarts (1968), it became possible to record from single cells in the central nervous systems of alert, behaving animals, and this technique began to be applied to the oculomotor system in the late 1960s. Because the entire oculomotor system is contained in the cranial vault, all of its circuits became accessible to exploration with microelectrodes, and such investigations, which have been going on now for over ten years, have given us a rich supply of new data. This happy situation is not yet possible for the study of the control of limb movements because recordings within the cranial vault from such structures as the motor cortex, basal ganglia, and cerebellum describe the behavior of neurons that appear to be rather distantly related to events in the spinal cord, and mechanical instability has so far prevented extensive recordings from the complex, signal-processing circuits in the spinal cords of behaving animals.

In summary, a number of features—technical, functional, and conceptual —combine in the oculomotor system to permit the gathering of large amounts of interpretable data. One is thus in a position to get on with the business of interpreting these data. Since the eye movement control system is just that—a control system—it is natural to explain its workings in the language developed over the last fifty years by those who design, describe, and analyze control systems. Thus, interpreting the data means drawing the wiring diagram and specifying the signal processing in some format, such as transfer functions.

THE OCULOMOTOR PLANT

Physiological Observations

When it became possible to record from single nerve cells in alert monkeys, the motoneuron became the obvious first target of the oculomotor neurophysiologist. It was found first that the discharge rate of these cells depended upon eye position (Fuchs & Luschei 1970, Robinson 1970, Schiller 1970, Henn & Cohen 1973). For an abducens motoneuron, for example, the discharge rate was higher the farther the monkey looked ipsilaterally; in the opposite direction, the discharge rate decreased and often became zero at some contralateral eye position. When the animal made saccades, motoneurons usually burst at high rates in association with those movements that were in the pulling direction of the muscle and were inhibited during saccades in the opposite direction. Henn & Cohen (1973) divided the motoneurons they observed into four categories: (a) tonic cells, whose firing rates were modulated with changes in eye position but not eye velocity; (b) purely phasic cells, which were very active during saccades, but not with variations in eye position; (c) predominantly phasic cells; and (d) predominantly tonic cells. The activity of the latter two types was partly phasic and partly tonic in different proportions.

To proceed from this qualitative description to a description suitable for mathematical analysis, one must describe the behavior of a motoneuron in terms of its instantaneous discharge rate, $R_m(t)$, in spikes sec⁻¹ and relate it to instantaneous eye position, E(t), in degrees, measured in the plane of action of the muscle being considered. Independent studies by Fuchs & Luschei (1970) by Robinson (1970), and by Schiller (1970) found that the behavior of all ocular motoneurons could be described by the equation

$$R_{\rm m} = R_{\rm o} + kE + r\frac{dE}{dt}.$$
 1.

When the monkey looks straight ahead, where E is defined as zero, and fixates so that eye velocity, dE/dt is also zero, the motoneuron discharges at a constant rate, R_o , with a typical value of 100 spikes/sec. If the monkey fixates (so that dE/dt remains zero) at some angle E in the pulling direction of the muscle, called the on-direction, the discharge rate increases by the amount, kE. In the opposite, or off-direction, the rate decreases by kE. This change in rate represents the change in muscle force required to oppose the elastic elements in the orbit and hold the eye in its new position. If the eye is also in motion, an extra force, proportional to velocity, is required to overcome the viscous impedances in the muscles and orbit. This force is represented by the term, r(dE/dt). The behavior of the typical motoneuron

may be found by substituting into Eq. 1 values for R_0 , in spikes sec⁻¹, k, in (spikes sec⁻¹)deg⁻¹, and r, in (spikes sec⁻¹)(deg sec⁻¹)⁻¹, that are the means of a large population of cells observed in many laboratories:

$$R_{\rm m} = 100 + 4 E + 0.95 \frac{dE}{dt}.$$
 2.

Thus, if the monkey fixates 30 deg in the on-direction, the typical motoneuron fires at 220 spikes sec⁻¹. If the monkey fixates at 25 deg in the opposite direction, the discharge rate is zero. If the eye passes through zero position at a velocity of 100 deg sec⁻¹ in the on-direction, the rate will be 195 spikes sec⁻¹.

The Transfer Function

There are large differences from cell to cell in the values of the parameters R_{0} , k, and r, which probably are related to the different types of muscle fibers found in eye muscles. The physiologist is usually drawn to examine these differences, hoping to find them sufficiently large to justify subdivisions and classifications. To understand how the eve behaves during the operation of some oculomotor subsystem, however, it is necessary to regard the eyeball and its muscles simply as a device, or physical plant, to be controlled, and one need only be able to predict how it will respond to any signal that reaches the motoneurons. For this purpose one makes the assumption that the activity of the entire motoneuron pool can be approximated by the behavior of the typical motoneuron described by Eq. 2, thereby emphasizing the similarities rather than the differences in the motoneuron pool. It is simply impractical, in actual simulation, to represent the plant with an equation for each motor unit. The approximation represented in Eq. 2 is reasonable since there are no qualitative differences in behavior from cell to cell and the distributions of R_{or} , k_{r} and r over the population are broad and flat, thus indicating that motor units cannot even be usefully divided into quantitatively different subgroups. Of course, the total force on the eye depends on the number of fibers recruited into activity and the strength of each particular fiber as well as the discharge rate, but Eq. 1 and 2 make no pretense at describing internal forces: they describe only the input and output behavior. If one specifies a given eye motion, Eq. 2 tells one what most motoneurons are doing. If one took the population distributions of R_{0} , k, and r into account, one would know what all the motoneurons were doing, but we are essentially saying that this amount of detail is unnecessary. What is most important is that if any signal reaches the motoneurons, we can predict, with fair accuracy from Eq. 2, what eye movement, E(t), it will produce.

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The practice of allowing the typical neuron to represent the activity of a pool of neurons is common in neurophysiology in general (although the pitfalls of doing this are obvious) and this is true of the oculomotor system. As we see later, there exist groups of cells, such as burst neurons, vestibular neurons, and gaze Purkinje cells, in which cells differ from one another only quantitatively, and it seems reasonable to put these cells together and regard them as forming a pool carrying a single signal—that carried by the average cell. The fact that many oculomotor signals appear to be carried in this way, at least at the premotor levels of the brain stem and cerebellum, constitutes another powerful advantage in studying the oculomotor systems. At these levels one need not worry about complex, spatial interactions between neighboring cells, as one must in, say, the inner and outer plexiform layers of the retina; one need only deal with groups of similar cells, between which flow rather simple analogue signals coded in firing rate. The practice, however, of representing the information carried in a neural pool by the signal of a typical cell does raise several issues. There is always the possibility that some target nucleus may receive fibers from only a special fraction of cells in a given nucleus and that fraction may carry a signal quantitatively rather different from the typical cell. Also, when the typical cell is driven to silence. it is no longer representative, since other, atypical cells are still transmitting a signal. In fact, the typical cell will not reflect the nonlinearities of a system with much accuracy. Thus, one must use the typical signal with caution.

Given these precautions, Eq. 2 may be said to describe the signal carried by the ocular motor nuclei. It is, however, in the form of a differential equation. The fact that it is only a linear, first-order differential equation is another, fortuitous advantage of the oculomotor system, but this form is not the most convenient one for describing how a system will respond to a variety of presynaptic signals arriving at the nucleus. While there are many ways to describe such behavior, the one that has been in most common use for the last 40 years is that of frequency analysis. If one delivers a sinusoidal stimulus of frequency s to a system that is approximately linear, the response will be also a sinusoid of frequency s. The ratio of the amplitude of the response sinusoid to the stimulus sinusoid is called the gain of the system. There will also be a phase shift between the two signals. The ratio between the response and stimulus can be conveniently represented by a complex number, G(s), which contains both the gain and phase information. The complex gain depends on frequency and is called the transfer function of the system. Its form indicates how the system will deal with stimuli of all possible frequencies. Nonperiodic stimuli are made up of sums of component sinusoids so that even for such stimuli the transfer function indicates how the system will alter the frequency components of the stimu-

lus to produce those of the response. The system may be thought of as a device that operates on its input signal to produce a different output signal.

The method of finding the transfer function of a system from its differential equation involves the use of Laplace transforms. Such mathematical manipulations are the subject of many engineering textbooks and are beyond the scope of this review, but the system at hand—the oculomotor plant—can serve as an illustrative example. Since we are mainly interested in the modulation of the discharge rate, R_m , in Eq. 1 around that in the primary position, R_o , it is convenient in Eq. 1 to replace $(R_m - R_o)$ by the modulation ΔR_m , which results in

$$\Delta R_{\rm m} = kE + r \frac{dE}{dt}.$$
3.

In these terms the transfer function G(s) for the oculomotor plant has the form

$$G(s) = \frac{E(s)}{\Delta R_{\rm m}(s)} = \frac{1}{sT_{\rm c} + 1}.$$
 4.

The most important parameter in Eq. 3 is not so much k and r themselves but their ratio, r/k, which is the time constant, T_e . This parameter describes how rapidly the eye will respond to changes in the central command. If, for example, the innervation ΔR_m suddenly changes from one value to another—called a *step command*—the eye will respond with an exponential movement with the time constant T_e . In the amount of time T_e , the eye will have traveled to within 37% (e^{-1}) of its final displacement. From the values of r and k given in Eq. 2, the value of T_e is 0.24 sec.

If the input is sinusoidal at a high frequency (s large), the transfer function is approximately $(sT_e)^{-1}$. This is the equation of an integrator: the output lags the input by 90° and for every increase in frequency of, say, a factor of ten, the output will decrease by the same factor. At a very low frequency (s small), the gain is constant (at 1.0) and the output will follow the input exactly. The ratio between the two should actually be k^{-1} , or 0.25. But a liberty has been taken in Eq. 4 by adjusting the scale factor so that the gain is 1.0 in the low frequency range, which is the range that contains the signal components of most eye movement commands. The reason for this choice is that little is known about amplification factors within the nervous system—that is, the ratio between the modulation of the typical cell in a neuron pool and that of a typical cell in a presynaptic pool—and one can avoid this problem by describing the rate modulations in terms of the physical variables they represent, in this case eye position, rather than in spikes sec⁻¹. In fact, the modulation of one motoneuron pool is not the total drive to the eye; ΔR_m should represent the difference in drive between the motoneuron pools of two antagonistic muscles, and it is simplest to express this drive in terms of the steady-state eye deviation it produces. The boundary between the high and low-frequency behavior in Eq. 4 occurs when, in the denominator, sT_e changes from being less than 1.0 to greater than 1.0. This occurs when the frequency equals $(2\pi T_e)^{-1}$. In the present case, this frequency is 0.66 Hz.

Before incorporating the transfer function of Eq. 4 into models of the oculomotor system, it was necessary to demonstrate that it describes the plant during all types of eye movements, to exclude possibilities that certain types of eye movements, such as vergence, might be made by a special subset of muscle fibers (Keller & Robinson 1972). Other studies showed that Eq. 4 also describes the plant for vestibularly induced eye movements (Skavenski & Robinson 1973) and that there was no stretch reflex that could possibly influence the nature or parameters of the transfer function (Keller & Robinson 1971). Thus, the oculomotor plant processes all signals alike, according to the transfer function in Eq. 4, regardless of the type of eye movement required. Keller (1973) found that closer inspection revealed a small relationship between R_m and eye acceleration, which becomes apparent when the eye changes velocity abruptly, as in a saccade. Thus, a better differential equation to describe the plant for high-frequency signals is

$$R_{\rm m} = 100 + 4 E + 0.95 \ \vec{E} + 0.015 \ \vec{E}, \qquad 5.$$

where \dot{E} and \dot{E} denote the first and second time derivatives of *E*. This equation may also be rewritten in the form of a transfer function:

$$\frac{E(s)}{\Delta R_{\rm m}(s)} = \frac{e^{-s\tau}}{(sT_{\rm e_1}+1)(sT_{\rm e_2}+1)}$$
 6.

where T_{e_i} is similar in value to T_e in Eq. 4 and T_{e_i} is a second, smaller time constant with a value of about 16 msec. The term containing T_{e_i} causes the eye to respond even more poorly to input signals that contain frequency components above 10 Hz. The term in the numerator is the Laplace representation of the latency or pure delay, τ , (about 8 msec, e.g. Fuchs & Luschei 1970) between changes in neuronal activity and changes in eye position. Eq. 6 describes the eye movement a bit more accurately when the input changes quickly, but, for the purposes of analyzing the behavior of some proposed model of an oculomotor subsystem, one need only choose this transfer function if its complexity is warranted by the nature of the input signals considered and the accuracy of simulation desired.

There have, of course, been reductionist attempts to relate the observed

behavior described by Eq. 4 or 6 to the mechanical elements of the globe and eye muscles (Clark & Stark 1974, Collins 1975; for a review, see Robinson 1980), but, if one is to analyze one or several entire oculomotor subsystems, the emphasis must be on obtaining as simple a description of the plant as possible, commensurate with observed behavior. Thus, all details of motoneuron size, conduction velocities, muscle fiber types, muscle force-velocity and length-tension relationships are subordinated, or placed inside a "black box," and, at least at one level, it is sufficient if we can say what the plant does in response to any stimulus (the transfer function of the black box), if not how it does it. Consequently, we may regard the study of the oculomotor plant as complete at this level and pass on to a consideration of the signals that impinge upon it.

OCULOMOTOR SIGNALS

When it became possible to record from neurons in the brain stems of alert animals (usually rhesus monkeys), many researchers began to explore the oculomotor system, and the 1970s saw a burgeoning of such investigations. In these studies the behavior of cells could be related to eye movements, but one did not know the anatomical connections of the cells so one could only guess where the signals one observed came from and where they went. Very recently it has become possible to answer these questions (e.g. Yoshida et al 1979). It is possible, but difficult, to record intracellularly from cells or axons in alert animals to discover the signal the cell or axon carries during normal eye movements and then inject a tracer that fills the cells' processes. Such methods will undoubtedly produce important results in the 1980s, but as of this writing we are left with a variety of cell types in the cerebellum and brain stem, characterized by the signals they carry, and one can only try to guess how the cell groups might be interconnected. It should be stated at once that a workable arrangement has yet to be found and it would seem that important parts of the circuit are still undiscovered.

Nevertheless, it is interesting to look at the collection of signals observed so far to at least appreciate the sorts of problems that oculomotor physiologists currently face. As one might guess from Eq. 1, because the motoneurons receive their signals from premotor neurons, the signals observed on the latter often consist, in part, of various components proportional to eye position and eye velocity. The only general rule that has emerged so far is that the eye position components (the E signal) is independent of which subsystem moved the eye. Thus, if the eye went from fixation at 5 deg left to 10 deg right, a central neuron would change its discharge rate (if it carried an eye position component) from one value to another regardless of whether the movement were a saccade, a pursuit, vestibular, or optokinetic movement. On the other hand, the velocity components can depend very much on which system commanded the movement. For example, a neuron might participate vigorously (in a manner related to eye velocity) if the movement were a pursuit movement, but not if it were a saccade, and vice versa. This behavior suggests that the various occulomotor subsystems generate their own eye velocity commands, by visual or vestibular afferent signals according to the purpose of each subsystem, and they are then added together at the input of some element that converts this sum into a single eye position command.

Fortunately, the signal components seen on most central oculomotor neurons are analogues of various physical variables coded in discharge rates. The variables are such as: E, eye position in some plane of interest (usually horizontal); \dot{E} , eye velocity; \dot{E}_{p} , eye velocity commanded by the pursuit system; \dot{E}_{r} , eye velocity commanded by rapid eye movement systems (saccades and quick phases of nystagmus); H, head position in space; \dot{H} , head velocity; G, eye position in space; \dot{G} , eye velocity in space; and \dot{e} , the velocity of image motion on the retina. The following signals are those most commonly observed on oculomotor pathways.

Burst-Tonic Cells

Burst-tonic cells are found in a variety of locations, such as the vestibular nucleus (Keller & Daniels 1975), prepositus nucleus (Lopez-Barneo et al 1979), and the interstitial nucleus of Cajal (Büttner et al 1977, King & Fuchs 1977). The term "tonic" has been used, somewhat unfortunately, to denote a discharge rate component proportional to eye position, E, just as in Eq. 1. The term "burst" comes from the vigorous discharge that occurs during saccades in a certain, preferred, direction. The actual discharge rate, however, can be approximated by

$$R_{\rm bt} = R_{\rm o} + kE + r_{\rm p} \dot{E}_{\rm p} - r_{\rm v} \dot{H} + r_{\rm r} \dot{E}_{\rm r}.$$
 7.

 R_{o} , in this as in other equations, is the discharge rate when the eye and head are stationary and the eye looks straight ahead. The term kE indicates, as in Eq. 1, that should the monkey fixate in the on- or off-direction, the neuron will increase or decrease its discharge rate. If, for example, the monkey looked 30 deg in the on-direction and k were 2.5 (spikes sec⁻¹)(deg sec⁻¹)⁻¹, the rate increase would be 75 (spikes sec⁻¹). If at any gaze angle the eyes were also moving in pursuit, the rate would change by the amount $r_p \dot{E}_p$. If, however, the eyes were moving at the same velocity during the execution of the vestibulo-ocular reflex, the extra discharge rate, $-r_v \dot{H}$, might be different even though eye velocity, \dot{E} , was equal to $-\dot{H}$, because the coefficients, r_p and r_v , describing the sensitivities to pursuit and vestibularily

induced eye velocities, can be different (e.g. Keller & Daniels 1975). Similarly, the term $r_r \dot{E}_r$ describes the additional modulation of the discharge rate related to eye velocity during rapid eye movements such as saccades and quick phases.

The coefficients r_p , r_v , and r_r are, in general, not equal except in the special case of the motoneuron. In that case, since the eye will either be making a pursuit (\mathring{E}_p) , a vestibular $(-\mathring{H})$, or a rapid (\mathring{E}_r) eye movement, and r_p , r_v , and r_r all have the same value, all the velocity terms can be replaced by a single term $r\mathring{E}$, as in eq. 1. The signal components of burst-tonic cells illustrate rather well that the individual oculomotor subsystems generate their own eye velocity commands but that they are combined before integration to produce a single eye position component. Burst-tonic cells that are not motoneurons appear to carry all of the components of the latter's signal and are probably close to the final output of the oculomotor system. It seems reasonable to suppose that many of them are a source of input to the motoneurons. These are, however, by no means the total source of such input since it is known that other cells (e.g. tvp cells, burst cells; see below) also project directly to motoneurons and provide important parts of their signal.

Primary Vestibular Afferents

The vestibulo-ocular reflex is a very important reflex common to all vertebrates. It allows animals to see and move at the same time by rotating the eyes backward in the head, when the head moves, so that the visual axes remain stationary in space. More concisely, the reflex makes eye velocity in the head, \dot{E} , approximately equal to $-\dot{H}$ so that \dot{G} (their sum), which is eye velocity in space, is kept close to zero. The signal \dot{H} is obtained from the semicircular canals. In the squirrel and rhesus monkey, within the frequency range of 0.03 to 3.0 Hz, the signal sent by the canals to the vestibular nucleus, encoded in the discharge rate R_{v_1} of the typical primary afferent fiber, is approximately

$$R_{\rm v_1} = 90 + 0.4 \ \dot{H}$$
 8.

(Goldberg & Fernandez 1971, Miles & Braitman 1980). When the head is still, the resting discharge rate is 90 spikes sec⁻¹. If the head should start to turn at, say, 100 deg sec⁻¹ in the plane of the canal, the rate will increase or decrease by 40 spikes sec⁻¹, depending on which direction the cupula of the canal deflects the haircells of the crista. Note that while head acceleration, H, is the raw stimulus that causes the endolymph to move, the hydraulics of the canal (i.e. the very large viscous resistance to endolymph flow) cause the cupula deflection, and so R_{yy} to reflect head velocity. From the

standpoint of signal processing, the canals integrate: they are stimulated by head acceleration but produce an output signal proportional to its integral, namely head velocity.

At very low and high frequencies, the canal behavior departs from Eq. 8. The major cause of this departure is the elasticity of the cupula that causes it to return slowly to its resting position after the start of a rotation of constant velocity. This causes R_{y_i} also to return exponentially to its resting rate of 90 spikes sec⁻¹ with a time constant, T_c , of about 4 sec in cat (Melvill Jones & Milsum 1971) and 5.7 sec in squirrel monkey (Fernandez & Goldberg 1971). Although one could continue to describe this behavior with differential equations, the equations would be cumbersome, especially when one considers, as we shall, two additional departures from Eq. 8, one at low and one at high frequencies. Such equations give less insight to those familiar with mathematical representations than the transfer function, which can reveal how the canals operate on input signals and produce output signals for either sinusoidal or transient stimuli. Consequently, the canal behavior is best described by the transfer function, which relates its input, \dot{H} , to its output, the change, ΔR_{y_0} of the afferent discharge rate from the resting rate:

$$\frac{\Delta R_{\rm vi}(s)}{\dot{H}(s)} = \frac{sT_{\rm c}}{(sT_{\rm c}+1)}.$$
9.

Just as with Eq. 4, the scale factor in Eq. 9 has been adjusted so that the gain is 1.0 in the high frequency range (when sT_c is larger than 1.0) and ΔR_{v_1} is no longer measured in spikes sec⁻¹ but in terms (deg sec⁻¹) of the head velocity that causes it. At lower frequencies (when sT_c is less than 1.0, which is below 0.03 Hz if T_c is 6 sec), Eq. 9 indicates that the gain is approximately sT_c . This operator describes differentiation; the gain decreases in direct proportion to a decrease in frequency and the phase shift approaches a 90° lead—the canal output now reflects head acceleration rather than velocity. The step response of Eq. 9 describes the slow return of the discharge rate to its resting level during per- or post-rotatory stimulation.

Equation 9 describes the major dynamic behavior of the canals—that they transduce head velocity over most of the spectrum of head movements but fail to do so at rather low frequencies—and may be used for many purposes in simulating systems involving the canals, such as the vestibuloocular reflex. For other situations involving very high frequencies (brief transients) or low frequencies (rotations of long duration), a more complete description is

$$\frac{\Delta R_{v_1}(s)}{\dot{H}(s)} = \frac{sT_c}{(sT_c+1)} \frac{sT_a}{(sT_a+1)} (sT_z+1)$$
 10.

(Fernandez & Goldberg 1971). The term containing T_a describes peripheral adaptation with a time constant, T_a , of 80 sec. The term containing the time constant, T_z , which has the value 0.49 sec, describes a high-frequency, phase lead term. For purposes of simulation, one may use the canal signal predicted by either Eq. 9 or 10, depending on the stimulus being considered and the accuracy required.

Second-Order Vestibular Neurons

Primary vestibular afferents may be the only neurons in the brain to carry a purely vestibular signal (i.e. Eq. 9) since all second-order cells observed so far in the vestibular nuclei of alert animals carry other signals as well, which converge on these cells from central sources. The most prevalent signal is associated with the optokinetic system. There exists a visual pathway (at least in subprimates) from the retina to the nucleus of the optic tract (Collewijn 1975, Hoffmann & Schoppmann 1975) and thence through the nucleus reticularis tegmenti pontis to the vestibular nuclei (Precht & Strata 1980). This pathway allows cells in the vestibular nucleus to be driven by optokinetic stimuli (Dichgans et al 1973, Henn et al 1974, Waespe & Henn 1977). It is shown below that this signal is proportional to head velocity as determined by the visual system, so it may be denoted by \dot{H}_{ok} . If one uses the simpler canal model of Eq. 9, the signal ΔR_{vy} , which is proportional to the discharge rate modulation of many second-order cells, can be written

$$\Delta R_{v_2} = \frac{sT_c}{(sT_c+1)}\dot{H} + \dot{H}_{ok}.$$
 11.

Experiments reveal that the signal \mathring{H}_{ok} created by rotation of an animal in the light is approximately equal to $[1/(sT_c + 1)]\mathring{H}$. In response to a sudden rotation at a constant velocity, this signal rises slowly (with time constant T_c) to a sustained level. This signal, then, just complements the canal signal, which, in this instance, rises instantly and then falls slowly back to zero. These signals are illustrated in Figure 2, where the nature of \mathring{H}_{ok} is discussed in more detail. When this signal is substituted into Eq. 11 for \mathring{H}_{ok} , one has

$$\Delta R_{v_2} = \frac{sT_c}{(sT_c+1)} \, \mathring{H} + \frac{1}{(sT_c+1)} \, \mathring{H} = \mathring{H}.$$
 12.

This equation shows how, when vision is available during head rotation, the transient or high-frequency response of the canals (first term) is supplemented by the sustained or low frequency optokinetic response (second term) so that the total signal carried by those vestibular neurons is proportional to head velocity at all frequencies from the lowest (including zero) to the highest. In connection with Figure 2, below, it is shown that \dot{H}_{ok} does not depend entirely on vision, but can affect the vestibulo-ocular reflex even in the dark.

Tonic Cells

Tonic cells are found in the reticular formation in the region of the abducens nucleus (Keller 1974) and the prepositus nucleus (Lopez-Barneo et al 1979). These cells carry a signal component proportional to eye position but do not burst during saccades, which may suggest that they do not carry any eye velocity signals at all. Closer inspection, however, shows that some tonic cells do have an additional modulation during pursuit (\dot{E}_p) and vestibularly induced movements $(-\dot{H})$. Thus, their discharge rate, R_t , may be described by

$$R_{t} = R_{o} + kE + r_{p}E_{p} - r_{v}H.$$
13.

This equation is similar to Eq. 7 for burst-tonic cells, except that the burst term, $r_r E_r$, is missing. For some cells, however, r_p and r_v are zero so that such cells do reflect only eye position with no eye velocity signal components. In the reticular formation these cells are evidently small and hard to hold with a microelectrode and so have not been adequately studied. This is unfortunate since they may represent the output of an important element called the neural integrator, to be described subsequently.

Burst Cells

There are cells scattered in the pontine and mesencephalic reticular formations that discharge vigorously during saccades or quick phases with components in some particular direction and are otherwise silent. There are a variety of such cells (Luschei & Fuchs 1972). Long-lead burst cells discharge well in advance of an impending saccade and their discharge rate is usually poorly correlated with any specific aspect of the eye movement, such as saccade size or eye velocity. On the other hand, medium-lead burst neurons discharge at rates that are clearly related to rapid eye velocity in a certain direction (Keller 1974, Cohen & Henn 1972, van Gisbergen et al 1981). Thus, the behavior of their discharge rate, R_r , may be roughly approximated by

$$R_{\rm r} = r_{\rm r} \dot{E}_{\rm r}.$$
 14.

These cells do not discharge during other types of movements, such as pursuit or fixation. Closer inspection of instantaneous discharge rate (van Gisbergen et al 1981) confirms that R_r is also related to eye acceleration, as Eq. 5 would suggest, and is also influenced by either a nonlinearity or a nonstationarity in the plant. The evidence to date indicates that mediumlead burst neurons contact motoneurons monosynaptically (Igusa et al 1980), cause a burst in motoneurons (and burst-tonic cells), and through them create saccadic eye movements. Since these burst neurons discharge at about 1000 spikes sec⁻¹ during large saccades, when eye velocity is near 1000 deg sec⁻¹ (in the monkey), r_r has a value of roughly 1.0 in that animal.

Pause Cells

There are cells in the reticular formation, especially clustered near the midline at the level of the abducens nerve rootlets (Keller 1974, Raybourn & Keller 1977), that fire at a fairly constant rate but pause during all rapid eye movements. Thus, their discharge, $R_{\rm p}$, might be expressed,

$$R_{\rm p} = R_{\rm o} - r_{\rm ps} |\dot{E}_{\rm r}|.$$
 15.

The absolute value sign around E_r indicates that inhibition occurs for saccades in any direction, a characteristic that causes these cells also to be called omnidirection pause cells. Of course, as in all these equations, R cannot be negative. It is currently thought that pause cells inhibit burst cells and that the former must be turned off to allow the latter to create saccades (Keller 1977, King & Fuchs 1977).

Tonic-Vestibular-Pause Cells

A subset of cells in the vestibular nucleus send their axons via the medial longitudinal fasciculus (mlf) to the oculomotor nucleus to complete the vertical vestibulo-ocular reflex. The discharge rate, R_{tvp} , of the typical cell is,

$$R_{\rm tvp} = 130 + 2.5 \ E + 0.47 \ \dot{E}_{\rm p} - 0.98 \ \dot{H} - |\dot{E}_{\rm r}|$$
 16.

(King et al 1976, Pola & Robinson 1978). This equation came from recordings from the fibers of these cells in the mlf, but, subsequently, many similar cells have been observed in the vestibular nucleus (Lisberger & Miles 1980) with activity related to horizontal as well as vertical movements. The term "tonic" refers, as usual, to the eye position signal, 2.5 E, "vestibular" refers to the signal component, -0.98 H, and the last term indicates that the cell pauses during all rapid eye movements; hence the name tonic-vestibularpause, or tvp. Most, if not all, of these cells are also second-order vestibular neurons and so constitute the middle portion of the three-neuron arc: the backbone of the vestibulo-ocular reflex. The surprising feature about the behavior described by Eq. 16 is that eye movement signals (2.5 *E*, 0.47 $\dot{E}_{\rm p}$ and $-|E_{\rm r}|$) emerge from some central structure to converge on these second-order vestibular cells, thereby starting the process immediately, in the vestibulo-ocular reflex, of converting the canal signal (Eq. 8) to the motor signal (Eq. 2).

Gaze-Velocity Purkinje Cells

There is a class of Purkinje cells in the monkey flocculus that discharges in relation to eye velocity in space, \mathring{G} , (Miles & Fuller 1975, Lisberger & Fuchs 1978). The discharge rate, $R_{\rm gPc}$, of the typical cell is

$$R_{\rm gPc} = 79 + 0.9 \, (\dot{H} + \dot{E}) = 79 + 0.9 \, \dot{G}.$$
 17.

When the monkey makes pursuit movements with the head still (\mathring{H} zero), the rate modulates in proportion to eye velocity, \mathring{E} . When the monkey is rotated but cancels its vestibulo-ocular reflex by fixating a target rotating with it (\mathring{E} zero), the rate modulates with head velocity, \mathring{H} . During head rotation in the dark, when the vestibulo-ocular reflex causes \mathring{E} to be about -0.9 \mathring{H} , the modulation falls almost to zero. Since the sum of \mathring{H} and \mathring{E} is \mathring{G} , the activity of the cell reflects eye velocity in space.

Retinal Image Slip

Several oculomotor subsystems are designed to prevent images from slipping about on the retina due to self motion or the motion in space of visual targets, presumably to improve vision. There are cells in the retinas of animals, such as rabbit and cat, called direction-selective cells, that respond to retinal image slip (e.g. Oyster et al 1972). These cells discharge most vigorously when images slip across the retina in one particular direction and the discharge rate is then a function (usually nonlinear) of the slip velocity, \dot{e} . Thus, the cells carry information of both the direction and speed of the retinal slip. A rather oversimplified, linear, description of the discharge rate, R_{ds} , of these cells in their direction of sensitivity is

$$R_{\rm ds} = R_{\rm o} + a\dot{e} = R_{\rm o} + a(\ddot{W} - \dot{G}),$$
 18.

where a is some constant of proportionality. The second half of Eq. 18 simply expresses the fact that the velocity with which images move across

the retina (\hat{e}) is the difference between the velocity of visual objects in space and the velocity of the eye in space or gaze (G). The optokinetic system is mainly concerned with the motion of the entire visual environment relative to the observer so that the velocity of the visual objects in space in that case is the velocity of the seen world, W. In nature, the entire seen world never moves en bloc so that \hat{W} is normally zero and retinal slip. \hat{e}_{i} is created by motion of the eye in space (\dot{G}). For a subject inside an optokinetic drum, W is the drum velocity. The pursuit system of foreate animals is concerned with motion of objects moving within the visual environment. In that case, \dot{e} refers to the retinal motion of the image of a particular, selected target to be tracked and \hat{W} should be replaced by \hat{T} , the velocity of that target in space. For the optokinetic system, the signal \dot{e} is relayed from the retina to the nucleus of the optic tract in cats and rabbits. In primates, the retina apparently does not produce an \dot{e} signal and the role of cortical and subcortical visual pathways in generating the optokinetic \hat{e} signal is not yet clear. In such foveate animals, however, the striate cortex appears to be essential in generating the \dot{e} signal for pursuit.

A Synthesis

The problem that remains is to propose neural circuits containing cells that carry the above signals and that also explain the overall organization of eve movements. Such proposals will become the working hypotheses to be shaped by subsequent anatomical and physiological findings until the circuits of the oculomotor system are correctly understood. In some oculomotor subsystems, considered below, this hope is close to realization; in others, much more study, theoretical as well as experimental, is needed. For example, many of the intermediary signals are still missing, as evidenced by the continual discovery of new oculomotor nuclei and the discovery of cells carrying new combinations of signal components in well-known nuclei (e.g. the vestibular nuclei), as well as the more newly discovered nuclei. Many of the cell groups described above (e.g. tonic cells) have not been studied in sufficient detail to allow quantitative relationships to be established with other cell groups. Some of the signal components described above are rather nonlinear and have been approximated as linear only for simplicity of discussion. Consequently there is still much guess-work in proposing which cell groups drive which other cell groups.

There are a few connections that seem likely. It is probable, for example, that burst-tonic cells receive much of their input from burst cells and tonic cells, although the appropriate anatomical connections are not yet established. On the other hand, there are puzzles: Purkinje cells in the flocculus, on anatomical evidence, are generally believed to inhibit cells in the vestibular nucleus; yet, in the monkey, no cells in the latter nucleus have been

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observed, so far, that appear capable of receiving the signal carried by gaze-velocity Purkinje cells described by Eq. 17. There are many similar problems and new data will be required to solve them. Of course, as one works backwards from the motoneuron in the visual-oculomotor subsystems, one soon reaches the interface between the motor and the sensory systems. In some cases, as in the optokinetic system, the action of the visual system is easy to describe, as in Eq. 18, and its link to the oculomotor system is easy to guess (see below). In the saccadic system, on the other hand, the signal given to the brain-stem saccadic circuits comes from a process of visual pattern recognition and cognitive target selection that is not understood at all. Furthermore, the specification of target locations is coded retinoptically. That is, in both the visual cortex and superior colliculus, the location of a visual target, with respect to the fovea, is indicated by which cells are active in these structures, according to the well-established retinoptic maps. Yet the final saccadic command, represented by the activity of burst neurons (Eq. 14) is a temporally-coded signal (intensity of discharge rate for a desired duration). How the transformation takes place between retinoptic specifications of target location by the position of a cell within a population, regardless of the exact nature of its discharge rate, to the specification of saccade size by the time course of the discharge rate of a cell, regardless of its exact location in a pool of similar neurons, is one of the major problems in understanding the oculomotor system. Consequently, one can only expect to proceed centrally just so far with the type of analogue signals listed above before coming to a point where the coding of signals changes and becomes more complicated.

A discussion of the details of the problems one encounters in trying to fit all of the above signals into a hypothetical network that describes the flow of the signal processing in the premotor circuits is beyond the scope of this review. Equations 1 through 18 are presented only to represent the state of our current knowledge of signals in the oculomotor system and to emphasize that the recent progress in oculomotor physiology has brought us close to one of the final goals of neurophysiology—understanding how the nervous system processes signals to produce behavior.

SIMPLE PREMOTOR CIRCUITS

If we are not yet certain how the cells described above are interconnected to perform their tasks, we at least know what those tasks are. Consequently, one can model parts of the oculomotor system at a higher level of organization in which relatively simple neural networks are represented by a transfer function that describes what must be done, although we do not yet understand just how it is done. One reason for proceeding in this way is to make

it very clear, in the unambiguous language of mathematics, just what signal processing must be done by the neural networks so that one can then propose hypothetical networks that can be tested experimentally. Another reason is to permit the oculomotor theorist to use such premotor circuits as building blocks in efforts to model larger sections of the system, including one or more entire oculomotor subsystem. Such considerations are best illustrated by example.

The Vestibulo-Ocular Reflex

The vestibulo-ocular reflex, shown in Figure 1, is a good example of modeling at this level. As described earlier, this reflex causes the eyes to rotate in the direction opposite to a head rotation so that the direction of gaze in space is kept constant and the location of images of the visual environment on the retina are not perturbed by head motion. The dynamic behavior of this reflex has been well studied and the transfer function that relates eye position in the head, E, to head position in space, H, is

$$\frac{E(s)}{H(s)} = -g \frac{sT_{\text{vor}}}{(sT_{\text{vor}}+1)} \frac{sT_{\text{a}}}{(sT_{\text{a}}+1)}.$$
19.

The term containing T_a represents the peripheral adaptation of the canals already encountered in Eq. 10. The term containing T_{vor} is related to the behavior of the cupula and is similar to the term involving T_c in eq. 10. When a subject is rotated in the dark at a constant velocity, slow-phase eye velocity decreases exponentially with the time constant T_{vor} . The difference between T_c and T_{vor} will be explained shortly. At frequencies for which sT_a and sT_{vor} are both larger than 1.0 (above about 0.03 Hz), Eq. 19 has a value close to -g: the gain of the reflex (the minus sign simply indicates that eye motion is opposite to head motion). If g were 1.0, the eye movements would be perfectly compensatory. In cat and monkey, g is about 0.9. In man, it is around 0.6 during mental arithmetic, but rises to 0.95, even in the dark, if the subject tries to use the reflex by looking at imagined targets (Barr et al 1976). The gain, g, is also under some form of adaptive control (not shown), in which the cerebellum plays some part, to calibrate the gain, g, just after birth and maintain it during growth, disease, and aging (Ito et al 1974, Robinson 1976).

Since the behavior of the canals is known (Eq. 10), the behavior of the plant is known (Eq. 6), and the overall behavior is known (Eq. 19), one can deduce what must occur in the central pathways of the reflex and this is illustrated in Figure 1. The plant transfer function (Eq. 6) is shown on the right in Figure 1. The canal transfer function, on the left, has been modified

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Figure 1 A model of the vestibulo-ocular reflex. The transfer function of the semicircular canals (*left*) and eyeball (*right*) are shown just below as transformations A and D. Transformation B describes how the time constant of the cupula, $T_{\rm ex}$, is replaced by the larger time constant, $T_{\rm vor}$, of the vestibulo-ocular reflex. Transformation C describes the neural integrator (NI) in parallel with a direct signal path which effectively cancels the main lag of the plant ($T_{\rm e_1}$). The bottom equation is the product of these four transformations and the overall gain factor -g. The terms in the square brackets affect behavior at high frequencies (greater than 3 Hz) and their effects approximately cancel out. Figures at the top illustrate how a neural pulse from burst neurons (B) is integrated to produce a step on tonic cells (T) so that the motoneurons ($\Delta R_{\rm m}$) can transmit a pulse-step waveform to create a saccade. At upper left are shown a normal saccade (N) and three types of abnormal saccades commonly seen in the clinic.

by abandoning the notation of discharge rate, ΔR_{v_1} in Eq. 10 and denoting the canal output as an internal signal, \dot{H}_c , reflecting head velocity, as reported by the canals. Also, head position, H, is used as the input instead of head velocity, H (or, in operator notation, sH). As Figure 1 indicates, there are two major signal transformations that occur in the central pathways. The first step (transformation B, Figure 1) creates an apparent increase in the cupula time constant. It is known, in the cat, that the cupula time constant, T_c , is about 4 sec (Melvill Jones & Milsum 1971), but the main time constant of the entire reflex, T_{vor} , is about 15 sec (Robinson 1976). A similar situation is seen in the monkey: T_c is about 6 sec (Fernandez & Goldberg 1971), but Tvor is around 16 sec (Waespe & Henn 1977, Buettner et al 1978). Waespe & Henn (1977) also discovered that the transformation of the major system time constant from $T_{\rm c}$ to $T_{\rm vor}$ occurs directly on second-order vestibular neurons so it must be effected by a signal that converges on those neurons from some central source. The signal is indicated as \dot{H}_{ok} in Figure 1 because, as is argued below, there is good

evidence that this signal is associated with the optokinetic system when vision is available, as well as with the vestibular system, and it is the same signal that appears in Eq. 11. The effect of the signal \dot{H}_{ok} , in the dark, is to create the transfer function marked *B* in Figure 1. When this function is multiplied by the transfer function for the canals (marked *A* in Figure 1), the terms containing T_c cancel out so that the resultant contains only the parameter T_{vor} . Consequently, the behavior of the overall reflex does not reflect the canal time constant, T_c , but a larger time constant, T_{vor} , created by central signal processing. Thus, transformation B simply describes what must be done to account for the experimental observations. How this transformation might actually be accomplished by neural circuits is described when the optokinetic system is discussed. In any event, the result of the transformation by \dot{H}_{ok} is to create an improved representation of head velocity, \dot{H}' , which, after multiplication by -g, becomes an eye movement command, \dot{E}' .

The second, and more important, transformation of the central pathways creates the motoneuron signal (Eq. 1) from the eye-velocity command, $\vec{E'}$. The eve-velocity component in Eq. 1 indicates that there must be a direct projection of the canal signal, \vec{E}' , to the motoneurons. The origin of the eye position signal in Eq. 1 requires more explanation. The signal, in the dark, must obviously be created from the eye velocity signal, the only one available, which can only be done by integrating it—that is simply a restatement of the experimental observations-but how and where the integration is done is not known. Tonic cells (T, Figure 1; Eq. 13) mainly carry an eye position signal and could represent the output of the neural integrator (NI, Figure 1). They are located in the paramedian pontine reticular formation and this region is known to be vital for eye movements based on the results of lesions (Goebel et al 1971). The cerebellum is also necessary for correct operation of the integrator (Carpenter 1972, Robinson 1974), particularly the flocculus (Zee et al 1978). It would appear that the integrator is formed by some neural circuit involving links between the vestibulocerebellum and the pontine reticular formation. Various neural models have been proposed for the integrator (e.g. Kamath & Keller 1976) but they remain speculative.

The transfer function of the integrator shown in Figure 1 is that of a leaky integrator with a time constant T_n . The transfer function of a perfect integrator is 1/s (T_n infinitely large). Such an integrator would store the integral of a transient input signal and produce a constant output indefinitely in the absence of any new input. A leaky integrator would not hold a signal indefinitely; its output in the absence of any new input would slowly return to zero, exponentially, with the time constant T_n . In the normal situation, T_n is about 25 sec (Becker & Klein 1973), which is so large that for most practical purposes the integrator may be regarded as essentially

perfect. If we make this assumption, the parallel combination of direct and integrator path in Figure 1 has the transfer function,

$$\frac{\Delta R_{\rm m}}{\dot{E}'} = T_{\rm e_{\rm i}} + \frac{1}{s} = \frac{sT_{\rm e_{\rm i}} + 1}{s}$$
 20.

and this is shown as transformation C in Figure 1. To produce the correct ratio between the \dot{E}' and E' signals at the motoneuron, the gain of the direct path must be $T_{\rm el}$. In terms of transfer functions, this gain causes the numerator in Eq. 20 to cancel the term in the denominator of the plant transfer function which contains its major time constant, $T_{\rm el}$. In this way the sluggishness of the plant, which would otherwise fail to respond appropriately to any oculomotor signals in the frequency range above about 0.7 Hz, is compensated so it does not interfere with the vestibulo-ocular reflex.

When all the transfer functions, A to D in Figure 1, are multiplied together, the overall function, shown at the bottom, results. This equation differs from the experimentally determined behavior (Eq. 19) by the terms in the square brackets, all of which affect performance at high frequencies (above about 3 Hz). Presumably these terms more or less cancel out since observations indicate that up to 7 to 8 Hz, which seems to be the upper limit for naturally occurring head movements, the gain is relatively independent of frequency and the phase shift is small. Thus, the descriptions of the signal processing done by the various sensory, motor, and central parts of the reflex in Figure 1 appear to be correct even though, in the two central transformations, we do not know the exact neural circuitry. It must be kept in mind that Figure 1 only describes what is done centrally, not how it is done. It shows, for example, the \dot{E}' and E' signals arriving at the motor nucleus by separate pathways, yet Eq. 16 shows that the middle leg of the three-neuron arc-from second-order vestibular neurons to the motoneurons—already carries an eye position signal (2.5 E) in addition to the vestibular eye-velocity signal (-0.98 \dot{H}). Presumably the integrator sends part of the E signal back to the vestibular nucleus to join the eye velocity command, in addition to a direct projection to the motoneurons (Pola & Robinson 1978). Thus, Figure 1 does not propose an actual neural wiring diagram. No doubt future research will delineate the actual circuit, but even then there will be some utility in the diagram in Figure 1 since it presents an equivalent circuit that indicates most clearly what the real circuit does. Such a model of the vestibulo-ocular reflex is also useful for models of more complex eye movement systems in which this reflex plays only a part.

Saccades

Figure 1 also suggests how rapid eye movements (saccades or quick phases) are made. Although we do not understand how visual targets are selected and their coordinates sent to the lower motor machinery, it does seem possible to propose a scheme for the generation of saccades in premotor circuits close to the motoneuron. As already mentioned, burst cells, described roughly by Eq. 14, also produce an eye velocity command just as do the semicircular canals. Because of the shape of the burst, it produces a high-velocity movement of short duration rather than the slower, smoother movement of the vestibular signal. In order to translate the burst into a saccade, the motoneurons, according to Eq. 1, must receive both the velocity command (rE) directly, and its time integral, the eye position (kE), to hold the eye in its new position after the saccade. In this case, as shown by the wave forms at the top of Figure 1, the velocity command is the burst, B, which must project directly to the motoneurons. The integral of the burst is the step seen on tonic cells (T), which are presumed to project also to the motoneurons. The latter then carries the sum of the two signals, the burst-step shown by the waveform $\Delta R_{\rm m}$ in Figure 1. Thus, exactly the same signal processing-a direct and an integrating pathway having the same relative gains—is needed as in the case of the vestibulo-ocular reflex and the question arises as to whether the two systems share a single integrator, as shown in Figure 1. Such an arrangement would certainly seem economical, but there are also fairly good experimental and theoretical reasons to indicate that it is correct (Robinson 1975). Separate integrators would require that there exist cells that are modulated in proportion to Efor certain types of eye movements, such as quick phases, but not for others, such as slow phases. Yet, as already pointed out, all the cells described above carry a signal that reflects eye position regardless of the type of movement that carried the eye to that position. Such cells always reflect, for example, the change in eye position created by both the slow and fast phases of nystagmus. This same argument indicates that all other conjugate eve movement systems, such as the pursuit system, also share a common integrator. Optokinetic movements obviously also share this integrator since, as indicated by Eq. 11, their velocity command leaves the vestibular nucleus along with the canal signal.

Several pathological eye movements can be explained by the scheme shown in Figure 1. If the neural integrator is abnormally leaky (if T_n has decreased to, say, 2 sec), as occurs with cerebellar degeneration, the eyes of a patient will drift back from an eccentric position exponentially with the time constant T_n . To maintain eccentric fixation, the patient must make repeated eccentric saccades creating a pattern called gaze nystagmus (waveform GN, Figure 1). Occasionally the transmission of the pulse or step to the motoneurons is affected by some disease with the result that the pulse is too large or too small for the step. In that case, the pulse initially carries the eyes beyond, or causes it to fall short of, the steady-state position commanded by the step, to which the eyes then drift exponentially with the time constant T_{e_1} (Easter 1973, Bahill et al 1975b). The result is an overshoot or undershoot saccade shown by the waveforms OS and US in the upper right of Figure 1. Such waveforms do not occur normally because the size of the step, relative to the pulse, is adaptively adjusted by some cerebellar-dependent mechanism to eliminate such post-saccadic drifting movements (Optican & Robinson 1980). Over- and undershoot saccades are seen in patients only when the result of the lesion is beyond the capabilities of this repair mechanism. The saccade circuit in Figure 1 is useful, then, in that it offers a mechanistic explanation for a number of eye movement disorders frequently seen in the clinic or in lesioned animals (Zee & Robinson 1979a).

The circuit in Figure 1 is, at the moment, a useful description of some of the elementary signal processing that occurs immediately prior to the motor neurons. It must, however, be regarded as a working hypothesis until the actual neural circuits have been determined. Certainly, one of the major issues raised by Figure 1 is the neural integrator: Where is it and how does it work?

COMPLEX PREMOTOR CIRCUITS

It is the nature of neurophysiology that experimental data are difficult to obtain and usually fall far short of what one needs to explain the behavior of some neural system with any degree of certainty. Yet the desire to explain at least some aspect of neural behavior, however scant the evidence, leads the curious and frustrated investigator to try to extrapolate from the meager data available and the function, if known, of a particular system, and to propose hypotheses for how such a system might be wired together. Such a hypothesis usually consists of a specific circuit topology and the transfer functions required to produce the observed responses given the appropriate stimuli. To be useful, the hypothesis should be quantitative so that it can be tested by solving its equations-usually by computer simulation-to verify its predictions numerically. Such a mathematical hypothesis is usually called a model. The plausability of a model is related to the number of experimental observations it can simulate and the number of assumptions it requires. The main usefulness of a model is to make predictions that can be tested experimentally. If verified by sufficient testing, the model becomes an accepted theory for explaining the system's known behavioral repertoire.

In oculomotor physiology, it is interesting and useful to try to guess how

the visual system may project to the premotor circuit in Figure 1 and use it to effect visually guided eye movements. It is in the invention and testing of such models that the concepts and practice of the analysis of control systems plays an important part. Despite all the transfer functions in Figure 1, the signal processing in that circuit is rather simple and there are not even any feedback loops. In visually guided movements, feedback plays a large role and models for their control utilize the concepts and analysis of feedback. Perhaps the simplest visually guided system is the optokinetic system. Recent experiments have provided enough clues to allow us to make a good guess about how this system is wired together.

Models of the Optokinetic System

The scheme in Figure 2 illustrates the general format for all models proposed for the optokinetic system and, inside the dashed lines, a specific model for its central processing. The vestibulo-ocular reflex is included because it and the optokinetic system are so intertwined in structure and function that it is not useful to consider one without the other. For present purposes, however, it may be greatly simplified from the system shown in Figure 1. It is sufficient to describe the canal dynamics by Eq. 9 and all the elements affecting the responses at very high and low frequencies may be ignored. The optokinetic system is only concerned with the velocity, not the position, of eye, head, and retinal images. Consequently, by using eye and head velocities, rather than positions, as the variables of interest, the action of the neural integrator need not be shown explicitly. The actual eye response is often nystagmoid but the quick phases, which only change eye position, may also be ignored and eve velocity may be taken as that of the slow phases. The summing junction on the right in Figure 2 expresses the fact, already indicated in Eq. 17, that eye velocity in space, \dot{G} , is the sum of eye velocity in the head, \vec{E} , and head velocity in space \hat{H} . The summing junction on the left indicates, as in Eq. 18, that the rate of image slip on the retina, e, is the difference between the velocity of the visual environment, \dot{W} , (usually zero) and eye velocity in space, \ddot{G} .

The path \mathring{G} indicates that the optokinetic system is a negative feedback system. Retinal slip is an error signal and the function of the feedback system is to try to keep eye velocity in space, \mathring{G} , equal to the velocity of the visual world, \mathring{W} , which will then minimize the error \mathring{e} . In normal situations, of course, the visual world does not move. The optokinetic system did not evolve to track a moving visual world, but the visual world always moves relative to the head when an animal rotates in space and it is this situation with which the optokinetic system was designed to deal. As the waveforms in Figure 2 illustrate, when an animal begins to rotate at a constant velocity, the vestibulo-ocular reflex initially generates compensa-

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Figure 2 A model of the optokinetic system (OKS) and its connection with the vestibuloocular reflex. As shown by the waveforms, the optokinetic signal (\dot{H}_{ok}) is inserted in the vestibular nucleus (vn) and supplies the sustained activity during rotation at a constant velocity (\dot{H}) while the canal supplies the transient activity (\dot{H}_{o}) . The general transfer function between \dot{H}_{ok} and retinal slip velocity (\dot{e}) in the nucleus of the optic tract (not) is characterized by a gain G_{ok} and a time constant T_{okan} . The specific circuit shown in OKS utilizes a corollary discharge path $(k\dot{E}')$ from the vn to the nucleus reticularis tegmenti pontis (nrtp) and thence back to the vn, forming a positive feedback loop. Switch S_1 illustrates how going from light (L) to dark (D) opens the feedback loop \dot{G} .

tory eye movements but, as the cupula returns to its resting position, the canal signal \mathring{H}_c falls back to zero. As it does, eye velocity is no longer adequate, so a retinal slip is created that activates the optokinetic system and produces a rising signal, \mathring{H}_{ok} , that supplements \mathring{H}_c . Their sum, \mathring{H}' , is thus a much better approximation to \mathring{H} than is \mathring{H}_c and eye velocity will be appropriate for the sustained, as well as the transient, portion of the rotation. It is this action that is described by Eq. 12.

The fact that the optokinetic signal appears on second-order neurons in the vestibular nucleus (vn, Figure 2) to augment the canal signal is discussed above in connection with Eq. 11. The visual input, \dot{e} , has been traced to the nucleus of the optic tract (not) in the pretectum in rat, cat, and rabbit (Cazin et al 1980, Hoffman & Schoppman 1975, Collewijn 1975), and this structure was shown to be essential for optokinetic responses. The situation remains unexplored in primates where there is clearly a cortical involvement in optokinetic responses (Atkinson 1979). The question has been: How does the signal \dot{e} become transformed into \dot{H}_{ok} and where are the neural pathways? First, one can at least characterize the nature of the $\dot{e}-\dot{H}_{ok}$ transformation. As indicated by the equation in Figure 2, it is largely described by a gain, G_{ok} , and a time constant, T_{okan} . These parameters have been determined experimentally. Although \dot{W} is usually zero, it is convenient to study

the optokinetic system in the laboratory if the animal or subject is stationary, in which case the visual environment is made to move by enclosing the subject inside a rotating drum. In this case, drum velocity, \dot{W} , may be considered the input, eye velocity, \dot{E} , the output, and the ratio \dot{E}/\dot{W} as the gain—more technically, the steady-state, closed-loop gain—of the system. Experimentally, this gain is about 0.7 to 0.8 depending on drum speed and species. The gain $\dot{H}_{\rm ok}/\dot{e}$ of the forward path (the part enclosed in the dashed lines in Figure 2) is the parameter $G_{\rm ok}$, and the relationship between it and closed-loop gain, \dot{E}/\dot{W} , is

$$\frac{\dot{E}}{\dot{W}} = \frac{G_{\rm ok}}{1+G_{\rm ok}}.$$
21.

 $G_{\rm ok}$ must be about 3 to 5, according to this equation if eye velocity is to be 70 to 80% of drum velocity. The other feature of the optokinetic system is a lag with a time constant of $T_{\rm okan}$. The value of $T_{\rm okan}$ may be measured by driving the system to a steady state by rotating the drum at a constant speed and then turning off the lights. This opens the feedback loop, as suggested by the switch S_1 in Figure 2, since the retina can no longer transmit \dot{e} to the system. In this situation, nystagmus, called optokinetic after-nystagmus (OKAN), continues due to the activity stored in the lag element and \dot{E} slowly falls back to zero. If one approximates this decline by an exponential, its time constant, $T_{\rm okan}$, is about 15 to 20 sec (Cohen et al 1977).

The specific circuit shown in the box marked OKS in Figure 2 is only one of several models proposed for the optokinetic system. The simplest, topologically, is that the \dot{e} signal projects from the not to the vn by a feed-forward path with a transfer function characterized by G_{ok} and T_{okan} as just discussed (Collewijn 1972, Schmid et al 1979). Specifically, such models do not contain internal, feedback pathways such as the one marked kE' in Figure 2. Such models deal reasonably well with responses to stimulation by optokinetic drums but fail to reflect important interactions with the vestibular system. These models cannot, for example, account for the transformation of the main vestibular time constant from T_c to T_{vor} described as transformation B in Figure 1. In contrast, the studies of Cohen et al (1977) and Raphan et al (1979) revived the old ideas of ter Braak from the 1930s (e.g. Rademaker & ter Braak 1948) that there was a common circuit-called a velocity storage element by Cohen and Raphan-that was shared by the optokinetic and vestibular systems and that carried a signal proportional to the nervous system's current estimate, H', of head velocity based on both visual and vestibular information. The signal in this element,

if one likes functional interpretations, may be thought to represent the action of the inertia of the body: When the circuit is excited, it perseverates that activity, in the absence of new information, according to Newton's First Law of Motion (a body set in motion continues moving at a constant velocity unless acted upon by a force). In approximating such behavior, the circuit creates OKAN and transforms T_c into T_{vor} . This notion of a common velocity storage element would account naturally for the many similarities between the responses of the vestibulo-ocular reflex and the optokinetic system (Takemori 1974). For example, it explains why, in most circumstances, T_{okan} and T_{vor} are equal.

Two models have been proposed for this storage element. One suggests the existence of a storage element with a time constant of T_{vor} (or T_{okan}) that was fed by the primary vestibular afferent signal (H_c) and sent its output (\dot{H}_{ok}) to the vn forming a feed-forward path in parallel with the primary afferents (Raphan et al 1979). The alternate model shown in Figure 2 uses a positive feedback loop to achieve the same result (Robinson 1977). This model is discussed in more detail because it illustrates how simple properties of feedback can be used in analyzing the behavior of a model. The rationale of the model is that the visual system desires to augment the canal signal by determining head velocity independently. To do this in Figure 2, a copy of eye velocity, \dot{E} , (k is close to 1.0), is added to the velocity of the world with respect to the eye, e, to recreate the velocity of the world with respect to the head, $\dot{W}_{\rm h}$. This positive, internal feedback is similar to the efference copy notion of von Holst & Mittelstaedt (1950) and the corollary discharge of Sperry (1950) that has stimulated the ideas of Young (1977) in applying it to the oculomotor system. Since the visual world is presumed stationary, $-W_h$ is taken by the nervous system to be the velocity of the head in space. This signal is denoted by \dot{H}_{v} to indicate that it is head velocity according to the visual system. If this signal is to be used to augment the canal signal at low frequencies, its high frequency components must be removed, by the lag element $1/(sT_0 + 1)$ in Figure 2, so as not to duplicate the canal signal in the high-frequency range. For this purpose T_0 must have a value close to $T_{\rm c}$.

Recent evidence suggests that this model might resemble the actual neural circuit. It has been discovered that the nucleus reticularis tegmenti pontis (nrtp) is a major relay station between the not and the vn in rat and cat (Cazin et al 1980, Precht & Strata 1980). Moreover, the vn projects back to the nrtp so as to form a positive feedback loop (W. Precht, unpublished observations). If one interprets the vn-nrtp projection as an eye velocity command \vec{E}' rather than a canal signal, it would appear possible that the addition of \vec{e} and \vec{E}' to form $-\dot{H}_v$ occurs in the nrtp. There remain many aspects of the neurophysiology and anatomy of this circuit to be explored

so, at the moment, one must still regard it as only a reasonable working hypothesis.

The theory of the operation of the circuit in Figure 2 can be seen by applying the feedback equation (similar to Eq. 21) to the relationship between \dot{H}_{ok} and \dot{e} , taking into account that the feedback is positive,

$$\frac{\dot{H}_{\rm ok}}{\dot{e}} = \frac{\frac{-1}{(sT_0+1)}}{1 - \frac{k}{(sT_0+1)}} = \frac{-\frac{1}{(1-k)}}{[s\left(\frac{T_0}{1-k}\right)+1]} \stackrel{\Delta}{=} \frac{-G_{\rm ok}}{sT_{\rm okan}+1} \cdot 22.$$

Thus, G_{ok} is identified (by definition, \triangleq) with the term 1/(1-k). If G_{ok} is, for example, 4.0, k would have the value 0.75. The value of T_{okan} is given by $T_0/(1-k)$. If T_0 were 4 sec (a typical value for T_c for laboratory animals such as the cat) then T_{okan} would be 16 sec. Thus, the model provides reasonable gains and time constants for the basic open- and closed-loop responses of the optokinetic system to visual stimulation. Of more interest is demonstrating the circuit's effect on the vestibulo-ocular reflex by deriving the transfer function between \dot{H}_c and \dot{E}' in the dark, again taking the positive feedback loop into account,

$$\frac{\dot{E}'}{\dot{H}_{c}} = \frac{-1}{1 - \frac{k}{(sT_{0} + 1)}} = -\frac{1}{(1 - k)} \frac{(sT_{0} + 1)}{[s(\frac{T_{0}}{1 - k}) + 1]} \stackrel{\Delta}{=} -\frac{T_{vor}}{T_{0}} \frac{(sT_{0} + 1)}{(sT_{vor} + 1)} = 23.$$

The last step on the right defines T_{vor} as $T_0/(1-k)$ which, therefore, also has the same value as T_{okan} . If T_0 is equal to T_c , Eq. 23 describes the operator needed to transform T_c to T_{vor} illustrated by transformation *B* in Figure 1. Thus, even a rather simple application of feedback theory allows one to demonstrate that the neural connections in Figure 2, originally proposed to simulate optokinetic behavior, can also account for the increase in the apparent time constant of the canals seen even when vision is not available.

The scheme in Figure 2 is obviously oversimplified. There are known to be nonlinearities in the system (e.g. Collewijn 1972, 1975) that might account, for example, for the fact that during OKAN the decrease in \vec{E} with time often departs markedly from an exponential waveform. There is another problem especially evident in the rabbit: When its optokinetic system

is examined by opening the feedback loop by mechanically holding one eye (Collewijn 1969) or electro-optically stabilizing images on the retina (DuBois & Collewijn 1979), values for $G_{\rm ok}$ in the region of 50 to 100 are observed that would imply, from Eq. 21, a steady-state, closed-loop gain of 0.98 to 0.99. But the latter value is actually only about 0.8 at best, which, as indicated, requires that $G_{\rm ok}$ be only 5.0. The cause of this large discrepancy is unknown. In both the cat and primates, some of the optokinetic drive (\dot{e}) is apparently obtained by a cortical pathway (Hoffmann 1979, Atkinson 1979) about which little is known. In primates, which have a well-developed pursuit system, the question arises of where, in Figure 2, the pursuit command might be injected: Before or after the corollary discharge signal \dot{E} is fed back? Obviously more research is needed to settle these problems but the scheme in Figure 2 at least offers an interesting starting point for such studies.

The scheme in Figure 2 has also been able to provide a hypothetical explanation of a clinical entity called periodic alternating nystagmus (PAN) (Leigh et al 1981). The storage element in Figure 2 produces OKAN and prolongs the time course of per- and post-rotatory nystagmus. But after each of these phenomena, which are often called phase I of the entire nystagmus pattern, E reverses with a prolonged tail of low-velocity nystagmus called phase II, and that may be followed by yet another reversal, phase III, of even smaller velocity. To create phase II, it is generally supposed that some adaptive mechanism attempts to repair the original vestibular or optokinetic nystagmus by building up an opposing signal in the vestibular system and, in so doing, it creates an eye velocity bias in the opposite direction that is unmasked when phase I has disappeared. If such a repair mechanism is added to Figure 2, the positive feedback loop causes the system to generate damped oscillations during post-stimulus nystagmus, producing not only phase II but phase III as well. If, as is evident in several ways in PAN, such patients can no longer utilize retinal slip (the \dot{e} signal) to generate following eye movements or prevent inappropriate slow eye movements, control over the parameter k may also be lost and, due to some unknown aspect of the lesion (thought to exist in the caudal-dorsal brain stem, flocculi, or both), k might drift to a value above 1.0. If that happens, the damped oscillations just described become undamped, sustained oscillations and resemble PAN with remarkable accuracy. The model also predicts the changes in the amplitude and phase of the PAN oscillations when such patients are subjected to rotatory, vestibular stimuli (Leigh et al 1981). These findings by no means validate the model but do indicate that the model in Figure 2 constitutes a hypothesis that can explain a rather large number of phenomena associated with the optokinetic system and optokinetic-vestibular interaction in both normal and pathological situations.

A Model of the Saccade Generator

Another visually guided system, which has received far more attention than the optokinetic system, is the saccadic system. When we look about, the nervous system must perceive a visual object with the peripheral retina, select it from all other objects and construct a command for the lower brain-stem circuits that will move the eye where we want it. We know very little about how any of this is done, especially target selection. In terms of brain-stem circuits, however, one can speculate on the more specific question of how the burst neurons in Figure 1 are governed so that the intensity (in spikes sec⁻¹) and duration of the burst is just correct to move the eyes by an amount appropriate to the retinal error of the selected target. A theory has been proposed for this task that uses a local feedback scheme (Zee et al 1976, Zee & Robinson 1979b, van Gisbergen et al 1981) and it is interesting to examine it in the context of this review because it certainly represents an application of control system's theory to the oculomotor system. There is, however, no question that the following hypothesis is still rather speculative and must be regarded as an interesting idea that needs further investigation. Its value, at the moment, is that it requires no unreasonable assumptions and seems to account for a large amount of normal and pathological saccadic behavior.

Until recently it has been assumed that saccades were generated in a retinotopic coordinate system. That is, if a target appeared 10 deg to the right of the fovea, the activity evoked at the retinal location would be translated, by some unspecified network, into the pulse carried by burst cells in such a manner that the burst had the correct intensity and duration to create a 10 deg saccade to the right. This proposed system would operate in a manner that was independent of initial eye position, being concerned only with changes in position. Yet it would appear that other motor systems probably use internal copies of eye position in the head and head position on the body, to create an internal representation of the location of a seen target in space to which, say, the hand is directed by a command in a body-oriented coordinate system. Most body movements must be directed by signals in such a reference frame. It may therefore be the case that the input to the saccade-generating circuit is, similarly, a signal proportional to desired eye position in the head: E_d in Figure 3. Several studies support this idea (Hallet & Lightstone 1976, Crommelinck et al 1977, Mays & Sparks 1980). The virtue of the idea is that it then becomes quite simple to construct a scheme for timing the saccadic pulse automatically by feedback. At the right in Figure 3 the neural integrator (NI), parallel feed-forward path, and plant are shown just as in Figure 1; for saccades it is best to use the plant transfer function of Eq. 6. The output of the neural integrator is an internal signal, E', proportional to instantaneous eve position. If, as shown in Figure



Figure 3 A model for generating saccades. An internal copy of eye position (E') from the neural integrator (NI) is hypothesized to feed back through inhibitory tonic cells (T_i) to be compared with a signal from higher centers proportional to desired eye position (E_d) . The difference is motor error (e_m) which drives left and right burst cells (B_L, B_R) . Pause cells (P) inhibit burst neurons. A trigger signal (trig) inhibits the pause cells to initiate a saccade. Inhibitory burst interneurons (B_i) keep pause cells off (latch) until e_m is zero, the burst is over, and the eye is on target. This model provides a hypothetical explanation for a large number of normal and abnormal saccadic behaviors.

3, this signal were compared to desired eye position, $E_{\rm d}$, and their difference, motor error, e_m , were allowed to drive the burst cells, the eye would always be driven until E' matched E_d and e_m became zero, at which point the burst would end and the eyes would stop on target. In this way the burst amplitude and duration would automatically be always just appropriate to the desired saccade size. All that is required is an inhibitory, tonic-cell interneuron (T_i , Figure 3) to close the feedback loop.

Figure 3 shows left and right burst cells, $B_{\rm L}$ and $B_{\rm R}$, driving the neural integrator in push-pull and being driven by separate feedback loops. The relationship between the instantaneous discharge rates $B_{\rm L}$ and $B_{\rm R}$ and motor error, $e_{\rm m}$, is shown in the boxes in Figure 3. In the monkey this relation rises steeply as e_m , increased from zero and, for most cells, saturates around 1000 spikes sec⁻¹ when e_m reaches 10 to 20 deg. It is the shape of this curve that allows the model to simulate saccades of all sizes with the correct waveform and peak velocities and durations that match experimental data. If one analyzes this feedback scheme, however, one discovers that the system is unstable. This odd situation comes about because saccades, to be useful, must be both fast and brief. The first feature requires a high gain so that even a small motor error of, say, 5 deg can cause a typical burst

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neuron to discharge at 700 spikes sec^{-1} and move the eye at a peak velocity of about 300 deg sec⁻¹. The second feature requires a wide bandwidth. The result is that the gain around the loop is greater than 1.0 at frequencies where the phase shift exceeds 180 deg, which, according to feedback theory, insures instability and oscillations. The neural integrator creates a constant 90° phase lag at all frequencies. Any delays in the loop, which are all lumped into τ_1 , will create another 90° lag at the frequency 1/(4 τ_1). It would be reasonable to suppose that synaptic and recruitment delays around the loop could easily amount to 10 msec. This value for τ_1 causes a total phase shift around the loop of 180° at the frequency 25 Hz. According to theory, the system should oscillate near this frequency. In less technical terms, the system oscillates because e_m does not become zero until 10 msec after the eye has reached the target. Since the burst cells do not stop in time, the eye goes past the target before it stops. This creates an error, $e_{\rm m}$, in the opposite direction so the contralateral burst cells are activated to bring the eye back on target. But they make the same overshoot mistake and the process continues, resulting in oscillations. The fact that the model predicts saccadic oscillations is interesting because there are several situations, normal and pathological, in which oscillations, discussed below, do occur.

Nevertheless, it seems startling to propose that nature had deliberately designed a control system to be unstable. A simple solution, however, which allows the high gain-wide bandwidth features to be retained, is to turn the circuit off when it is not in use. The pause cells (Eq. 15) seem to represent just such a mechanism. It is generally believed (and indirectly supported by anatomical studies) that pause cells inhibit burst cells so that saccades cannot occur so long as the former are active. Consequently, one might propose that saccades are initiated by turning off the pause cells. It is proposed that a trigger signal (trig, Figure 3) momentarily silences the pause cells and releases the burst cells to initiate a saccade to the position $E_{\rm d}$. If, however, the trigger pulse disappears before the saccade is over, the pause cell would be allowed to reinhibit the burst cells and stop the saccade. To prevent this, it is proposed that an inhibitory burst interneuron exists $(B_i, Figure 3)$ that can prevent the pause cell from firing so long as either the left or right burst cells are active. This pathway (latch, Figure 3) allows an on-going saccade to run to completion before the pause cells are released to once again disable the pulse generator.

This model has the following interesting features:

- 1. It produces saccades of all sizes that automatically have the correct velocity and duration.
- 2. It simulates the wave-shape of instantaneous burst rate for saccades of all sizes and direction.

- 3. It is compatible with the results of stimulating the pause cells during a saccade, which can stop the saccade momentarily in midflight (Keller 1977).
- 4. By decreasing the slope and amplitude of the burst-rate function $[B(e_m)$ in Figure 3], one can describe slow saccades seen in certain neurological disorders thought to affect the pontine reticular formation (Zee et al 1976).
- 5. If the primary saccade is over before the trigger signal is over, another small saccade in the opposite direction will occur as the system, without inhibition from the pause cells, starts to oscillate. Such movements do occur and are called dynamic overshoot. In the case of microsaccades, which have a short duration, inhibition of pause cells by the trigger signal may permit several, back-to-back, microsaccades to occur. Such microsaccades. The model in Figure 3 mimics all these naturally occurring examples of saccadic oscillations (van Gisbergen et al 1981). If the pause cells can be kept off for many seconds, continuous saccadic oscillations occur similar to voluntary nystagmus.
- 6. There are patients whose abnormal eye movements can be described as an exaggeration of all the movements just mentioned in 5: very large dynamic overshoot and episodes of spontaneous oscillations called ocular flutter. Increasing the delay τ_1 and putting a lag in the latch circuit in the scheme in Figure 3 can simulate these abnormal movements (Zee & Robinson 1979b).

It has recently been demonstrated in monkeys that there is a very close relationship between instantaneous motor error $e_m(t)$ and instantaneous burst rate B(t), which supports the idea that burst cells are driven by motor error e_m as indicated in Figure 3 by the relationship $B(e_m)$ (van Gisbergen et al 1981). This is the best neurophysiological evidence to date to support the hypothesis expressed in Figure 3. An obvious advantage of the hypothesis is that the feedback and latch circuits require only cell types (burst and tonic) that are already known to be present. An obvious disadvantage is that signals such as E_d have not been observed with microelectrodes and the model also fails to provide a role for other types of burst cells called long-lead burst neurons seen in the superior colliculi (Mays & Sparks 1980) and pontine reticular formation (Cohen & Henn 1972), although it is generally believed that such cells must play some role in shaping the burst delivered by those burst cells shown in Figure 1.

This model is seductive in its ability to mimic many properties of the physiology and neurophysiology, both normal and pathological, of saccades. It is especially seductive to the oculomotor neuro-ophthalmologist who must deal with such a bewildering array of eye movement disorders

in the clinic that any reasonable hypothesis is better than none at all. Fortunately, many basic scientists in this field are seduced not at all, and for them the model is a challenge to be tested. For this purpose, one of the major virtues of a model should be appreciated: by its nature it is completely unambiguous. There is no way to misinterpret what is being proposed, so the testing of it can be equally unambiguous. To the oculomotor theoretician, the scheme is a challenge to produce a better model that can simulate all the phenomena listed above and more. As an example of the usefulness of the model in suggesting new experiments, it would never have occurred to van Gisbergen et al (1981) to look for, and find, a unique relationship between burst rate and motor error if the model in Figure 3, which evolved from an effort to explain clinical observations (Zee et al 1976), had not existed. In fact, constructing a model always makes one ask questions that would otherwise never have been thought of.

DISCUSSION

The examples offered in this review are intended to illustrate that the theory and practice of control systems analysis is not only useful in oculomotor neurophysiology but is rapidly becoming an essential tool. Clearly, the models in Figures 2 and 3 could not have been conceived, let alone tested, without the concepts and tools of control theory. In oculomotor physiology, we are approaching the stage of complexity where hypotheses will, of necessity, entail control systems analysis. Even if, for example, the scheme proposed in Figure 3 proves to be incorrect, the scheme that replaces it will certainly not be simpler and its conception and testing will require more, not less, systems analysis. When we start to study the interactions between subsystems such as those shown in Figures 1, 2, and 3, the dependence on quantitative analysis will increase. In fact, in visually guided systems such as the saccadic system, as we move above the level where movement commands are coded in discharge rate to those where the spatial distribution of activity within a population of cells becomes important, as in the superior colliculus, quantitative models will become more and more necessary and complex. In short, as neurophysiology grows up and addresses the main problem of how nervous tissue processes signals (or how the brain thinks), it must, in the end, come to grips with information processing and feedback regulation. It is hard to imagine how this will come about without using some form of the analytic techniques designed and utilized by those who have studied these phenomena from the time their examination was first recognized as a scientific discipline. It is now generally conceded that the facts that the nervous system is built with neurons and its effectors for movement are muscles do not constitute any reason for supposing that it should not be analyzed by theories of signal processing and feedback control, although one may readily admit that our analytical techniques must become more sophisticated to cope with higher brain functions. Clearly, if integrative neurophysiology of the mammalian brain is not to stagnate as a discipline incapable of interpreting its own data, it must progress from being descriptive to being interpretive, and it would appear that the oculomotor system is one of the first areas in which this transition is becoming clear. Thus, the question of whether control systems analysis is useful in the eye movement control system is perhaps inappropriate. The question is simply how rapidly can new data be acquired to fill in, verify, modify, and expand the systems models already being proposed.

In this review, specific models are described in some detail because there is no better way to allow the reader to judge whether the use of modeling is or is not useful in describing the neural circuits that control eve movements. Unfortunately, this practice has prevented the review of other models of oculomotor performance. Most of those models, however, describe the behavior of entire visuo-motor subsystems. Such models of eye tracking -(Fender & Nye 1961, Dallos & Jones 1963, Stark et al 1962) or, in particular, saccadic tracking (Westheimer 1954, Young & Stark 1963, Robinson 1973, Wheeless et al 1966, Becker & Jürgens 1979, Bahill et al 1975a), or pursuit tracking (Robinson 1965, Yasui & Young 1975, Murphy et al 1975, Steinbach 1976, Miles 1977, Young 1977, Kowler et al 1978, Pola & Wyatt 1980) usually described the strategies of the human operator as a tracking machine, which is of interest to the psychologist and those concerned with man-machine systems. But these models had little direct impact on the neurophysiologist since they usually shed no light on how neural networks processed signals. These studies began in the early 1960s but fizzled out in the mid- to late 1970s because they could not cope with the complexities of trying to model the decision-making activities of high-order mental processes, and offered nothing testable for the electrophysiologist. They did, however, have a more subtle, long-range impact on oculomotor neurophysiology by formalizing the tasks of oculomotor subsystems and pointing out the general operations that must be done, such as integrating, amplifying, and sampling. They pointed out to us that the oculomotor control system was just that-a control system-and reminded us that there were established techniques for analyzing its behavior. It was their influence that caused the description of the canals in 1971 by Fernandez & Goldberg and of the oculomotor plant in 1970 by myself to be couched in terms of transfer functions. Those studies suggested that the qualitative, anecdotal descriptions that often characterized neurophysiological investigations of complex interactions had to be replaced by quantitative descriptions of some sort if they were to be useful in explaining behavior.

Thus, the major achievement of the behavioral models of the 1960s was to focus our attention on the need for quantitation and analysis but the more

recent efforts to propose specific neural circuits, as in Figures 1 through 3, are more exciting because they address the basic issue of how neural circuits actually do process information. The specific models examined in this review are only examples of many phenomena that would benefit from modeling. How are the planes of motion sensitivities of the six semicircular canals transformed, in the vestibulo-ocular reflex, to the planes of rotation of the extraocular muscle pairs? How can long-lead burst neurons in the deep lavers of the superior colliculi and in the pontine reticular formation be used to generate the burst seen on medium-lead burst cells, to challenge the model in Figure 3? Why are the velocities of saccades sometimes slowed (Morasso et al 1973), and sometimes increased (Haddad & Robinson 1977), during combined eye-head movements in various species? What constitutes appropriate pursuit and optokinetic stimuli in primates and how do they interact? Are the central commands for saccades generated in a polar or Cartesian coordinate system, or in a totally different system? These and many other questions are emminently suitable for attack by modeling.

The current modeling activity in oculomotor neurophysiology is a healthy sign because it is a measure of this discipline's vigor and growth. It marks the transition from gathering data to interpreting it. Many more data are still needed—they are the sine qua non of the models—and as they become available the use of control or systems theory will become increasingly important because, in the end, it will be the models, not the data, that will tell us how the oculomotor system works.

Literature Cited

- Atkinson, J. 1979. Development of optokinetic nystagmus in the human infant and monkey infant: An analogue to development in kittens. In *Developmental Neurobiology of Vision*, ed. R. D. Freeman, 27:277-87. New York: Plenum. 446 pp.
- Bahill, A. T., Bahill, K. A., Clark, M. R., Stark, L. 1975a. Closely spaced saccades. *Invest. Ophthalmol.* 14:317-20
- Bahill, A. T., Clark, M. R., Stark, L. 1975b. Glissades-eye movements generated by mismatched components of the saccadic motoneuronal control signal. *Math. Biosci.* 26:303–18
- Barr, C. C., Schultheis, L. W., Robinson, D. A. 1976. Voluntary, non-visual control of the human vestibuloocular reflex. *Acta Oto-Laryngol.* 81:365-75
- Becker, W., Jürgens, R. 1979. An analysis of the saccadic system by means of double step stimuli. Vision Res. 19:967-83
- Becker, W., Klein, H. M. 1973. Accuracy of saccadic eve movements and mainte-

nance of eccentric eye positions in the dark. Vision Res. 13:1021-34

- Buettner, U. W., Büttner, U., Henn, V. 1978. Transfer characteristics of neurons in vestibular nuclei of the alert monkey. J. Neurophysiol. 41:1614–28
- Büttner, U., Büttner-Ennever, J. A., Henn, V. 1977. Vertical eye movement related unit activity in the rostral mesencephalic reticular formation of the alert monkey. *Brain Res.* 130:239-52
- Carpenter, Ř. H. S. 1972. Cerebellectomy and the transfer function of the vestibulo-ocular reflex in the decerebrate cat. Proc. R. Soc. London Ser. B 181:353-74
- Cazin, L., Precht, W., Lannou, J. 1980. Pathways mediating optokinetic responses of vestibular nucleus neurons in the rat. *Pflügers Arch.* 384:19-29
- Clark, M. R., Stark, L. 1974. Control of human eye movements. I. Modelling of extraocular muscles. II. A model for the extraocular plant mechanism. III. Dynamic characteristics of the eye track-

ing mechanism. Math. Biosci. 20:191-265

- Cohen, B., Henn, V. 1972. Unit activity in the pontine reticular formation associated with eye movements. *Brain Res.* 46:403-10
- Cohen, B., Matsuo, V., Raphan, T. 1977. Quantitative analysis of the velocity characteristics of optokinetic nystagmus and optokinetic after-nystagmus. J. Physiol. London 270:321-44
- Collewijn, H. 1969. Optokinetic eye movements in the rabbit: Input-output relations. Vision Res. 9:117-32
- Collewijn, H. 1972. An analog model of the rabbit's optokinetic system. *Brain Res.* 36:71-88
- Collewijn, H. 1975. Direction selective units in the rabbit's nucleus of the optic tract. *Brain Res.* 100:489-508
- Collins, C. C. 1975. The human oculomotor control system. In *Basic Mechanisms of Ocular Motility and Their Clinical Implications*, ed. G. Lennerstrand, P. Bach-y-Rita, 24:145–80. Oxford: Pergamon. 584 pp.
- Crommelinck, M., Guitton, D., Roucoux, A. 1977. Retinotopic versus spatial coding of saccades: Clues obtained by stimulating deep layers of cat's superior colliculus. In Control of Gaze by Brain Stem Neurons, ed. R. Baker, A. Berthoz, pp. 425-35. Amsterdam: Elsevier. 514 pp.
- 514 pp. Dallos, P. J., Jones, R. W. 1963. Learning behavior of the eye fixation control system. Inst. Electr. Electr. Eng. Trans. Automatic Controls AC-8:218-27
- Dichgans, J., Schmidt, C. L., Graf, W. 1973. Visual input improves the speedometer function of the vestibular nuclei in the goldfish. *Exp. Brain Res.* 18:319-22
- goldfish. Exp. Brain Res. 18:319-22 Dubois, M. F. W., Collewijn, H. 1979. The optokinetic reactions of the rabbit: Relation to the visual streak. Vision Res. 19:9-17
- Easter, S. S. Jr. 1973. A comment on the glissade. Vision Res. 13:881-82
- Evarts, E. V. 1968. A technique for recording activity of subcortical neurons in moving animals. *Electroencephalogr. Clin. Neurophysiol.* 24:83–86
- Fender, D. H., Nye, P. W. 1961. An investigation of the mechanisms of eye movement control. *Kybernetik* 1:81-88
- Fernandez, C., Goldberg, J. M. 1971. Physiology of peripheral neurons innervating semicircular canals of the squirrel monkey. II. Response to sinusoidal stimulation and dynamics of peripheral vestibular system. J. Neurophysiol. 34: 661-75

- Fuchs, A. F., Luschei, E. S. 1970. Firing patterns of abducens neurons of alert monkeys in relationship to horizontal eye movement. J. Neurophysiol. 33:382–92
- Goebel, H. H., Komatsuzaki, A., Bender, M. B., Cohen, B. 1971. Lesions of the pontine tegmentum and conjugate gaze paralysis. Arch. Neurol. 24:431-40
- Goldberg, J. M., Fernandez, C. 1971. Physiology of peripheral neurons innervating semicircular canals of the squirrel monkey. I. Resting discharge and response to constant angular accelerations. J. Neurophysiol. 34:635-60
- Haddad, G. M., Robinson, D. A. 1977. Cancellation of the vestibuloocular reflex during active and passive head movements in the normal cat: Soc. Neurosci. 3:155 (Abstr.)
- Hallett, P. E., Lightstone, A. D. 1976. Saccadic eye movements towards stimuli triggered by prior saccades. *Vision Res.* 16:99-106
- Henn, V., Cohen, B. 1973. Quantitative analysis of activity in eye muscle motoneurons during saccadic eye movements and positions of fixation. J. Neurophysiol. 36:115-26
- Henn, V., Young, L. R., Finley, C. 1974. Vestibular nucleus units in alert monkeys are also influenced by moving visual fields. Brain Res. 71:144-49
- Hoffman, K.-P. 1979. Optokinetic nystagmus and single-cell responses in the nucleus tractus opticus after early monocular deprivation in the cat. See Atkinson 1979, pp. 63–72
- Hoffman, K.-P., Schoppmann, A. 1975. Retinal input to direction selective cells in the nucleus tractus opticus of the cat. *Brain Res.* 99:359-66
- Igusa, Y., Sasaki, S., Shimazu, H. 1980. Excitatory premotor burst neurons in the cat pontine reticular formation related to the quick phase of vestibular nystagmus. *Brain Res.* 182:451-56
- Ito, M., Shiida, T., Yagi, N., Yamamoto, M. 1974. The cerebellar modification of rabbit's horizontal vestibulo-ocular reflex induced by sustained head rotation combined with visual stimulation. *Proc. Jpn. Acad.* 50:85–89
- Kamath, B. Y., Keller, E. L. 1976. A neurological integrator for the oculomotor control system. *Math Biosci.* 30:341–52
- Keller, E. L. 1973. Accommodative vergence in the alert monkey. Vision Res. 13: 1565-75
- Keller, E. L. 1974. Participation of the medial pontine reticular formation in eye movement generation in monkey. J. Neurophysiol. 37:316-32

- Keller, E. L. 1977. Control of saccadic eye movements by midline brain stem neurons. See Crommelinck et al 1977, pp. 327-36
- Keller, E. L., Daniels, P. D. 1975. Oculomotor related interaction of vestibular and visual stimulation in vestibular nucleus cells in alert monkey. *Exp. Neurol.* 46:187–98
- Keller, E. L., Robinson, D. A. 1971. Absence of a stretch reflex in extraocular muscles of the monkey. J. Neurophysiol. 34: 908–19
- Keller, E. L., Robinson, D. A. 1972. Abducens unit behavior in the monkey during vergence movements. *Vision Res.* 12:369-82
- King, W. M., Fuchs, A. F. 1977. Neuronal activity in the mesencephalon related to vertical eye movements. See Keller 1977, pp. 319–26
- King, W. M., Lisberger, S. G., Fuchs, A. F. 1976. Responses of fibers in medial longitudinal fasciculus (mlf) of alert monkeys during horizontal and vertical conjugate eye movements evoked by vestibular or visual stimuli. J. Neurophysiol. 39:1135–49
- Kowler, E., Murphy, B. J., Steinman, R. M. 1978. Velocity matching during smooth pursuit of different targets on different backgrounds. *Vision Res.* 18:603-5
- Leigh, R. J., Robinson, D. A., Zee, D. S. 1981. A quantitative hypothesis for periodic alternating nystagmus. Proc. NY Acad. Sci. In press
- Lisberger, S. G., Fuchs, A. F. 1978. Role of primate flocculus during rapid behavioral modification of vestibuloocular reflex. I. Purkinje cell activity during visually guided horizontal smooth-pursuit eye movements and passive head rotation. J. Neurophysiol. 41:733-63
- Lisberger, S. G., Miles, F. A. 1980. Role of primate medial vestibular nucleus in long-term adaptive plasticity of vestibuloccular reflex. J. Neurophysiol. 43:1725-45
- Lopez-Barneo, J., Darlot, C., Berthoz, A. 1979. Functional role of the prepositus hypoglossi in the control of gaze. In *Reflex Control of Posture and Movements*, ed. R. Granit, O. Pompeiano, pp. 668–79. Amsterdam: Elsevier. 827 pp.
- Luschei, E. S., Fuchs, A. F. 1972. Activity of brain stem neurons during eye movements of alert monkeys. J. Neurophysiol. 35:445-61
- Mays, L. E., Sparks, D. L. 1980. Dissociation of visual and saccade-related responses

in superior colliculus neurons. J. Neurophysiol. 43:207-32

- Melvill Jones, G., Milsum, J. H. 1971. Frequency-response analysis of central vestibular unit activity resulting from rotational stimulation of the semicircular canals. J. Physiol. London 219:191–215
- Miles, F. A. 1977. The primate flocculus and eye-head coordination. In *Eye Movements*, ed. B. A. Brooks, F. J. Bajandas, pp. 75-92. New York: Plenum. 223 pp.
- Miles, F. A., Braitman, D. J. 1980. Longterm adaptive changes in primate vestibulo-ocular reflex. II. Electrophysiological observations on semicircular canal primary afferents. J. Neurophysiol. 43:1426-36
- Miles, F. A., Fuller, J. H. 1975. Visual tracking and the primate flocculus. Science 189:1000-2
- Morasso, P., Bizzi, E., Dichgans, J. 1973. Adjustment of saccade characteristics during head movements. *Exp. Brain Res.* 16:492-500
- Murphy, B. J., Kowler, E., Steinman, R. M. 1975. Slow oculomotor control in the presence of moving backgrounds. *Vision Res.* 15:1263–68
- Optican, L. M., Robinson, D. A. 1980. Cerebellar-dependent adaptive control of the primate saccadic system. J. Neurophysiol. In press
- Oyster, C. W., Takahashi, E., Collewijn, H. 1972. Direction-selective retinal ganglion cells and control of optokinetic nystagmus in the rabbit. *Vision Res.* 12:183–93
- Pola, J., Robinson, D. A. 1978. Oculomotor signals in the medial longitudinal fasciculus of the monkey. J. Neurophysiol. 41:245-59
- Pola, J., Wyatt, H. J. 1980. Target position and velocity: The stimuli for smooth pursuit eye movements. Vision Res. 20:523-34
- Precht, W., Strata, P. 1980. On the pathways mediating optokinetic responses in vestibular nuclear neurons. *Neuroscience* 5:777-87
- Rademaker, G. G. J., ter Braak, J. W. G. 1948. On the central mechanism of some optic reactions. *Brain* 71:48-76
- Raphan, T., Matsuo, V., Cohen, B. 1979. Velocity storage in the vestibuloocular reflex arc (VOR). *Exp. Brain Res.* 35:229–48
- Raybourn, M. S., Keller, E. L. 1977. Colliculoreticular organization in primate oculomotor system. J. Neurophysiol. 40:861-78
- Robinson, D. A. 1965. The mechanics of hu-

man smooth pursuit eye movement. J. Physiol. London 180:569-91

- Robinson, D. A. 1970. Oculomotor unit behavior in the monkey. J. Neurophysiol. 33:393-404
- Robinson, D. A. 1973. Models of the saccadic eye movement control system. Kybernetik 14:71–83
- Robinson, D. A. 1974. The effect of cerebellectomy on the cat's vestibulo-ocular integrator. Brain Res. 71:195-207
- Robinson, D. A. 1975. Oculomotor control signals. See Collins 1975, pp. 337-74
- Robinson, D. A. 1976. Adaptive gain control of vestibuloocular reflex by the cerebellum. J. Neurophysiol. 39:954-69
- Robinson, D. A. 1977. Linear addition of optokinetic and vestibular signals in the vestibular nucleus. Exp. Brain Res. 30:447-50
- Robinson, D. A. 1978. The functional behavior of the peripheral oculomotor apparatus: A review. In Disorders of Ocular Motility, ed. G. Kommerell, pp. 43-61. München: Bergmann. 386 pp.
- Robinson, D. A. 1980. Models of the mechanics of the orbit. In Models of Oculomotor Behavior and Control, ed. B. L. Zuber, W. Palm Beach, Fla: CRC Press. In press
- Schiller, P. H. 1970. The discharge characteristics of single units in the oculomotor and abducens nuclei of the unanesthetized monkey. Exp. Brain Res. 10:347-62
- Schmid, R., Zambarbieri, D., Sardi, R. 1979. A mathematical model of the optokinetic reflex. Biol. Cybernetics 34:215-25
- Skavenski, A. A., Robinson, D. A. 1973. Role of abducens neurons in the vestibuloocular reflex. J. Neurophysiol. 36:724-38
- Sperry, R. W. 1950. Neural basis of spontaneous optokinetic response produced by visual inversion. J. Comp. Physiol. Psychol. 43:482-89
- Stark, L., Vossius, G., Young, L. R. 1962. Predictive control of eye tracking movements. Trans. Inst. Radio Eng. Prof. Grp. on Human Factors in Elect. HFE-3:52-56
- Steinbach, M. J. 1976. Pursuing the perceptual rather than the retinal stimulus. Vision Res. 16:1371-76
- Takemori, S. 1974. The similarities of optokinetic after-nystagmus to the vestibular

and the second

nystagmus. Ann. Otol. Rhinol. Laryngol. 83:230-38

- van Gisbergen, J. A. M., Robinson, D. A., Gielen, S. 1981. A quantitative analysis of the generation of saccadic eye movements by burst neurons. J. Neuro*physiol*. In press
- von Holst, E., Mittelstaedt, H. 1950. Das reafferenzprincip. Naturwissenschaften 37:464-76
- Waespe, W., Henn, V. 1977. Neuronal activity in the vestibular nuclei of the alert monkey during vestibular and optokinetic stimulation. Exp. Brain Res. 27:523-38
- Westheimer, G. 1954. Eye movement responses to a horizontally moving visual stimulus. AMA Arch. Ophthalmol. 52:932-41
- Wheeless, L. L. Jr., Boynton, R. M., Cohen, G. H. 1966. Eye movement responses to step and pulse-step stimuli. J. Opt. Soc. Am. 56:956-60
- Yasui, S., Young, L. R. 1975. Eye movements during after-image tracking under sinusoidal and random vestibular stimulation. See Collins 1975, pp. 509-13
- Yoshida, K., McCrea, R. A., Berthoz, A., Vidal, P. 1979. Morphological and physiological characteristics of burst inhibitory neurons in the alert cat. Soc. Neurosci. 5:391 (Abstr.)
- Young, L. R. 1977. Pursuit eye movementswhat is being pursued? See Crom-melinck et al 1977, pp. 29-36 Young, L. R., Stark, L. 1963. Variable feed-
- back experiments testing a sampled data model for eye tracking movements. Inst. Electr. Electr. Eng. Trans. Prof. Grp. on Human Factors in Elect. HFE-4:38-51
- Zee, D. S., Optican, L. M., Cook, J. D., Robinson, D. A., Engel, W. K. 1976. Slow saccades in spinocerebellar degeneration. Arch. Neurol. 33:243-51
- Zee, D. S., Robinson, D. A. 1979a. Clinical applications of oculomotor models. In Topics in Neuro-Ophthalmology, ed. H. S. Thompson, pp. 266–85. Baltimore: Williams & Wilkins. 377 pp. Zee, D. S., Robinson, D. A. 1979b. An hypo-
- thetical explanation of saccadic oscillations. Ann. Neurol. 5:405-14
- Zee, D. S., Yamazaki, A., Gücër, G. 1978. Ocular motor abnormalities in trained monkeys with floccular lesions. Soc. Neurosci. 4:168 (Abstr.)

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