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Limits on the comprehension of rotational motion: mental imagery of rotations with oblique components

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Abstract. Mental imagery of rotational motion across variation in the orientation of a square to an axis of rotation, the orientation of the axis to the environment/viewer, and the starting orientation of the rotation were investigated in three experiments. The experimental method included specifying the particular rotations that subjects should consider and obtaining exact predictions of the outcomes of the rotations. When the square was normal to the axis and the axis was normal to the environment/viewer, performance was excellent. When either of these relationships was oblique, performance was quite good. When both of these relationships were oblique, nearly every subject made large errors on every problem. The difficulty of the double-oblique rotations was reduced when the initial orientation of the square was not canonical. Current views of the comprehension of rotational motion are discussed. It appears that the comprehension of rotational motion can be understood as an organization of the symmetric space traced out by the motion. People succeed in organizing this space when it is aligned with a principal spatial reference system.

1 Introduction

We can learn much about how people comprehend rotational motion by having them predict the outcomes of various forms of rotation. Two basic properties of simple rotational motion are the orientation of a rotating object with respect to the axis of rotation and the orientation of the axis of rotation with respect to the spatial reference system, as illustrated in figure 1. In recent years, studies have shown that people perform less well in mental-rotation tasks when the axis of rotation is oblique to the spatial reference system (Just and Carpenter 1985; Parsons 1987a, 1987b; see also Green 1961). Other studies have shown that both the orientation of the axis of rotation to the reference system and the orientation of the object to the axis affect the comprehension of rotation. Thus, Shiffrar and Shepard (1991) found, in a study of motion recognition, that when both of these orientations were normal, recognition performance was best. Performance was somewhat worse when one of the orientations was oblique, and performance was worse again when both orientations were oblique. Massironi and Luccio (1989) obtained similar results in a study of mental rotation.

In the present paper, I report three experiments on mental imagery of rotational motion. The experiments demonstrated strong constraints on people's ability to visualize rotational motion; constraints much stronger than most previous studies would suggest. These constraints are related to the orientation of the object to the axis of rotation, the orientation of the axis to the environmental reference system, and the starting orientation of the rotation.

A number of features differentiate the present experiments from most studies of mental rotation. Three features seem most important. First, the rotational motions to be considered were specified to the subjects. Most previous experiments have left the choice of a particular form of rotation to the subjects, with a resulting ambiguity as to what specific transformation was imaged (see Parsons 1987a, 1987b). Second, subjects were asked to produce the orientations that would be the outcomes of the specified rotations. Such unaided imagery is analogous to using a free-recall method in the study of memory, or absolute judgment in the study of perception. In contrast, in most studies of mental rotation experimenters have had subjects choose the correct response from a pair of qualitatively distinct alternatives. Typically, subjects discriminate between a rotated object and a rotation of a distractor object (eg the object reflected through a plane). Although these experiments appear to have demonstrated various forms of analog mental rotation (eg Cooper and Shepard 1973; Shepard and Metzler 1971), it can be unclear how accurately people are able to image specific motions (see Folk and Luce 1987). The third feature of the present experiments is that the tasks were ostensibly very simple and concrete. The displays were simple three-dimensional (3-D) constructions, few trials were administered, and subjects were encouraged to take their time. Hence, any limits on the ability to perform these tasks would seem to be robust.

Massironi and Luccio (1989) have reported a pair of experiments in which they employed a methodology similar to the present one. The experimenters specified particular rotational motions and had subjects demonstrate specific answers. However, there are some basic differences in stimuli and an important difference in outcome. Massironi and Luccio showed subjects ink drawings of quadrilaterals with rotation axes that were in the plane of the paper. The present experiments explore rotation of a concrete object (a wooden square), and the object and the rotation axes are in a variety of 3-D orientations. Regarding results, Massironi and Luccio found that if a quadrilateral was oblique to an axis of rotation, the problem was always difficult (ie more than half the responses were large errors). In the present experiments, such rotation problems were not difficult if the axis of rotation was vertical. The results of Massironi and Luccio (1989) were probably influenced by the two-dimensional (2-D) spatial organization of their stimuli (as the authors suggest).

The present experiments also have important features in common with Hinton's (1979) tipped-cube task. Hinton asked subjects to imagine a cube so that a main diagonal through the cube was standing vertically. Subjects were told to hold one finger at the top of the diagonal and to point with the other hand to the remaining corners of the cube. This is a simple task to administer, and the results with subjects who have

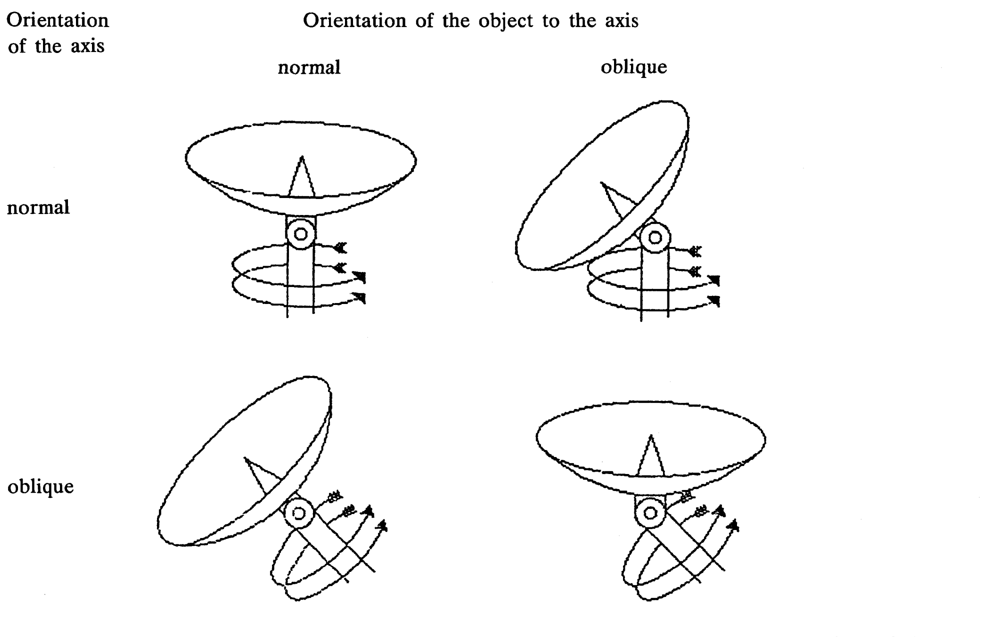

Figure 1. This hypothetical receiver dish exhibits variation both in the orientation of the rotating dish to the axis of rotation and in the orientation of the axis of rotation to the environment.

no special training are reliable: virtually no one gives the correct answer. A common answer is to outline a regular octahedron with the square cross section horizontal. However, the cube is a hexahedron. The general failure in this task is significant given that the cube is a simple and familiar form, standing the cube on a main diagonal requires only a simple rotation of the cube, and the subject concentrates on just the one task, taking as long as he or she requires. General failure of ostensibly straightforward spatial cognition would seem to establish an important class of experimental results.

It would seem that the failure to image the tipped cube implies, among other things, an inability to visualize the more obvious rotations that would tip the cube. The simplest rotation that would take a normally resting cube and tip it onto a diagonal is a rotation in which most of the edges and surfaces of the cube are oblique to the axis of rotation. After preliminary work it appeared that it should be possible to demonstrate, with a simple task that concerned the outcome of a hypothetical rotation, the same nearly complete failure that is observed in the tipped-cube task.

The research of Rock and his colleagues also provides a context for the present work (Rock and DaVita 1987; Rock et al 1981; Rock et al 1989). These authors found that relatively unstructured curvilinear objects were not recognized, nor accurately imagined, in new orientations whether these new orientations were produced by translation or rotations. Thus, a boundary condition on people's ability to imagine rotations was identified. The work reported here explores boundary conditions on the imagination of rotation related to oblique orientations within the structure of the rotations.

The present experiments were designed to examine the rotation of squares about rods. A planar object was chosen because it is a simple but natural class of rotational motions. For instance, texts in mathematics, physics, and engineering typically introduce rotational motion in terms of rotation in the plane. Many natural rotations either are rotations essentially of planes (eg swinging doors) or can be closely approximated by them (eg the motions of wheels or of satellite dishes). An informal experiment with eighteen subjects suggested that discs and squares behaved the same in the type of experiment reported here. Squares (with stripes along one side of one edge) were adopted to facilitate identification of reference points about the perimeter of the shape.

Experiment 1 was a one-trial experiment with a simple concrete display. With a square oblique to a rod and the rod oblique to the environment (and viewer), virtually no subject could predict the outcome of a 180° rotation of the square about the rod. In experiment 2 both the orientation of the square to the rod and the orientation of the rod in the environment were varied. Both factors affected performance. However, performance with the double-oblique rotations was markedly poor. Experiment 3 had three basic results. First, even when subjects were encouraged to take the axis of rotation as a spatial reference system, the double-oblique rotations were very difficult. Second, the vertical axis of rotation was superior to the frontal-horizontal axis for most subjects. Third, if the square was not displayed in a canonical orientation relative to the environment, the difficulty of the double-oblique rotations was reduced. Overall, these results are consistent with recent suggestions of how people represent rotational motion. I will point out, however, important areas in which theory can be developed further.

2 Experiment 1

The purpose in experiment 1 was to demonstrate that with a simple object oblique to an axis of rotation, and the axis oblique to the environment, it is possible to produce an elementary problem about the outcome of a rotation that most people cannot solve correctly.[1] A planar object was used so that the spatial relationships would be quite basic. A square was chosen so that subjects would be able to identify specific locations about the perimeter of the object. To ensure that there would be no ambiguities in understanding the problem, a concrete 3-D stimulus display was presented, rather than a picture of one, and the axis of rotation was a prominent rod running through the square. To know how accurately the rotation could be imaged, subjects were required to demonstrate the exact orientation that would result from a given amount of rotation. Finally, to ensure that subjects gave a complete effort in solving the rotation problem, only one problem was given to subjects, and they were informed that the problem was difficult.

2.1 *Method*

2.1.1 *Subjects*. Twenty-seven Harvard University undergraduates, graduate students, and research assistants volunteered to participate. The undergraduates had responded to signs requesting volunteers for experiments in mental imagery and visual perception. They were paid for their participation.

2.1.2 *Materials*. The stimulus apparatus is illustrated in figure 2. Two thin wooden supports suspended a wooden rod (85 cm by 1 cm) 43 cm above a rectangular table. The rod was horizontal and at a 45° angle to the edge of the table. At the center of the rod was a 10.8 cm wooden square. The square was vertical and parallel to the front edge of the table (and thus at 45° to the rod). The rod and square were painted flat white, except that the square had a black stripe 0.5 cm wide along its upper front edge. The supports were left as plain wood. A square identical to the one on the rod lay on the table. This square was for the subjects to use in illustrating their answers.

2.1.3 *Procedure*. The subject sat facing the square and went through the following sequence. He or she read a set of written instructions, paraphrased the instructions to the experimenter, developed an answer to the rotation problem, illustrated the answer

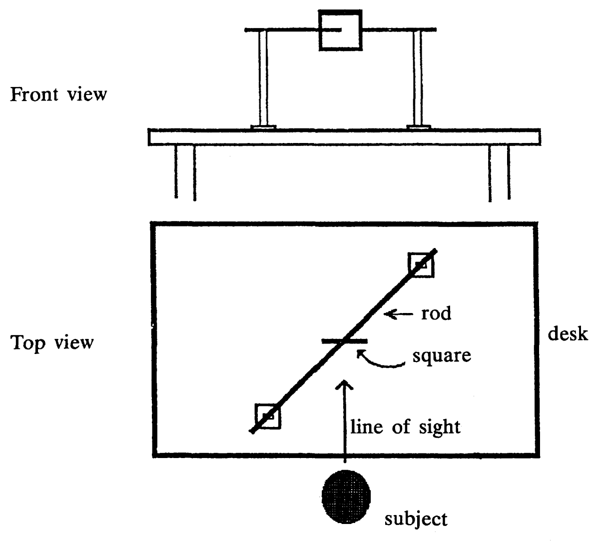

Figure 2. Stimulus arrangement in experiment 1.

[1] In all of the experiments reported here, when the axis of rotation is oblique to the planes of the environment, it is also oblique to the line-of-sight of the viewer. However, the viewer-reference system generally is not mentioned, because research has shown that the viewer-reference system is not generally critical to the comprehension of rotation (Corballis et al 1978; Hinton and Parsons 1988; Pani and Dupree 1992).

for the experimenter, provided a confidence rating, and described the strategy used to derive the answer.

The specific rotation problem was this: "The square is rigidly attached to the rod. The question for you to answer is, if there were a crank at the end of the rod, and the crank were turned 180 degrees (ie through a half circle), in what orientation would the square be? In other words, if the rod turned like an axle, so that a point on the rod rotated around to the opposite side of the rod, in what orientation would the square be?" The subject took as long as was needed to develop an answer.

The subject was instructed to answer the question "in your head" and only to hold up the illustration square to illustrate a completed answer. To enforce this procedure, subjects were told that the first orientation in which the illustration square was held up would be taken as their answer. Subjects had been told that the problem was difficult and that they should take their time and be careful.

After indicating an answer, the subject rated his or her confidence in the answer on a 1 to 7 scale, where '1' indicated "absolutely confident" and '7' "I don't know". The experimenter measured the time from the completion of the paraphrase of the instructions to when an answer was indicated, using a silent stopwatch. Subjects had been instructed that they would be timed, but that deriving their best answer was the important consideration.

The orientation of the illustration square was recorded in terms of 'surface orientation', whether it faced front or back, and 'spin'. Two analog values encoded surface orientation. With the subject holding the illustration square, the experimenter imagined it to be at the center of a sphere. Surface orientation was the point on the sphere where a perpendicular line through the center of the square would emerge from the sphere (ie the point that the square faced). The data sheet showed a projection of a sphere with latitude and longitude indicated in 45° intervals. With the aid of these intervals as references, the experimenter placed a mark anywhere on the sphere to provide an analog indication of surface orientation. To allow only a half sphere facing the experimenter to be used to measure surface orientation, the experimenter indicated whether it was the front of the illustration square (ie the side with the stripe) or the back that faced forward.

Finally, the experimenter indicated the orientation of the illustration square with respect to the imaginary perpendicular through its center, or spin. If the striped edge was at the top and the edges of the square coincided with horizontal and vertical planes of the environment, spin was said to be 0°. A circle demarcated in 45° intervals (like the spokes of a wheel) provided a convenient set of reference points for the experimenter to make an analog indication of spin. The measurements of spin and of surface orientation taken together are the same as the roll-pitch-yaw description of orientation common in engineering and the physical sciences (eg Craig 1989). What is called spin here corresponds to 'roll', and the latitude and longitude that encode surface orientation correspond to pitch and yaw, respectively. (Alternatively, one could say that this system expressed orientation in spherical coordinates with an additional component for spin.)

The two assistants who tested subjects in this experiment were well practiced with the recording system before the experiment was conducted. The assistants had no knowledge of the hypotheses or prior experimental results that had led to this experiment.

2.2 *Results*

2.2.1 *Subject responses.* The experimenter codings were converted to decimal values in the roll-pitch-yaw descriptive system. This generally could be accomplished by eye, although a ruler or protractor was often used for verification. Conversion to decimals could be completed confidently in intervals of 2.5°.

The subjects' responses are illustrated in figure 3. Only one subject gave the correct answer (shown in the lower-left corner of figure 3; if the reader has difficulty seeing that this is the correct answer, consider the top view of the stimulus apparatus shown in figure 2, and imagine placing the 45° angle between the square and the rod on the other side of the rod). One additional subject was incorrect by only 22.5°. Ten subjects (37%) said the surface would be in the frontal plane (as it began), with the stripe at the bottom (see figure 3). This answer is incorrect by 90°; in addition, it is an orientation that never occurs during the specified rotation. Eight subjects (29.6%) said the surface would be at 45° to the viewer, and perpendicular to the axis of rotation, with the stripe at the bottom (see figure 3). This answer is incorrect by 45° and does not maintain the surface at a slant to the axis (and thus clearly is an impossible outcome of the rotation). The remaining eight subjects (29.6%) gave seven different answers (see figure 3). In general, these answers were similar to the two more frequent incorrect responses.

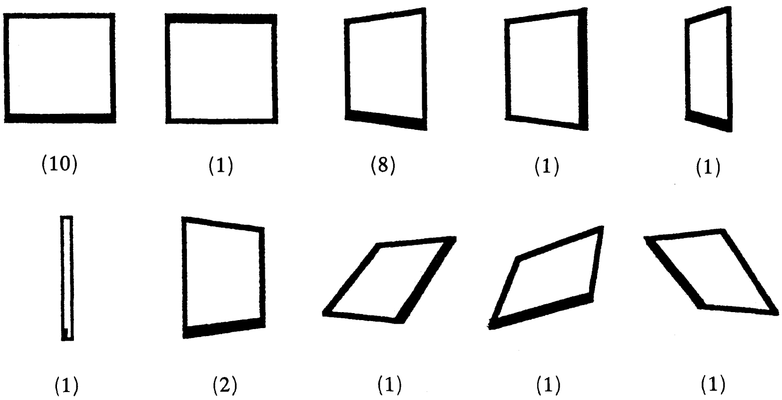

Figure 3. Subject responses in experiment 1, with the frequency of each response in parentheses. The correct answer is the leftmost figure in the bottom row.

2.2.2 *Response time and confidence ratings.* The mean response time was nearly 2 min (111 s). This duration verifies that people took the task seriously, and that it was difficult for them. Actually, the measurements of response time appear to be underestimates. Many subjects seemed to begin solving the problem as they paraphrased the instructions for the experimenter, and this time was not within the recorded interval.

Concerning the two modal responses (see figure 3), it seemed that the answer in which the square is said to end in the frontal plane (as it began), but to be upside down, was merely an educated guess. In contrast, the answer in which the square is said to end at 45° to the viewer seemed a more sophisticated answer; at least subjects understood that the surface orientation of the square would change. In this light, it is interesting that the eight subjects who gave the 'sophisticated' answer required nearly 2.5 times longer than nine of the ten subjects who gave the educated guess (2 min 33 s compared with 1 min), $t_{15} = 2.02$, $p < 0.05$). (A response time was not available for the remaining subject.)

The mean confidence rating was 3.44 on a 7 point scale, with a standard deviation of 1.68. No trends related to confidence ratings emerged in the data.

2.2.3 *Self reports.* The subject provided little systematic description of strategies or mental processes. The extreme difficulty of the task probably made it impossible for people either to rely on a particular strategy or to describe the mental processes which they had used. In the self reports that were obtained, some form of mental imagery of rotation was often mentioned, as were various geometric strategies. Perhaps the clearest statement was from the person who gave the correct answer. She said that she noted the acute angle between the rod and surface, and then reasoned that the angle would be reproduced on the opposite side of the rod, but with the surface turned upside down. This would appear to be an analytical geometric strategy, one in which analog mental rotation does not play a part. Given the apparent simplicity of this analytical strategy, it is interesting that more subjects did not adopt it (or, at least, did not succeed with it).

2.3 *Discussion*

In this simple concrete task, a square surface was oblique to a rod and the rod was oblique to the standard planes of the environment. Only one subject out of twenty-seven correctly predicted the outcome of a 180° rotation of the square about the rod, despite a mean solution time that approached 2 min. Nearly all of the answers were incorrect by 45° or more. There were two common answers. In one, the plane of the square was left unchanged, as if the square had rotated as a wheel rotates about its axle. In the second, the square was put perpendicular to the rod. Many of the answers were patently impossible outcomes (eg that the square would end perpendicular to the rod). The nature of the qualitative errors will be discussed more fully below.

Perhaps the most obvious strategy in a task such as this is to image the spatial transformation that is suggested. With general failure of the kind observed in this experiment, it is clear that all of the strategies adopted by the great majority of the subjects were unsuccessful. With these results to anchor the high end of a scale of difficulty, it was important to explore more fully the effect on mental imagery of the orientation of the square to the rod and the orientation of the rod to the room and viewer.

3 Experiment 2

Experiment 2 was designed to investigate the range of performance across variations of the two basic components of simple rotation. Two orientations of the square to the rod (normal and slanted 45°) were combined factorially with three orientations of the rod to the environment (vertical, oblique to two standard planes, and oblique to all three standard planes). In addition, subjects were asked to predict the outcomes of three different amounts of rotation (0°, 90°, and 180°).

3.1 *Method*

3.1.1 *Subjects.* Subjects were twelve undergraduates (six males and six females) at the University of Massachusetts, Amherst. They volunteered to participate in a study of mental rotation for course credit.

3.1.2 *Materials.* The stimuli were wooden squares attached to wooden rods, as in experiment 1. The variety of stimuli is illustrated in figure 4. There was a yellow stripe on one edge and side of each square. For any given orientation of the rod-and-square system, the stripe was always at the topmost edge of the square relative to the environment. There were two orientations of the square to the rod: normal and slanted 45°. There were three orientations of the rod to the room and viewer: these were (i) vertical, (ii) horizontal but at 45° to the frontal plane, and (iii) 45° to all standard planes. For each combination of orientation of the square to the rod and orientation of the rod to the room, subjects were asked to predict the outcomes of 0°, 90°, and 180° rotations. With two levels of the rod-to-square orientation, three levels of the rod-to-room orientation, and three amounts of rotation, there were eighteen rotation problems for each subject.

The rods and squares were suspended in a wooden framework behind a large white screen (80 cm by 60 cm). The subjects viewed the display through a 24 cm diameter circular hole in the screen. A flat sheet of black cardboard was placed behind the rods and squares, 46 cm from the screen. The center of the square was suspended between the center of the viewing port and the background. The rods and squares were illuminated by fluorescent room lighting and a 175 watt bulb with a reflector positioned behind the screen. In sum, subjects sat in front of a flat screen, looked through a large viewing port, and saw a well illuminated 3-D display against a homogeneous dark background.

The viewing port could be covered by a large white circle mounted on a metal post. The post slid back and forth easily within a track. Also attached to the post at the center of the circle was a magnetic assembly that held a metal square like those in the stimuli. The square was attached to a metal ball that in turn was held by a magnet. Thus, when the viewing port was covered, the metal square could be manipulated to show how the stimulus square would be oriented after a rotation. Beginning from the frontal plane, the illustration square had $\pm 90°$ of movement in depth, and could not be turned around so that the back faced front. Subjects were instructed to use the square to show only the slant and spin of the surface. Whether the square faced front or back would be specified separately (ie as if the square were transparent and the stripe could be on either side).

A thin steel rod through the center of a tennis ball was used to show subjects the direction in which a hypothetical rotation was to take place. Prior to display of the actual stimulus, the rod was held up at the angle of the stimulus rod and then rotated in the desired direction.

3.1.3 *Procedure*. At the beginning of the experimental session, subjects sat in front of the viewing port with the post slid back. The stimulus shown in the central position

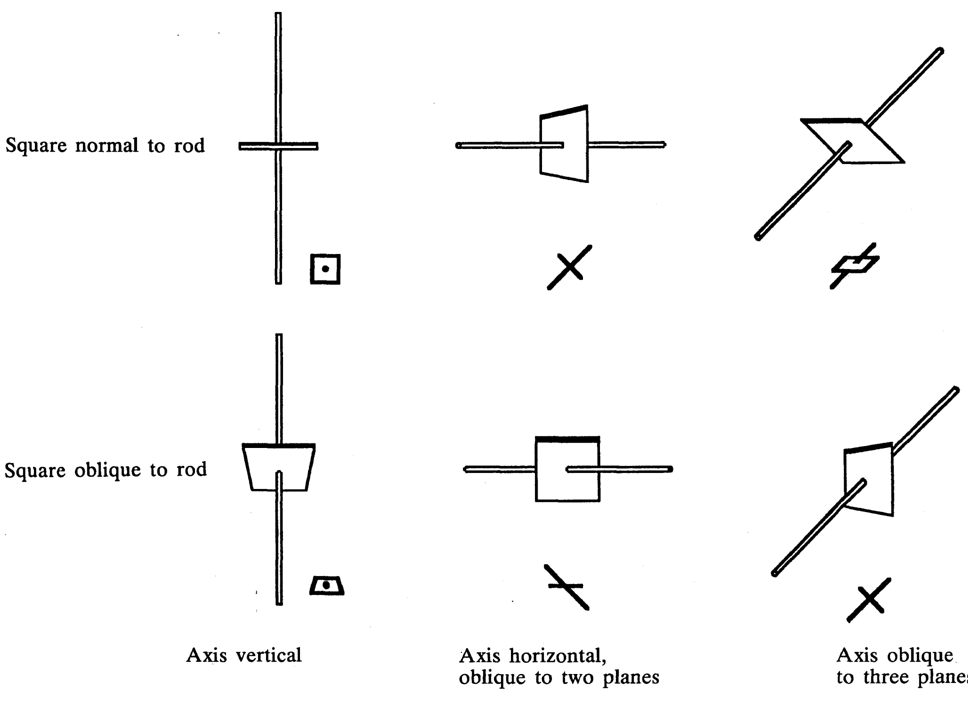

Figure 4. The rod-and-square displays of experiment 2. The larger pictures are the subjects' views; the smaller pictures are the top views.

of the top row in figure 4 was displayed. The experimenter read a set of instructions, while demonstrating the procedure with the equipment. Any questions asked by the subjects were then addressed. The instructions were similar to those in experiment 1. Three additional features were particularly important. First, the extent of 0°, 90°, and 180° rotations were explained with the use of diagrams. Second, the various angles of the rod that the subject might see were demonstrated. Third, the two orientations of the square to the rod were demonstrated. The experimenter suggested that this distinction was important to the outcome of the rotations. As part of this explanation, he briefly demonstrated both the normal-object and the oblique-object rotations about rods held at three different orientations (ie he briefly demonstrated motions similar to the ones subjects would be asked to consider).

Individual trials began with the screen covered and the illustration square in the frontal plane, with the stripe at the top. The experimenter held up the tennis ball and rod in the same orientation as the stimulus rod. He spun the ball in the direction of the desired rotation and instructed the subject how much rotation should be considered. When the subject was ready, he or she opened the screen, looked at the display, and developed an answer to the rotation problem. The subject took as long as he or she needed, then closed the screen and set the square to the predicted orientation. The experimenter then asked whether the stripe would be on the front or the back and recorded the answer. The orientation of the illustration square was recorded photographically by a camera supported on a tripod. No feedback was given.

The time from when the viewing port was opened to when the viewing port was closed was recorded with a silent stopwatch. The subject was told that the time would be recorded, but that the important consideration was to be as accurate as possible.

The three amounts of rotation were blocked for each display and administered in increasing order. That is, in the first trial for each display, the subject attempted only to reproduce the initial orientation of the square. The next trial contained a 90° rotation and the last trial of the block contained a 180° rotation. The order of stimulus displays was constrained so that the values of the two orientation variables changed with every completion of a block of three trials. The order of the blocks of stimulus displays was counterbalanced across subjects so that every combination of orientation variables occurred in every ordinal position.

3.2 *Results*

3.2.1 *Subject responses.* The photographic records of subjects' settings of the illustration square were converted to decimal coordinates. The 3-D orientation of each response was then computed and the results expressed in the roll-pitch-yaw descriptive system. These orientations then were compared with the correct orientations. The difference between actual and correct responses was calculated as the amount of rotation about a single fixed axis that would take the response orientation into the correct orientation. Specifically, the correct orientation was adopted as a reference system, and the response orientation was related to that system by a rotation matrix. The arccosine of the trace of that matrix gave the amount of rotation about a fixed axis that would align the two orientations. (That a simple rotation can relate any two orientations in space is expressed in Euler's theorem; eg Craig 1989; Gasson 1983). These error data, averaged across subjects, are presented in figure 5 for each of the 18 types of rotation. The primary statistical analysis was an analysis of variance with three repeated-measures factors: orientation of the square to the rod, orientation of the rod to the environment, and amount of rotation.

The error was substantially greater in all conditions in which the square was oblique to the axis, rather than normal to it, $F_{1,11} = 115.7$, $p < 0.001$. In addition, the error increased substantially as the axis was more oblique to environmental reference planes, $F_{2,11} = 64.9$, $p < 0.001$. There was also a clear interaction between these two variables, $F_{2,22} = 16.6$, $p < 0.001$. Even with the square oblique to the axis of rotation, error remained within the range of the normal-square rotations, as long as the axis of rotation was vertical. Only when the square was oblique to the axis and the axis was oblique to the planes of the room did the mean error become large (74° for the nonzero rotation problems).

The mean error for the conditions with the square normal to the rod and the rod oblique to all standard planes was 21°. However, this error apparently was not due to difficulty in understanding the rotation itself. The error at 0° of rotation (19.3°) was nearly as great as the error at 90° (24.5°) and 180° (22.1°)—see figure 5. Thus, the subjects found it difficult to encode and retain the three-plane oblique orientation of the square. This is a generalization of the well-known oblique effect to 3-D surface orientation (eg Appelle 1972; Howard 1982; Olson 1970; Palmer and Hemenway 1978).[2]

With the opportunity for various qualitative strategies in the solution of the rotation problems, mean error may not be the best index of relative difficulty. Perhaps certain problems lead to certain incorrect strategies, and some incorrect strategies just happen to produce larger errors than others. To eliminate this problem, the data were converted to all-or-none values. A tolerant criterion of ≤30° of error was used for scoring a response as correct. As seen in figure 6, this mode of scoring did not

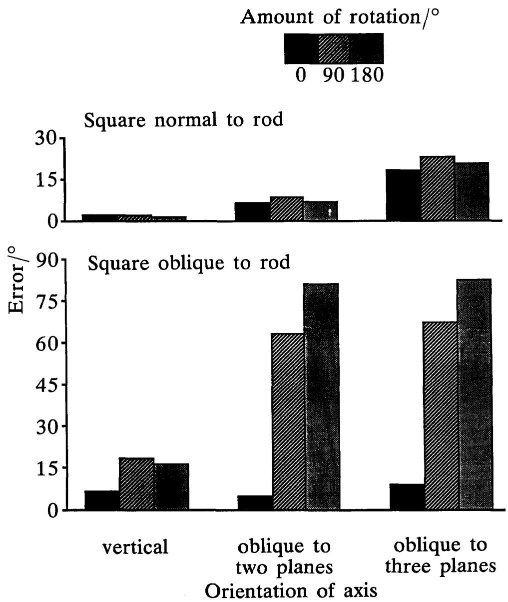

Figure 5. Mean error in experiment 2 as a function of the orientation of the square to the rod, orientation of the rod to the environment, and the amount of specified rotation.

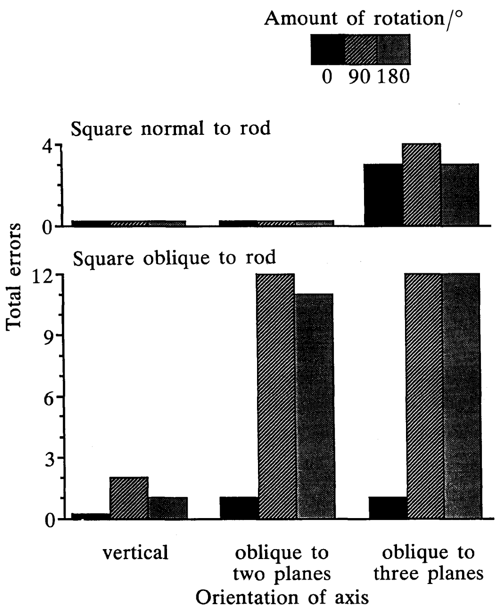

Figure 6. Error totals (maximum possible, 12) in experiment 2 as a function of the orientation of the square to the rod, orientation of the rod to the environment, and the amount of specified rotation.

[2] The fact that subjects generally did not accurately reproduce the orientations that were fully oblique to the room suggests that one difficulty in imagining rotations may simply be imagining oblique orientations. This appears to have had little effect on the present data, however. First, for those rotations for which subjects made many large errors (ie the double-oblique rotations), the correct responses for the 180° rotations were not oblique. Second, the observed error for the 90° rotations is more than three times as large as that for the rotations that demonstrate the oblique effect (65° compared with 21°).

change the pattern of data substantially. (Note that by this scoring criterion, two of the twenty-seven subjects in experiment 1 gave the correct answer.)

3.2.2 *Qualitative analysis of errors*. For the double-oblique rotations, 62.5% of the responses suggested that the plane of the square would not change with rotation (as if the rotation were that of a wheel rotating about its axle). Only these wheel-rotation errors emerged as a common category. In contrast, only 37% of the errors in experiment 1 were wheel-rotation errors, and there was a second type of common error (in which the square was said to end perpendicular to the rod). If, as suggested above, wheel-rotation errors are relatively unsophisticated, the high percentage of this error in experiment 2 may be due to the fact that with eighteen experimental trials, subjects took less time in each trial than they did in the single trial of experiment 1 (a mean of 9.3 s versus a mean of 111 s, for the comparable rotation problems).

3.2.3 *Response time*. In experiments in which the majority of responses in various cells are errors, response time becomes a general index of subjects' experienced level of difficulty. With this in mind, the mean response time for nonzero rotations ranged from 2.8 s to 14 s across the six stimulus types, as illustrated in figure 7. The pattern of response times indicated an additive effect of the orientation of the square to the axis of rotation ($F_{1,11} = 35.88$, $p < 0.001$) and the orientation of the axis to the room ($F_{2,22} = 3.74$, $p < 0.05$). The interaction between these two variables was not significant ($F < 1$). Rotations of 90° and 180° required more time than did rotations of 0°, $F_{2,22} = 4.89$, $p < 0.05$. This increased latency for nonzero rotations was more pronounced for the oblique-square rotation problems, $F_{2,22} = 7.86$, $p < 0.01$.

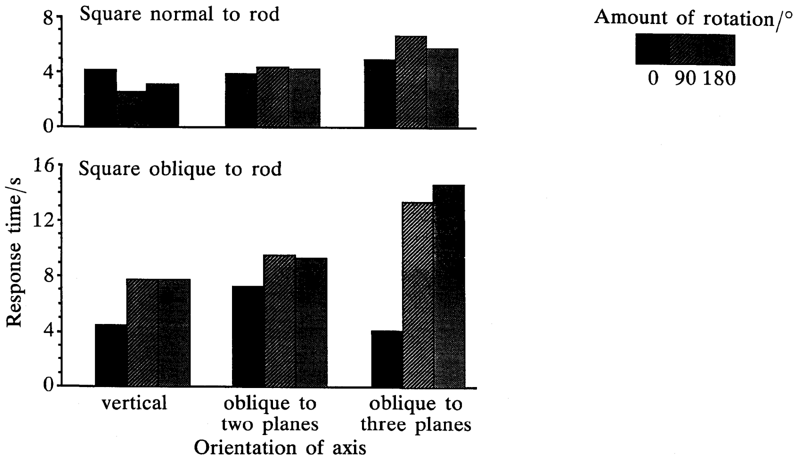

Figure 7. Response time in experiment 2 as a function of the orientation of the square to the rod, orientation of the rod to the environment, and the amount of specified rotation.

3.3 *Discussion*

The extreme difficulty of imagining double-oblique rotations, observed in experiment 1, was replicated. This was particularly clear in the size and frequency of errors. A simple summary of the data to this point is: (i) in the simple case, with the square normal to the axis of rotation and the axis vertical in the environment, subjects accurately predicted the outcome of the rotation; (ii) if either the square was normal to the axis or the axis was vertical in the environment, performance still was quite good; (iii) when the square was oblique to the axis of rotation and the axis was oblique to the environment, subjects were at a loss as to how the rotation would take place (the most common error was to suggest that the square would remain aligned with its initial orientation throughout the rotation, ie a wheel rotation); (iv) finally, even when a rotational motion was generally understood, surface orientations slanted to all three standard planes were difficult to reproduce accurately.

It is particularly interesting that when the square was oblique to the rod, comprehension of the hypothetical rotation was sensitive to the orientation of the rod. That is, for any given subject shown a rod with a square oblique to it, success or failure in the task depended simply on a 45° shift in the orientation of the rod to the room. This orientation sensitivity eliminates any hypothesis of the representation of rotational motion that suggests simply that people expect rotations to be wheel rotations. Instead, one must seek an explanation related to the orientation sensitivity observed in form perception (eg Palmer and Hemenway 1978; Rock 1973, 1983).

4 Experiment 3

A potentially important property of the displays in experiments 1 and 2 was the canonical orientation of the square in the double-oblique rotations. When the rod was horizontal and oblique to the walls, the square was frontal to the room and viewer (see figure 4). Even when the rod was oblique to all three planes of the room and the square was 45° to the walls, the square was still vertical (see figure 4). In contrast, the orientation of the square in the easier rod-vertical condition was not canonical (ie it slanted forward 45°). Other experimenters who have examined oblique-component rotations have used similar starting orientations (Just and Carpenter 1985; Massironi and Luccio 1989; Parsons 1987a, 1987b). Perhaps the canonical starting orientations of the double-oblique rotations contributed to the extreme difficulty of these rotation problems.

An additional issue leading to experiment 3 was whether the vertical axis is unique in making oblique-object rotation comprehensible. Perhaps any standard direction of the axis is sufficient. Work on form perception suggested that the vertical direction is a unique reference axis in perception (eg Palmer and Hemenway 1978; Rock 1973, 1983). If the vertical axis is also unique in the determination of mental rotation, this would provide further connection between mental rotation and the variables that affect form perception.

Methodological changes were introduced in experiment 3 to enable subjects to adopt the rod (ie the axis of rotation) as an object-centered reference system. Perhaps if there were encouragement to take the rod as defining its own vertical, whatever its orientation to the environment, subjects might succeed better with the double-oblique rotations (see Pani and Dupree 1992).

Four methodological features were adopted to encourage a rod-centered reference system. First, there was only one rod-and-square stimulus, and this fact was obvious to the subject. Thus, it was salient that a single rotational system was being reoriented through the various trials. Second, rather than always putting the stripe on the square at the topmost edge relative to the environment, the stripe now was at the top of the square relative to the rod. This emphasized the intrinsic directions of the rod-and-square system. Third, instructions were given with the rod-and-square displayed in a vertical orientation. Thus, subjects could come to think of the system as a vertical one that was reoriented at times. Finally, subjects were shown a large picture of a rocket-style spacecraft with a rod-and-square running lengthwise through the rocket and visible through a large opening. The instructions stated that the square was a transmitter dish and that rotation of the rod changes the direction of its transmission. Before every rotation problem, subjects demonstrated the direction of the rocket around the rod. This was intended to emphasize the rod-and-square as a self-contained system with its own set of intrinsic directions.

4.1 *Method*

4.1.1 *Subjects*. Subjects were eighteen undergraduates and advanced high-school students (nine males and nine females) in a summer-school course at Emory University. They volunteered to participate in a study of mental rotation for a fee of $5.00.

4.1.2 *Materials*. The stimuli are illustrated in figure 8. There was a single rod-and-square assembly, with the square at 45° to the rod. A yellow stripe was on one side of the square and at the top of the square when the rod was vertical. There were three basic classes of displays. There were standard directions of the rod, directions of the rod oblique in two planes and with the squares in canonical starting orientations, and the same two-plane oblique rods but with the squares in noncanonical starting orientations. For each of these three classes of stimuli there was an orientation of the rod that was in the sagittal plane (ie the plane that is vertical and edge on to the viewer) and a stimulus in the horizontal plane. Subjects were asked about rotations of 90° and of 180°. The two amounts of rotation were blocked. With three classes of display, two planes for each class, and two amounts of rotation, each subject had twelve trials.

The rod-and-square assembly was supported by two stands formed from 1.25 cm copper pipe painted dull gray. There was no longer a screen or a special background. The stimuli were presented on a desk that was against a wall.

An illustration square held by a magnet was again used. In this case, however, the square had a metal ball on each side. Subjects now could pick the illustration square off the desk, put it into the desired orientation, and then place it against the magnet. This allowed the illustration square to be placed so that the front faced back. The magnet was on a stand that could easily be moved by the experimenter.

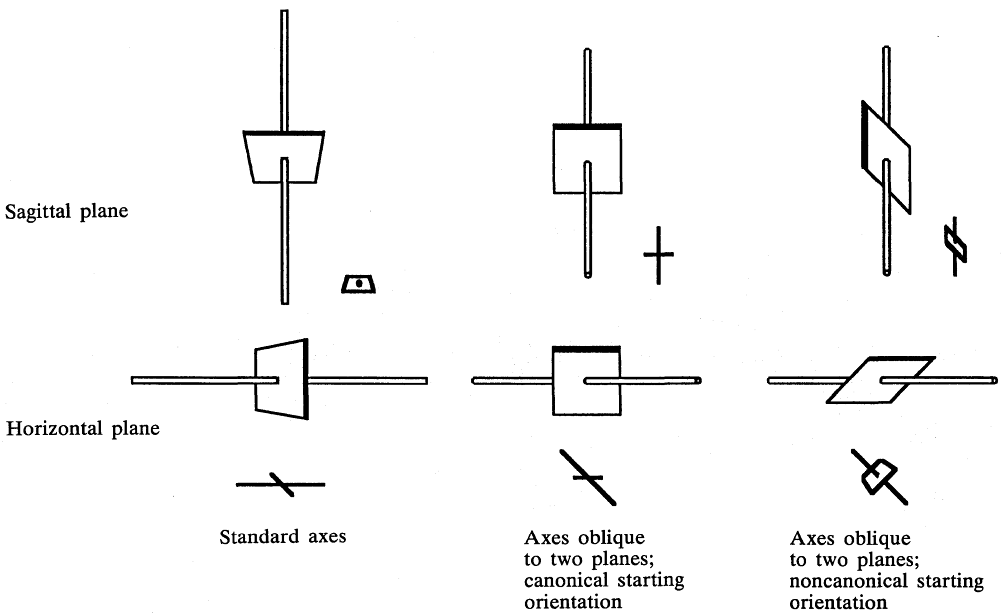

Figure 8. The rod-and-square displays of experiment 3. The larger pictures are the subjects' views; the smaller pictures are the top views.

4.1.3 *Procedure*. The experiment began with the subject reading a set of instructions. The rod-and-square assembly was oriented vertically on a large table. Behind the assembly was a large picture of a spacecraft flying at an oblique angle. In the picture, a rod-and-square assembly was visible through a large opening in the spacecraft. The rod ran vertically through the middle of the craft. As in experiment 2, the instructions alerted the subject to all of the variations of the stimuli that he or she would be seeing. The instructions also made it plain that the oblique angle of the square to the rod was important to the form of the rotation. In addition, it was pointed out that there would be only twelve trials, so that the subject would have ample time for each. After the subject had read the instructions, the experimenter paraphrased each major point. Then any questions by the subject were addressed.

The picture was placed to one side during the experiment, but remained visible. In each trial, the experimenter set up the appropriate stands in the appropriate orientation to hold the rod-and-square assembly. Then the subject would show with his or her hands the orientation of the imaginary spacecraft about the rod. The experimenter then took a 2.5 cm diameter dowel, held it up at the angle of the rod, and demonstrated the direction of the rotation. At the same time, he reminded the subject of the amount of rotation to be considered. The subject then took as long as he or she needed to develop an answer. When the illustration square was picked up, the experimenter slid the magnet over so that it stood just in front of, or beside, the stimulus square. The subject then placed the illustration square on the magnet and adjusted the final orientation of the square.

The description of the orientation of the illustration square was accomplished with the experimenter-rating system used in experiment 1. This was less precise than the photographic methods of experiment 2, but the results of that experiment suggested that great precision was probably unnecessary in experiment 3.

With the aid of a silent stopwatch, the experimenter recorded the duration of each trial from the moment that the instructions for the trial were completed to the moment that the subject picked up the illustration square.

4.2 *Results*

4.2.1 *Error measures.* As in experiment 1, the coding system of experiment 3 expressed the orientations of the illustration square in the roll-pitch-yaw descriptive system. The experimenter codings were converted to decimal values. These measurements of orientation then were compared with the correct orientations by the use of the system of experiment 2. That is, error was calculated as the amount of rotation about a single fixed axis that would take the response orientation into the correct orientation. These error data, averaged across subjects, are presented in figure 9 for each of the 12 rotation problems.

The experiment conformed to a factorial design with three within-subject factors: basic stimulus type (standard axes, oblique axes with canonical starting orientations, oblique axes with noncanonical starting orientations), plane of the rod (vertical and horizontal), and amount of rotation (90° and 180°). However, the major analyses were planned comparisons within that design.

With the amount of orientation error as a measure, the vertical axis of rotation appeared unique in being associated with accurate mental rotation of the oblique square about the rod. Specifically, the subjects were more accurate in demonstrating the outcome of a rotation about the vertical axis than about the frontal-horizontal axis, $F_{1,17} = 7.93$, $p = 0.01$. The subjects were not more accurate with the frontal-horizontal axis than with the comparable oblique axes (ie those in the horizontal plane).

As noted above, a continuous scale of error may not be always the best index of relative difficulty. Therefore, the error data were converted to all-or-none scoring, the results of which are illustrated in figure 10. With this index of error, subjects again were more successful with the vertical than with the frontal-horizontal axis,

$T_{12} = 7$, $p < 0.005$. In addition, however, the frontal-horizontal axis led to a large improvement of performance over the double-oblique rotations. That is, subjects were more successful with the frontal-horizontal axis than with the oblique axes in the horizontal plane; for the canonical starting orientation, $T_8 = 0$, $p < 0.005$; for the noncanonical starting orientation, $T_{14} = 24, p < 0.05$.

The intermediate difficulty of the frontal-horizontal axis of rotation raises the question of individual differences. In general, as subjects did better with rotation problems that involved the frontal-horizontal axis, they also did better with the other problems, $r = 0.42, p < 0.05$, for the number correct in the two sets of problems.

The comparisons above were the only ones to show a difference between the continuous measure of error and the all-or-none measure. Remaining comparisons are reported solely in terms of the continuous measure.

Subjects were substantially more accurate with the two standard axes taken together than with the oblique axes, $F_{1,17} = 19.55$, $p < 0.001$. Thus, despite the effort to emphasize an axis-centered reference frame, double-oblique rotation problems remained difficult.

Although all of the double-oblique rotation problems were difficult, subjects were more accurate with the double-oblique rotations that had noncanonical starting orientations than with those that had canonical orientations, $F_{1,17} = 4.45$, $p = 0.05$. This superiority of noncanonical orientations was uniform across subjects; it was not differentially associated with success with the frontal-horizontal axis, nor with overall performance.

The two amounts of rotation, 90° and 180°, were intended only as replications, and there was no overall difference between the two, $F_{1,34} = 2.519$, $p > 0.1$. It did appear, however, that the difference in level of error between the 90° and 180° rotations was relatively high for the double-oblique rotations that had canonical starting orientations, $F_{2,17} = 3.266$, $p = 0.05$. This elevated difference in error is due to the higher incidence of the wheel-rotation strategy for the double-oblique rotations with canonical starting orientations (see below). Wheel-rotation errors lead to 27° more error for a 180° rotation than for a 90° rotation.

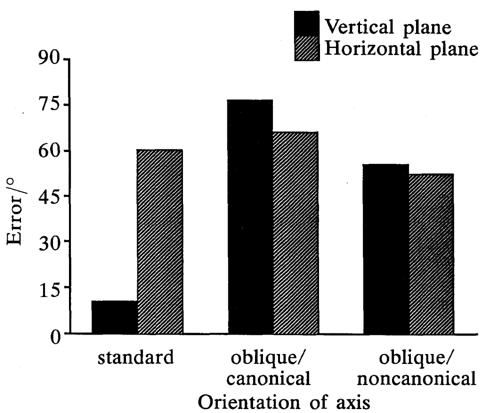

Figure 9. Mean error in experiment 3 as a function of the basic display type and the plane that included the axis of rotation.

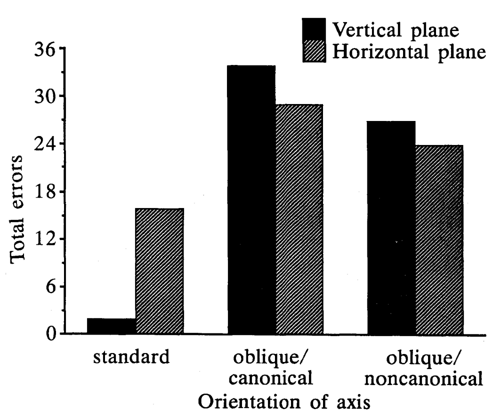

Figure 10. Error totals (maximum possible, 36) in experiment 3 as a function of the basic display type and the plane that included the axis of rotation.

4.2.2 *Qualitative analysis of errors.* The descriptive data were examined for regularities in the qualitative nature of subjects' errors. As with the all-or-none scoring, a response counted as an error if it was more than 30° from the correct answer.

There was a greater variety of common types of error in this experiment than in experiment 2. Categorization of these responses necessarily included judgments of what descriptions of orientation are theoretically interesting. Where there were not competing categories, a response was included in a category if the response was $\leqslant 25°$ from the canonical value of the category. Where there were competing categories, the canonical values were generally 45° apart and the dividing line was at 22.5° from both.

A large percentage of the errors (73.6%) can be accounted for with just four basic categories of error. Moreover, these errors were strongly associated with particular types of rotation problem. None of the orientations corresponding to these categories occurs at any point in the specified rotational motions. The first two types of error correspond to the two common errors in experiment 1. The four types of error were:

(i) There were 31 wheel-rotation errors, accounting for 24% of the total errors. This percentage is much smaller than in experiment 2, in which 62.5% of the errors were wheel-rotation errors. A large proportion of the wheel-rotation errors in experiment 3 (77.4%) were for the double-oblique rotations with canonical starting orientations. The remaining wheel-rotation errors were evenly split between errors with the frontal-horizontal axis and the remaining double-oblique axes (9.7% and 12.9%, respectively). The association between canonical starting orientations and wheel-rotation errors accounts for the larger percentage of wheel-rotation errors in experiment 2 than experiment 3 (ie the double-oblique rotation problems in experiment 2 had canonical starting orientations).

(ii) A common error type was that subjects said the square would end perpendicular to the rod (21 errors, 16.3% of the total). Note again that this orientation would require 'breaking the glue' attaching the square to the rod. With one exception, all of these errors were for double-oblique rotations in which the correct response was that the square would be upright in the frontal plane. (In particular, 9 of the errors were for 180° rotations with canonical starting orientations, and 11 of the errors were for 90° rotations with noncanonical starting orientations.)

(iii) A similar type of error was that subjects said the square would end parallel to the rod (22 errors, 17% of the total). Again, the majority of these errors (77.3%) occurred for double-oblique rotations in which the correct response was that the square would be upright in the frontal plane. (In particular, 7 of the errors were for 180° rotations with canonical starting orientations, and 11 of the errors were for 90° rotations with noncanonical starting orientations.)

(iv) Where correct answers would have been oblique in three planes, responses were often normalized so that they were oblique only in two planes (21 errors, 17.3% of the total). Responses were categorized as normalized if they were within 45° of the correct answer and the ratio of pitch error to yaw error (or vice versa) was at least two to one.

4.2.3 *Response time.* Response times, averaged across amount of rotation and subjects, are illustrated in figure 11. Response time for the vertical axis of rotation was shorter than for any other axis, $F_{1,17} = 18.42$, $p < 0.001$, for the comparison with the frontal-horizontal axis. Response times for rotation problems with standard axes were shorter than for problems with oblique axes, $F_{1,17} = 9.01$, $p < 0.01$. There was not a significant difference in response time between the double-oblique rotations with canonical starting orientations and those with noncanonical starting orientations, $p > 0.1$. The 180° rotation problems required more time than did the 90° problems, $F_{1,17} = 4.693$, $p < 0.05$. Finally, response times for axes of rotation in the horizontal plane were consistently larger than for axes in the sagittal plane, $F_{1,17} = 5.529$, $p < 0.05$. This result is consistent with the general superiority of the vertical over horizontal directions in perception.

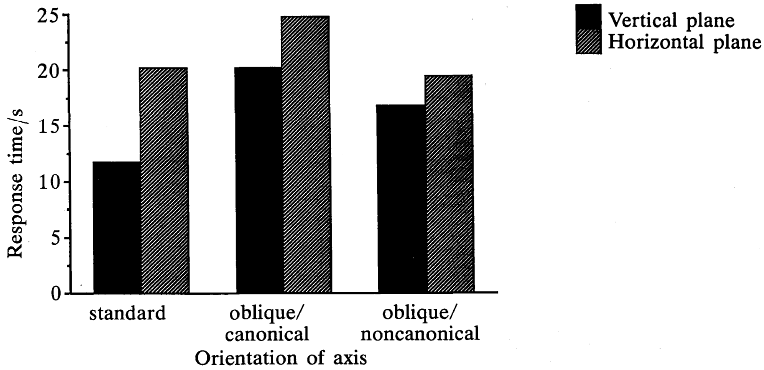

Figure 11. Response time in experiment 3 as a function of the basic display type and the plane that included the axis of rotation.

4.3 *Discussion*

It was found in experiment 2 that predicting the outcome of an oblique-object rotation depended on the orientation of the axis of rotation. If the axis was vertical, the rotation was generally predicted accurately. In experiment 3, there was a determined effort to lead subjects to think of the oblique-square rotation system as having an axis-centered vertical, regardless of the orientation of the axis to the environment. This effort was ineffective—the double-oblique rotations continued to be extremely difficult for subjects.

The results of experiment 3 indicated that the vertical axis of rotation is unique in leading to accurate prediction of the oblique-square rotations. The vertical-axis problems were solved correctly on virtually every trial. The frontal-horizontal axis of rotation was easier for many subjects than were the oblique-axis rotations. Whether only the vertical axis was effective, or standard axes in general were effective, appeared to depend upon individual differences in spatial ability. Subjects who did well with the frontal-horizontal axis tended to have generally good performance overall. It seems likely, therefore, that performance on rotation problems using the frontal-horizontal axis would correlate with other measures of spatial ability.

The results of experiment 3 also revealed that the initial orientation of a double-oblique rotation influences the difficulty of the problem. When the starting orientations of the square were not in a vertical plane of the environment, subjects made fewer errors, and different types of errors. That is, when the starting orientations were noncanonical, subjects were more likely to understand that the plane of the square would change with the rotation.

More than half of the responses in this experiment (59.7%) were errors of greater than 30° of orientation. Of the errors, 73.6% fell into four qualitative classes. All of these classes represent orientations that would not occur during the specified motions. In one third of the errors, corresponding to classes (ii) and (iii) in the aforementioned list, the square was said to end in an orientation that would not be oblique to the rod. In the following, brief explanations of the four classes of error are offered.

Consider that wheel-rotation errors only were common when the square was in a canonical orientation in the environment. In this situation, two basic spatial reference systems, the environment and the object, are aligned with each other. This mutual reinforcement would offer subjects a salient option for the orientation of the motion and would tend to distract from alternatives (see section 5).

The remaining three types of error can be accounted for by considering them variants of one basic strategy. In this strategy, subjects begin by establishing a general direction of the square after rotation by considering the outcome of a wheel-rotation of a vector perpendicular to the rod (and intersecting the sagittal plane of the square). The subjects must then relate the oblique angle of the square to the rotated vector. The three classes of errors arise from different ways of relating the oblique angle to the vector.

In certain rotation problems, the initial vector ends at 45° to the frontal plane of the display (and to the square, when there are canonical starting orientations). It appears that subjects mistake this oblique angle for the one they are seeking, and they take the vector as providing the correct orientation of the square. If the vector is taken to illustrate the orientation of the vertical edges of the square, the square is said to end perpendicular to the rod, producing the second type of error listed above. If the vector is taken to illustrate the direction in which the square will face, the square is said to end parallel to the rod, producing the third type of error.

In certain rotation problems, the initial vector ends in the frontal plane of the display (and coincident with the square when there are canonical starting orientations). Thus, the oblique angle of the square is not immediately given, and the subjects must provide it. They then incorrectly orient the square so that it has a 45° angle to the frontal plane of the environment (and to the starting orientation of the square, if the two are aligned), thus producing the fourth type of error. In all three of these common types of error, subjects make the mistake of not maintaining the oblique angle of the square relative to the rod. Instead, they place the angle relative to the frontal plane of the display (and to the square, when the two are aligned).

5 General discussion

In three experiments, simple 3-D constructions were used as displays, and subjects had a small number of problems to solve (just one problem in experiment 1). Subjects were asked to predict the outcomes of specific rotational motions of a square about a rod and to demonstrate their predictions. Performance ranged from fast and accurate responses for certain rotation problems to slow and highly inaccurate responses for other problems.

When the square was normal to the axis of rotation, the rotations were well understood. This is not to say, however, that the result of such a rotation could be demonstrated with precision. Orientations oblique to all three standard planes were reproduced inaccurately, although the subjects understood the properties of the motion itself. When the square was oblique to the axis, performance depended on the orientation of the axis. The motions were predicted well when the axis of rotation was vertical, but very poorly when the axis of rotation was oblique to the environment and viewer. For the more skilled subjects, both the vertical and the frontal-horizontal directions of the axis led to good performance. The difficulty of the double-oblique rotation problems was greater when the initial orientation of the square was vertical in the environment. When the initial orientation was noncanonical, subjects made fewer errors, smaller errors, and more sophisticated errors (ie fewer wheel-rotation errors). Over the three experiments, four qualitative categories described the majority of the errors.

The motions studied in these experiments were all relatively simple from a physical and mathematical standpoint, and yet there was a large range of performance. It is possible to explain this range with two basic statements. First, there is a single form of rotational motion that people generally are competent to perceive or imagine. Second, people are most efficient at perceiving or imagining this motion when it is aligned with the spatial reference systems in general use.

Regarding the first statement, Shepard has suggested that the psychologically simple form of rotational motion is rotation about a single fixed axis, the physically shortest rotation (Carleton and Shepard 1990a, 1990b; Shepard 1984, 1988; Shiffrar and Shepard 1991; see also Cutting and Proffitt 1982). Considering each point on an object, such a motion produces a set of parallel circular paths centered on and normal to the axis of rotation.

Regarding the second statement, it has been suggested that simple rotational motion is more recognizable or imaginable for people if it is aligned with either of two basic reference systems, the principal directions of the environment (especially the vertical) and the intrinsic directions of the object (Carleton and Shepard 1990a, 1990b; Just and Carpenter 1985; Massironi and Luccio 1989; Pani 1989; Pani and Dupree 1992; Parsons 1987a; Shiffrar and Shepard 1991). In contrast, the viewer-relative reference system is not generally important to the comprehension of rotation (Pani and Dupree 1992). Further, the geometrically most obvious reference system for a rotational motion, the axis of rotation, is not generally effective for people. If it was, people would not have special difficulty with double-oblique rotations. That is, moving the axis of rotation from the vertical to an oblique orientation leaves the orientation of the object and the motion relative to the axis unchanged. Even in experiment 3, in which the axis of rotation was made very obvious, subjects could still not predict the outcomes of double-oblique rotations.

These suggestions about the comprehension of rotation account for the basic pattern of the present findings in the following way. When the square is normal to the axis of rotation, the rotational motion is aligned with the object-relative reference system of the square (ie the plane of the square). Hence, the motion is salient and easy to comprehend. When the square is oblique to the axis of rotation, the rotational motion is oblique to the object-relative reference system; the orientation of the square is misleading in the attempt to comprehend the motion. However, if the axis of rotation is vertical, the rotational motion is aligned with the primary environmental reference system. Hence, the motion is again salient and easy to comprehend. Finally, when the square is oblique to the axis of rotation, and the axis is oblique to the environment, the rotational motion is oblique to both reference systems. Hence, the motion is not salient and people have difficulty comprehending it.

This theoretical framework also accounts for the common types of qualitative errors made by the subjects (see section 4.3). First, wheel-rotation errors are common when the starting orientation of the object is canonical. In this case, there is alignment of the two primary spatial reference systems, the object and the environment. The resulting salient orientation of the object would distract from recognition of the true orientation of the motion and would provide a tempting alternative—a motion aligned with both reference systems. The two basic features of the other common types of errors also are consistent with this view. First, subjects appear to begin problem solution by constructing a wheel-rotation of a vector about the rod. That is, subjects begin an analysis of the rotation by establishing a spatial component that is aligned with the plane of rotation. Second, in adding the oblique angle of the square to the rotated vector, subjects appear to mistakenly place the angle relative to the environmental reference system (and to the object, when the two are aligned) rather than to the axis of rotation.

There are a number of areas in which theoretical understanding of the comprehension of rotational motion is in need of further development. I will briefly mention three of these. First, it seems likely that the strong limits on performance found in the present experiments are related in part to the experimental methodology that was used. In the standard methodology for studying mental rotation (Cooper and Shepard 1973; Shepard and Metzler 1971; see also Shepard and Cooper 1982), hypothetical endpoints of the rotations are shown or implied to subjects, and the subjects' task is to discriminate between a rotated object and its enantiomorph. In a preliminary experiment not reported here, it was found that the difficulty of imagining double-oblique rotations is greatly reduced when these standard techniques are used (Pani and Kosslyn 1989, unpublished data). In more traditional areas of cognitive psychology (especially the studies of perception and memory) it is well established that different types of task lead to different types of cognitive processing and different levels of performance. It seems likely that the same will prove to be true for spatial cognition.

A second issue concerns the exact form of psychologically simple rotation. What particular spatiotemporal structures are organized when a person comprehends a rotation? One way to think of a simple rotation is that it produces planes of rotation. Every point on an object moves in a circular path about the axis. The planes of the circular paths are parallel to each other and perpendicular to the axis of rotation. Perhaps planar rotations, such as the motions of wheels, are the psychologically simple form of rotation. If so, then a person would look for reference parts on a rotating object (eg salient surfaces, edges, or corners) that move within planes of rotation. Such parts would serve in the organization of the motion of the whole object (see Hochberg and Gellman 1977; Just and Carpenter 1976). Figure 12 illustrates planes of rotation that would be useful for comprehending the rotations studied in the present experiments. Because these planes exist only through spatial integration over the rotation, such an organization of a rotation defines a 'motion space'.

An alternative (but consistent) embodiment of simple rotation is the radially symmetric surface swept out by the object during the rotation. For example, a solid rectangle rotating about its primary axis sweeps out a cylindrical surface as it rotates. In this form of organization, the motion space is quite similar to solids of revolution, to constraint manifolds (as discussed in theoretical kinematics), or to generalized cones (with straight axes; Biederman 1987; Binford 1971; Marr 1977, 1982). This shell-like motion space can include the planes of rotation as reference sections through the space. Whether a person organized a rotation in terms of a shell or simply in terms of planes of rotation might depend on the shape of the rotating object. It is relatively simple to imagine a solid rectangle rotating through a cylindrical space. It is much more difficult to imagine the hourglass shape produced by the rotation of the oblique square in figure 12. For that rotation, people might explicitly organize only the planes of rotation.

If people were to base comprehension of a rotation on the organization of a motion space, it also would be necessary to organize (or chunk) the phase relations between the different reference parts of the object as they traveled through the space (see Cutting and Proffitt 1982). Such phase relations are invariant throughout the rotation.

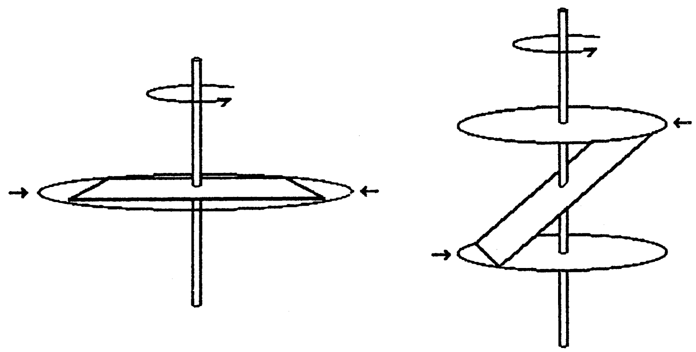

Figure 12. Planes of rotation that could be used to organize two rotations of a square. The small arrows indicate the opposite phase relations of the two edges within the planes.

For example, in the oblique-object rotations of the square studied in the present experiments (see figure 12), the top and bottom edges of the square (relative to the rod) always are opposite each other. If a motion space is organized, and the phase relations among reference parts are established, there is one remaining degree of freedom in comprehending the structure of a rotation: the radial position of the object within the motion space at different points of time. In other words, the parsing of a rotation into planes of motion and phase relations among reference parts reduces the kinematic structure of the motion to a wheel rotation. Experiments by Pani and Shippey (1992) suggest that the simplicity of the phase relations within the planes of rotation has a large effect on the perceived coherence of rotations.

A third theoretical issue concerns the general domain of spatiotemporal structures hypothesized to underlie the psychological structure of simple rotation. In their wide-ranging exploration of this topic, Carleton and Shepard (1990a, 1990b) emphasize three basic source domains. These are classical physics, classical kinematics (ie the geometry of motion independent of the action of forces), and extensions of kinematics to take account of intrinsic axes of the rotating objects. Classical physics and kinematics can be used to make predictions about what motions will be psychologically simple, and it is possible that representations of motion consistent with these systems have been internalized by humans through evolution (Carleton and Shepard 1990a, 1990b; Shepard 1984, 1988; Shiffrar and Shepard 1991). As Shepard and his colleagues note, however, there is evidence that the general principles of physical dynamics have not been internalized. People are rather poor at predicting the outcomes of motions across variation in the distribution of mass of the objects (eg Proffitt and Gilden 1989; Proffitt et al 1990), and it appears that people typically understand the laws of motion only for familiar events (eg Kaiser et al 1986; McCloskey 1983; McCloskey et al 1980).

Neither physics nor kinematics lead to specific predictions about the importance to humans of the alignment of rotational motions with spatial reference systems (see Carleton and Shepard 1990a, 1990b). However, a theory which accounts for these anisotropies is an extension of kinematics if the spatial reference systems can be equated with geometric structures, as when an object-relative reference system is equated with axes of rotational symmetry of the object (Carleton and Shepard 1990a, 1990b). In addressing this same issue, Shiffrar and Shepard (1991) speak of general geometric constraints on the perception of rotation. They point out that object-relative reference systems, and the reference system associated with the environmental vertical, have been shown to be important in a variety of spatial tasks, and that people have a general weakness in perceiving and imagining orientations oblique to reference systems. I would like to continue this discussion by suggesting that the geometric constraints on the comprehension of rotation are part of a set of regularities that determine the perceptual organization of 3-D space generally. In support of this view, consider the close similarity between constraints on the comprehension of rotational motion and constraints on form perception.

To characterize rotational motion, assume that the psychological organization of simple rotation includes attention to reference parts of an object moving through a motion space structured as planes of rotation (as in figure 12). This space is symmetric, ribbed with planar sections, and is aligned with and centered on the axis of rotation. Given such a structure, there are at least four types of basic similarity between the comprehension of rotation and form perception. First, the same spatial reference systems appear to be primary in defining the orientations of forms and rotations. Of the reference systems external to the object and motion, people generally use environmental reference systems rather than the viewer perspective (Corballis et al 1978; Hinton and Parsons 1988; Pani and Dupree 1992; Rock 1973, 1983).

The permanent environment is the most effective external reference system, but a local spatial frame also can be used (Koffka 1935; Palmer 1980; Pani and Dupree 1992). In addition to external reference systems, there are object-relative reference systems, and use of these is critical both to form perception and to the comprehension of rotation (see the discussion above; Hinton 1979; Marr and Nishihara 1978; Palmer 1989; Rock 1973). There is not a unique reference system associated with the comprehension of rotational motion (such as the axis of rotation). Second, people do not comprehend a rotation when the motion space is oblique to all basic reference systems, and people have a general weakness in perceiving and imagining oblique orientations in space (see experiment 2; Howard 1982; Palmer and Hemenway 1978; Shiffrar and Shepard 1991). Third, for the comprehension both of rotation and of form, the vertical direction in the environment is a more effective reference direction than is the horizontal direction (Palmer and Hemenway 1978; Pani and Dupree 1992; Rock 1973; Shiffrar and Shepard 1991). Fourth, the concepts of a motion space and the phase relations within it are attempts to explain why certain motions are psychologically simple. In taking this view, it becomes clear that simple rotations have much in common with simple forms. In particular, the motion space is a symmetric space with parallel sections, and people find symmetric and parallel structures to be relatively simple (eg Biederman 1987; Garner 1974; Goldmeier 1972; Koffka 1935; Palmer 1982, 1983, 1989). Regarding phase relations, Pani and Shippey (1992) found that rotations are comprehended particularly well when reference parts of an object in separate planes of rotation are in phase with each other. This occurs for all parts of an object just when the object is a generalized cone about the axis of rotation. A number of theorists have suggested that the generalized cone is the psychologically simplest volume (Biederman 1987; Binford 1971; Marr 1977, 1982).

The similarities between form perception and the comprehension of rotation correspond to constants of 3-D spatial organization. They include the choice of reference systems in the definition of spatial properties (see Pani and Dupree 1992), salient reference directions, the distinction between aligned (parallel) and oblique relationships among spatial components, and the importance of symmetry and phase relations among parts of objects. Because these properties of organization appear to be common and fundamental to the morphology of the world, their importance is consistent with the view of Shepard (1984, 1988) that people have internalized properties of the physical world that have been invariant over evolutionary time. Of course, to the degree that such organizational constants are part of the invariant morphology of the world, they are also consistent with a view that emphasizes perceptual learning. In either event, these aspects of perceptual organization define an ecological and psychological 3-D geometry that is distinguished from the more abstract principles in the general domain of mathematics.

In closing, it is noteworthy that the present research fits into a growing class of studies in which the boundaries of the typical person's spatial cognition are explored. These studies have begun to reveal a structure within spatial cognition, such that certain spatiotemporal forms appear simple and natural, while others appear complex and novel (eg Hinton 1979; Kaiser et al 1986; Massironi and Luccio 1989; McCloskey et al 1980; Proffitt and Gilden 1989; Proffitt et al 1990; Rock et al 1989). An important outcome of identifying the boundaries of everyday spatial knowledge and skill will be to aid definition of the spatial abilities uniquely associated with skill in the physical sciences, engineering, and architectural design.

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References

Appelle S, 1972 "Perception and discrimination as a function of stimulus orientation: The oblique effect in man and animals" *Psychological Bulletin* **78** 266 - 278

Biederman I, 1987 "Recognition by components: A theory of human image understanding" *Psychological Review* **94** 115 - 147

Binford T O, 1971 "Visual perception by computer", paper presented at IEEE Systems Science and Cybernetics Conference, Miami, FL

Carleton E H, Shepard R N, 1990a "Psychologically simple motions as geodesic paths. I. Asymmetric objects" *Journal of Mathematical Psychology* **34** 127 - 188

Carleton E H, Shepard R N, 1990b "Psychologically simple motions as geodesic paths. II. Symmetric objects" *Journal of Mathematical Psychology* **34** 189 - 228

Cooper L A, Shepard R N, 1973 "Chronometric studies of the rotation of mental images", in *Visual Information Processing* Ed. W G Chase (New York: Academic Press) pp 75 - 176

Corballis M C, Nagourney B A, Shetzer L I, Stefanatos G, 1978 "Mental rotation under head tilt: Factors influencing the location of the subjective reference frame" *Perception & Psychophysics* **24** 263 - 273

Craig J J, 1989 *Introduction to Robotics: Mechanics and Control* (Reading, MA: Addison-Wesley)

Cutting J E, Proffitt D R, 1982 "The minimum principle and the perception of absolute, common, and relative motions" *Cognitive Psychology* **14** 211 - 246

Folk M D, Luce R D, 1987 "Effects of stimulus complexity on mental rotation rate of polygons" *Journal of Experimental Psychology: Human Perception and Performance* **13** 395 - 404

Garner W R, 1974 *The Processing of Information and Structure* (Hillsdale, NJ: Lawrence Erlbaum Associates)

Gasson P C, 1983 *The Geometry of Spatial Forms* (West Sussex: Ellis Horwood)

Goldmeier E, 1972 "Similarity in visually perceived forms" *Psychological Issues* **8**(1) 1 - 135

Green B F, 1961 "Figure coherence in the kinetic depth effect" *Journal of Experimental Psychology* **62** 272 - 282

Hinton G, 1979 "Some demonstrations of the effects of structural descriptions in mental imagery" *Cognitive Science* **3** 231 - 250

Hinton G E, Parsons L M, 1988 "Scene-based and viewer-centered representations for comparing shapes" *Cognition* **30** 1 - 35

Hochberg J, Gellman L, 1977 "The effect of landmark features on mental rotation times" *Memory & Cognition* **5** 23 - 26

Howard I P, 1982 *Human Visual Orientation* (New York: John Wiley)

Just M A, Carpenter P A, 1976 "Eye fixations and cognitive processes" *Cognitive Psychology* **8** 441 - 480

Just M A, Carpenter P A, 1985 "Cognitive coordinate systems: Accounts of mental rotation and individual differences in spatial ability" *Psychological Review* **92** 137 - 172

Kaiser M K, Jonides J, Alexander J, 1986 "Intuitive reasoning about abstract and familiar physics problems" *Memory & Cognition* **14** 308 - 312

Koffka K, 1935 *Principles of Gestalt Psychology* (New York: Harcourt, Brace and World)

Marr D, 1977 "Analysis of occluding contour" *Proceedings of the Royal Society of London, Series B* **197** 441 - 475

Marr D, 1982 *Vision* (San Francisco: W H Freeman)

Marr D, Nishihara H K, 1978 "Representation and recognition of the spatial organization of three-dimensional shapes" *Proceedings of the Royal Society of London, Series B* **200** 269 - 294

Massironi M, Luccio R, 1989 "Organizational versus geometric factors in mental rotation and folding tasks" *Perception* **18** 321 - 332

McCloskey M, 1983 "Naive theories of motion" in *Mental Models* Eds D Gentner, A L Stevens (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 299 - 324
McCloskey M, Caramazza A, Green B, 1980 "Curvilinear motions in the absence of external forces: Naive beliefs about the motions of objects" *Science* **210** 1139 - 1141
Olson D R, 1970 *Cognitive Development: The Child's Acquisition of Diagonality* (New York: Academic Press)
Palmer S E, 1980 "What makes triangles point: Local and global effects in configuration of ambiguous triangles" *Cognitive Psychology* **12** 285 - 305
Palmer S E, 1982 "Symmetry, transformation, and the structure of perceptual systems", in *Organization and Representation in Perception* Ed. J Beck (Hillsdale, NJ: Lawrence Erlbaum) pp 95 - 143
Palmer S E, 1983 "The psychology of perceptual organization: A transformational approach", in *Human and Machine Vision* Eds J Beck, B Hope, A Rosenfeld (New York: Academic Press) pp 269 - 340
Palmer S E, 1989 "Reference frames in the perception of shape and orientation", in *Object Perception: Structure and Process* Eds B E Shepp, S Ballesteros (Hillsdale, NJ: Lawrence Erlbaum) pp 121 - 163
Palmer S E, Hemenway K, 1978 "Orientation and symmetry: Effects of multiple, rotational, and near symmetries" *Journal of Experimental Psychology: Human Perception and Performance* **4** 691 - 702
Pani J R, 1989 "Axis of rotation and orientation of the object in mental representation of rotation", poster presented at the 30th meeting of the Psychonomic Society, Atlanta, GA
Pani J R, Dupree D, 1992 "Spatial reference systems in the comprehension of rotational motion", submitted for publication; also presented as a poster at the 31st meeting (1990) of the Psychonomic Society
Pani J R, Shippey G, 1992 "On the perceptual organization of rotational motion: Influences of rotational symmetry, generalized cones, and orientation of the axis of rotation" in preparation
Parsons L M, 1987a "Visual discrimination of abstract mirror-reflected three-dimensional objects at many orientations" *Perception & Psychophysics* **42** 49 - 59
Parsons L M, 1987b "Imagined spatial transformations of one's body" *Journal of Experimental Psychology: General* **116** 172 - 191
Proffitt D R, Gilden D L, 1989 "Understanding natural dynamics" *Journal of Experimental Psychology: Human Perception and Performance* **15** 385 - 393
Proffitt D R, Kaiser M K, Whelan S M, 1990 "Understanding wheel dynamics" *Cognitive Psychology* **22** 342 - 373
Rock I, 1973 *Orientation and Form* (New York: Academic Press)
Rock I, 1983 *The Logic of Perception* (Cambridge, MA: Bradford/MIT Press)
Rock I, DaVita J, 1987 "A case of viewer-centered object perception" *Cognitive Psychology* **19** 280 - 293
Rock I, DaVita J, Barbeito R, 1981 "The effect on form perception of change of orientation in the third dimension" *Journal of Experimental Psychology: Human Perception and Performance* **7** 719 - 732
Rock I, Wheeler D, Tudor L, 1989 "Can we imagine how objects look from other viewpoints?" *Cognitive Psychology* **21** 185 - 210
Shepard R N, 1984 "Ecological constraints on internal representation: Resonant kinematics of perceiving, imagining, thinking, and dreaming" *Psychological Review* **91** 417 - 447
Shepard R N, 1988 "The role of transformations in spatial cognition", in *Spatial Cognition: Brain Bases and Development* Eds J Stiles-Davis, M Kritchevsky, U Bellugi (Hillsdale, NJ: Lawrence Erlbaum) pp 81 - 110
Shepard R N, Metzler J, 1971 "Mental rotation of three-dimensional objects" *Science* **171** 701 - 703
Shiffrar M M, Shepard R N, 1991 "Comparison of cube rotations around axes inclined relative to the environment or to the cube" *Journal of Experimental Psychology: Human Perception and Performance* **17** 44 - 54