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Leonardo's Constraint: Two Opaque Objects Cannot Be Seen in the Same Direction

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Given Leonardo's constraint that 2 opaque objects cannot be seen in the same direction, how are the regions of objects occluded to 1 eye included in perception? To answer this question, the authors presented 3-dimensional stimuli, similar to the ones that concerned Leonardo, and measured the visual directions of their monocular and binocular regions. When the distance between near and far objects was large, the nonfixated object was seen as double and blurry. Leonardo's constraint was met by seeing the near object as double and transparent or the distant object as double and superimposed. When the distance between near and far objects was small, the constraint was met by a perceptual displacement and compression of parts of the nonfixated object.

Leonardo da Vinci considered a painter's task to be the reproduction of the visual world onto canvas, and being a painter himself, he was interested in discovering nature's laws to apply them to his work (Richter, 1952). In searching for these laws, Leonardo noted that when looking with two eyes one can (a) see around and behind a small near object, thereby seeing more of both the near object and the background than is possible with either eye alone, and (b) see more through an aperture than when looking with one eye only. On the basis of these observations, he concluded that it is impossible for a painter to depict on canvas what he sees with two eyes. (See the caption of Figure 1 for a direct quote from Leonardo's notebooks.) In this article, we argue that it is also impossible for a human observer to see all objects in the visual world in their correct location at any given moment. This is so because the visual system, like a painter, cannot represent two objects as being in the same direction.

Because of its implications for depth perception, Leonardo's insight received some attention from early vision researchers and has recently seen a revival of interest by researchers. Wheatstone (1838) speculated that the concept of retinal disparity (two retinal images falling on slightly different positions in the two eyes) would have forced itself upon Leonardo if he had considered a cube instead of a sphere as the small near object shown in Figure 1. The view to each eye is noticeably different when viewing a cube rather than a sphere because of the corners of the cube. In reviewing early work on binocular vision, Boring (1942) discussed Leonardo's notes with respect to Wheatstone's discovery of retinal disparity as a cue to depth and coined the term *Leonardo's paradox*. According to Boring, the stimulus situation depicted in Figure 1 produces a paradox, because all of the background behind the sphere is seen even though the near object (sphere) is opaque. Without addressing the issue of transparency raised by Boring, recent researchers have shown that depth perception can depend on monocularly seen areas that lack retinal disparity (e.g., Anderson, 1994; Anderson & Nakayama, 1994; Gillam, Blackburn, & Nakayama, 1999; Gillam & Borsting, 1988; Grove, Gillam, & Ono, 2002; Grove & Ono, 1999; Häkkinen & Nyman, 1997; Howard, 1995; Lawson & Gulick, 1967; Nakayama & Shimojo, 1992; Ono, Shimono, & Shibuta, 1992; Shimojo & Nakayama, 1990). Depth perception that is attributed to unpaired monocular regions is called *da Vinci stereopsis* (Nakayama & Shimojo, 1990). See Appendix A for a demonstration that distinguishes between Wheatstone stereopsis and da Vinci stereopsis.

Boring's (1942) use of the term *paradox* highlights researchers' incomplete understanding of what an observer sees in the stimulus arrangement depicted in Figure 1. Despite the long list of studies regarding da Vinci stereopsis cited above, researchers' understanding of what we see in these stimulus arrangements remains incomplete. On the basis of the results of several informal demonstrations, Ono, Wade, and Lillakas (2002) argued that what observers see in these stimulus situations is better understood by also considering egocentric visual direction rather than visual distance alone. Although observers view the world from two distinct vantage points (the two eyes), directions of objects are perceived as though from a single vantage point, often called the *cyclopean eye*. It is also called the *binoculus*, the *visual egocenter*, the *double eye*,

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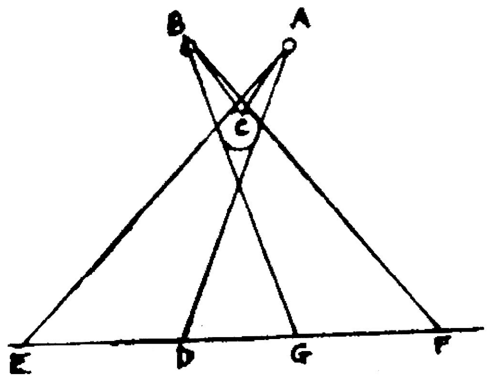

Figure 1. An illustration in one of Leonardo's notes described in Kemp (1989) under the heading "Why objects portrayed perfectly from nature do not appear to possess the same relief as appears in an object in nature." The note states, "It is impossible that a picture copying outlines, shade, light and colour with the highest perfection can appear to possess the same relief as that which appears in an object in nature. . . . Let the eyes be *a* and *b*, looking at an object *c*, with the converging central axes of the eyes as *ac* and *bc*. . . . The other axes, lateral to the central one, see the space *gd* behind the object, and the eye *a* sees all the space *fd*, and the eye *b* sees all the space *ge*. Hence the two eyes see behind the object and all the space *fe*. On this account, the object *c* acts as if transparent, by the definition of transparency, according to which nothing behind it is concealed" (Kemp, 1989, pp. 63–64).

the *projection center*, and the *center of visual direction*. The direction considered in this article is restricted to using a single point between the two eyes as the reference point. (For a recent discussion of visual direction using other reference points, see Mapp, Ono, & Howard, 2002.) The visual system's single reference point is analogous to the painter's station point, or *one-point perspective*, when depicting the visual world on a canvas (e.g., Alberti, 1435/1966; Haber, 1979). That is, using the inputs from two different vantage points, the visual system constructs a picture or cyclopean view with a common reference point from which visual direction is specified. The way in which the visual system constructs this picture can be understood, in part, in terms of the principles or laws of visual direction, originally proposed by Wells (1792) and Hering (1879/1942) and modified slightly by Ono and Mapp (1995). These laws predict the visual direction of isolated points in space but fail to predict the visual direction of features belonging to surfaces that are partially occluded to one eye. This is because the visual system is subjected to the same constraint as that imposed on a painter. Namely, two opaque objects cannot be seen or represented in the same direction. If two objects at different distances are seen in the same direction, the far object is seen through the nearer one that is transparent. In this article, this constraint is referred to as *Leonardo's constraint*.

Ono, Wade, and Lillakas (2002) discussed Leonardo's idea described in Figure 1 in the context of the status of visual science before and after Leonardo. They speculated on how the constraint is linked to processing direction and relied on informal demonstrations to make their case. In contrast, we conducted three formal experiments for this study. We examined how the visual system deals with Leonardo's constraint when constructing a representation of visual space in the two stimulus situations that concerned Leonardo. The results were analyzed with respect to (a) predictions based on the existing laws of visual direction and (b) possible revisions to the laws that may be needed to account for the new data. In Experiment 1, we considered the first stimulus situation that interested Leonardo. In this situation, the whole background was visible with two eyes as shown in Figure 1, but portions of it were occluded to each eye by the near object. A large distance separated the near object and the background. In Experiment 2, we examined a similar stimulus, but a small distance separated the occluder and the background. In Experiment 3, we examined a stimulus situation similar to another one that interested Leonardo, which involved seeing through an aperture. Instead of an aperture we used two occluders, and parts of the background between them were seen monocularly. As in Experiment 2, a small distance separated the occluders and the background.[1]

Experiment 1

We examined observers' perceptions when viewing the stimulus arrangement depicted in Figure 1. We asked them to report the appearance of the occluder and that of the background while fixating either the occluder or the background.

Method

Observers. Six adult observers from the York University (Toronto, Ontario, Canada) community participated. Four were experienced in psychophysical experiments, whereas two were not. Five observers, including the two inexperienced ones, were naive to the purpose of the experiment. All of them had normal or corrected-to-normal vision and normal depth perception.

Apparatus. To create the stimulus situation depicted in Figure 1, we placed a black vertical rod midway between a passage of text and an observer. The text was present on a 15-in. color computer monitor (Macintosh Color Display, Apple Computer, Cupertino, California). The distance from the monitor to the observer was 57 cm. A black vertical rod, either 0.6 cm or 2.5 cm in diameter, was located midway between the screen and the observer. The experiment was conducted in a normally lit room.

Procedure. The observers' task was to describe verbally their perception of the text and the rod under two conditions, fixating background and fixating occluder. In the fixating-background condition, the text (17 lines, 205 words, 14-point Times font) filled the computer screen. The observers were asked to fixate the middle of the background and describe the appearance of the rod and the text (i.e., whether the rod or the text was seen as single or double). Then they were asked to read the text continuously, out loud, for the remainder of the 5-min viewing time. Afterward, they described how the rod appeared while they were reading the text. In the fixating-occluder condition, the font was 72-point Times because all observers in a pilot study reported that a text with a smaller font size was unreadable. The text contained eight words and filled most of the screen. The observers were asked to fixate the rod and describe how the text appeared (i.e., whether the text was seen as single or double and how the letters in the text were seen when the text was seen as double). After giving their initial description, the observers were asked to report any changes in what they saw during the remainder of the 5-min viewing period (i.e., rivalry and/or seeing the letters as superimposed). All the observers per-

[1] Experiments 1 and 3 are formal experiments for the demonstrations described on the Web site http://www.perceptionweb.com/perc0102/ono.html that accompanies Ono, Wade, and Lillakas's (2002) study. On the Web page, Demonstration 3 corresponds to the current Experiment 1, and Demonstrations 4a and 4b correspond to the current Experiment 3.

formed the task once for each rod diameter in both fixation conditions. A chin rest and a forehead rest kept the head stationary.

Results and Discussion

In the fixating-background condition, all observers perceived the rod as blurry and double and the text as single, and all were able to read the text correctly. While reading the parts of the text that were seen with one eye only, observers reported that the rod either disappeared (the portion of the rod where the text was being read was not seen) or was seen as transparent (letters were seen as though through a blurry rod). The observers' perception of the rod and the text was the same for the 0.6-cm and the 2.5-cm diameter rod conditions.

The report that the parts of the text seen with only one eye were always readable could have resulted from observers using the context of the text to fill in any suppressed parts. To eliminate this possible explanation, we repeated this part of the experiment substituting a series of random letters (e.g., *zpl abjip simvncl*) for the text. The random letters, in 14-point Times, were presented in the center of the screen in two columns of five lines each. There were three or four groups of letters per line, and each group was three to seven letters long. Each reading of all the random letters took approximately 1 min. Six adults, five of whom were the same as in the first part of the experiment, participated and replicated the results of the fixating-background condition. That is, all observers read out all of the random letters correctly. An everyday example of this perception is viewing a scene through a wire mesh fence.

In the fixating-occluder condition, all observers saw the rod as single and were able to identify the letters. During the 5-min viewing period, three observers experienced binocular rivalry (i.e., alternating between the letter[s] seen by one eye and the other eye) as well as overlapping letters, while the other three did not experience any rivalry and continuously saw overlapping letters. As in the fixating-background condition, the observers' perception of the rod and the text was the same for the 0.6-cm and the 2.5-cm diameter rod conditions.

In both conditions, seeing a stimulus on the fixation plane as single and on a nonfixated plane as double is in accordance with Wells's (1792) and Hering's (1879/1942) laws of visual direction mentioned in the introduction. (For a recent discussion, see Mapp et al., 2002; Ono, Lillakas, & Mapp, 2003; Ono, Wade, & Lillakas, 2002.) The double vision on a nonfixated plane is due to the stimulus being displaced perceptually in two directions. Figure 2B illustrates the displacements in a composite drawing of Wells's

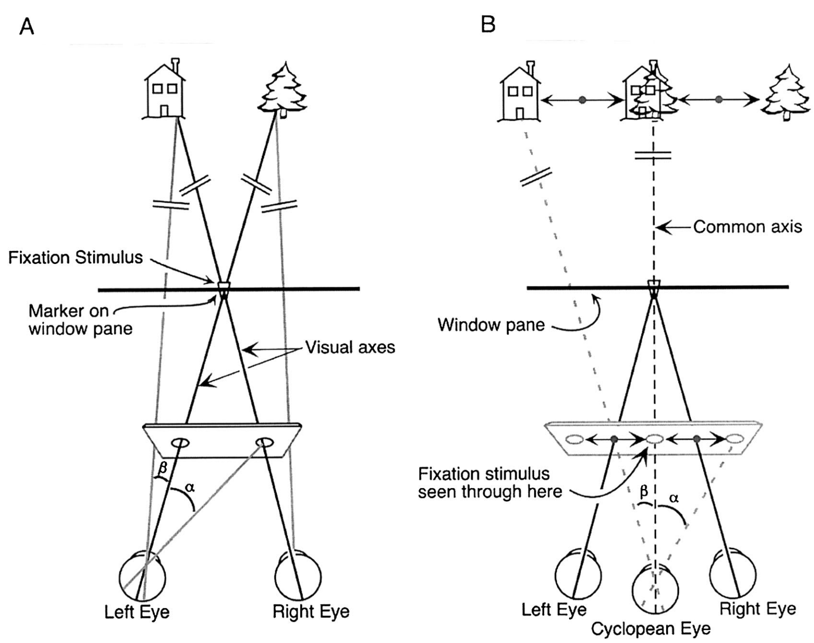

Figure 2. A composite illustration of the demonstration for Wells's (1792) propositions regarding visible direction and that for Hering's (1879/1942) principles of identical visual direction. A: Stimulus. B: Perception. Wells's demonstration consists of a card with two holes (3 cm apart) located at the midpoint between the fixation stimulus and the eyes in such a way that the right eye sees the fixation stimulus through the right hole and the left eye through the left hole. When an observer fixates on the fixation stimulus, she or he sees a fused hole in the median plane (straight ahead of the nose) and two monocular ones outside the visual axes (Angle α away from the median plane). Hering's demonstration consists of a fixation stimulus: for example, a mark on a windowpane 50 cm away, the right eye aligned to a chimney top and the left eye to a treetop. When an observer fixates on the fixation stimulus, she or he sees the chimney and the treetop in the median plane sometimes superimposed and sometimes rivaling. She or he also sees a monocular chimney and a monocular treetop outside the visual axes (Angle β away from the median plane). The angle between the visual axis of the right eye and the left hole on the card (also α) is not illustrated, and the visual axis of the right eye and the treetop (also β) is not labeled in Panel A. However, the outward displacements caused by the angles are illustrated in Panel B.

and Hering's classic demonstrations. In this figure, a binocular stimulus on a visual axis (a line of sight) either closer to or farther away from an observer than the intersection of the two visual axes is seen as double. One of the double images is displaced to the median plane (directly in front of the nose), and the other is displaced outside the two visual axes. The arrows in the figure point in horizontally opposite directions, indicating the two displacements. According to Wells's and Hering's laws of visual direction, a stimulus on a visual axis is seen on the common axis (a line passing through the intersection of the visual axes and the cyclopean eye), and the visual angle subtended by a stimulus and a visual axis (Angle α or β in Figure 2) is transferred unaltered to the cyclopean eye. In the two demonstrations, when a stimulus is seen double, one of the double images is displaced to the common axis, and the other is displaced outward.

The displaced stimuli on the nonfixated plane appeared blurred because they were not accommodated to; when two displaced stimuli were seen in the same direction (e.g., the treetop and chimney seen straight ahead in Figure 2), each was seen alternately (rivalry), or both were seen overlapped. Our results, in terms of rivalry and overlapping of two binocular stimuli were consistent with what is known in the literature. (See Blake, 2003, for a recent discussion and Howard, 2002, for a comprehensive review.) That is, a high-contrast (or focused) stimulus suppresses a low-contrast (or blurry) stimulus, and two low-contrast stimuli will not rival as frequently as two high-contrast stimuli. It is worth mentioning, however, that our experimental stimuli differ from those usually used to study these perceptual variables. Usually, an experimenter directly manipulates an experimental variable such as the contrast of the stimuli presented in a stereoscope. In this experiment, however, the rod and the background were located at different distances, and the observer accommodating on either the rod or the background modified the contrast mentioned above.

In both conditions the constraint was met. In the fixating-background condition, when part of the rod disappeared, each letter was seen in one direction. When it did not disappear, the letters were seen through a transparent or blurry rod. In the fixating-rod condition, when the letters rivaled, only one letter was seen in one direction at any given moment. When they perceptually overlapped, the superimposed letters were seen as one object in one direction and at one distance. In no condition did an opaque object appear behind another opaque object, in accordance with the constraint.

The major determining factors for the double vision in Experiment 1 are likely related to the fixation requirement and the large depth interval between the rod and text. Frequent eye movements in conditions similar to that of Experiment 1 would eliminate the double vision (Foley & Richards, 1972) or would make it less noticeable. Leonardo's constraint is not limited to situations where double vision is experienced, however. As we show in the next two experiments, it also operates when the background and the occluder are both seen as single.

Experiment 2

As in Experiment 1, observers fixated the background or the occluder, but in this experiment the distance between the occluder and the background was small enough for both to be seen as single. As discussed above, free eye movement would have allowed for the use of a stimulus with a greater depth interval, but steady fixation was required for the purpose of this experiment because visual direction is known to depend on eye position (e.g., Mapp et al., 2002; Ono & Mapp, 1995). The results of Experiment 1 showed that the stimuli on the nonfixated plane were perceptually displaced in two opposite directions. In Experiment 2 we examined the visual direction of a stimulus in the monocularly seen area(s) and that of the edge(s) of the occluder to ascertain how Leonardo's constraint operates in a visual field that is seen as single.

Method

Observers. Sixteen observers from the York University community participated. All had normal or corrected-to-normal vision and normal depth perception. Eight observers were experienced in psychophysical experiments. All but two of the experienced observers were naive to the purpose of the experiment.

Apparatus and stimuli. Stereograms were presented in a mirror stereoscope consisting of two 14-in. color computer monitors (Apple Color Plus Display, Apple Computer, Cupertino, California), which were the only source of light in the room. The mirrors were beam splitters and allowed presentation of two LEDs on the median plane placed directly above the fixation stimulus. Two sets of Polaroid filters, one in front of the eyes and another in front of the LEDs, made the two LEDs monocular, one to each eye, and served as Nonius[2] stimuli, used by observers to subjectively monitor their fixation. For Nonius stimuli in this experiment, one LED was optically presented to the left eye, slightly above the second LED, which was presented to the right eye. Because of this vertical separation, the LEDs did not combine perceptually and remained monocular. When the observer fixated the background accurately, the upper and lower monocular LEDs appeared vertically aligned. When the observer fixated the occluder, the two LEDs appeared misaligned.

Figure 3 shows a schematic illustration of the experimental conditions. The viewing distance was 75 cm, and each pixel subtended 1.5-min arc. The disparity of the black occluder (90-min $\times$ 322.5-min arc) with respect to the background was 15-min arc. There were two sets of stimuli (see Figures 3A and 3B). In each there were two fixation crosses (near and far), each subtending 15-min $\times$ 15-min arc, presented on the median plane one at a time. The monocular line was presented 7.5-min arc away from the edge of the occluder (at the edge of the monocularly seen area).

Procedure. To obtain a displacement score for the two primary conditions (monocular line and outer edge of the occluder), we created two subconditions for each, experimental and control. In the experimental condition, the visual directions of elements in the nonfixated plane were measured. In the control condition, the visual directions of elements in the fixated plane were measured. A chin rest kept the head stationary.

The method of adjustment was used to measure the visual direction of a line placed in the monocularly seen area(s) and that of the lateral edges of the occluder (see Figure 3). Observers were required to adjust the lateral position of the comparison stimulus until it appeared collinear with the feature being measured (i.e., the edge of the rectangle or the monocular line). To familiarize themselves with the task, observers practiced adjusting the location of the comparison line with the edge of the occluder and with the monocular line 12 times with fixation and the standard stimuli at the same distance.

In the experimental condition for the monocular line (see Figure 3A), observers fixated the fixation cross on the near plane and adjusted the comparison stimulus to appear aligned with the monocular line. In the control condition, observers fixated the far fixation cross and adjusted the comparison stimulus to appear aligned with the same monocular line. In the experimental condition for the edge of the occluder (see Figure 3B), observers fixated the far fixation cross and adjusted the comparison stim-

[2] *Nonius* is the latinized name of Pedro Nunes, who used the stimuli to access binocular fixation error.

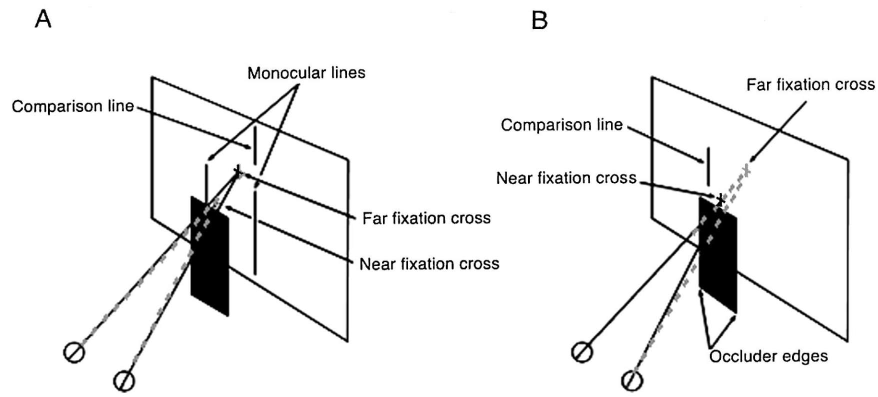

Figure 3. Schematic illustration of two pairs of conditions to measure displacements in Experiment 2. A: Measuring the displacement of the monocular line. A shows the adjustment of the comparison line to appear aligned with the monocular line in the experimental condition (near fixation cross) and the control condition (far fixation cross). B: Measuring the displacement edge of the occluder. B shows the adjustment of the comparison line to appear aligned with the edge of the occluder in the experimental condition (far fixation cross) and the control condition (near fixation cross). The relative dimensions are not scaled to the stimuli used.

ulus to appear aligned with an edge of the occluder. In the control condition, observers fixated the near fixation cross and adjusted the comparison stimulus to appear aligned with the edge. In both the experimental and control conditions, there were 20 trials in which the observer adjusted the horizontal location of the (binocular) comparison line.

The adjustment of the comparison line was performed on both the left and right sides of the occluder. There were six starting positions for the comparison line for each standard stimulus, three to the left and three to the right (±12-, ±13.5-, and ±15-min arc). For half the observers (four experienced and four inexperienced) the occluder was at the top of the screen, and the comparison line was at the bottom, opposite to that shown in Figure 3. The four different orders of presentation of the four stimulus conditions consisted of a balanced Latin square design, with each order of presentation used twice for both the experienced and the inexperienced group.

Results and Discussion

We defined the actual location of each standard stimulus as zero, the outward deviation from this point as positive, and the inward deviation as negative. These definitions allowed us to combine the results from the right side of the occluder with those from the left side. We calculated the mean of the 20 adjustments in each condition for each observer. We then computed each observer's displacement score for the monocular line and the edge conditions, which consisted of the mean value of the experimental condition minus that of the control condition. Each score served as the basic unit of analysis. The group means for the two conditions (separately for the experienced and inexperienced observers) are presented in Table 1. A two-way analysis of variance (ANOVA) with one within-subject factor revealed that the interaction of the two factors was not significant, $F(1, 14) = 1.60$, $p > .05$. The main effect of experimental conditions (the monocular line vs. the edge) was statistically significant, $F(1, 14) = 19.09$, $MSE = 1.89$, $p < .01$, $\eta^2 = .58$, but the main effect of groups (experienced vs. inexperienced observers) was not, $F(1, 14) = 0.09$, $p > .05$.

Averaging the values shown in Table 1 across the group factor indicated a displacement of 1.31-min arc in the monocular-line condition. In other words, when fixating on the occluder, the monocular lines appeared displaced outward by 1.31-min arc relative to when fixation was on the background. The edge condition had a displacement of −0.81-min arc. That is, when fixating on the background, the edges of the occluder appeared displaced inward by 0.81-min arc relative to when fixating on the occluder. Both displacements are in a direction that allows the monocularly seen areas to be included in perception.

The magnitudes of the obtained displacements are small, however, and a question can be raised as to whether the obtained amounts are smaller than what is theoretically required to include these monocularly seen areas in the cyclopean view. This question is difficult to answer because several assumptions are needed to compute the required displacement, and different assumptions lead to different predictions. Assuming that (a) the cyclopean eye is located on the horopter[3] midway between the two eyes, (b) the edges of the occluder are correctly localized, and (c) all the elements in the monocular zone are displaced equally, our calculations predict the displacement of the monocular line to be 3.75-min arc. In contrast with this predicted value, consider the minimum required displacement to see the monocular line without making any of the assumptions above. Because the monocular line was placed on the outer edge of the monocular zone, any displacement larger than the width of one pixel would allow the line to be seen. Clearly, these two extreme predictions indicate that the current status of the theory regarding Leonardo's constraint does not allow a precise prediction of the amount of displacement, whereas it does allow prediction of its direction.

In Experiment 3, we further discuss the theoretical significance of the obtained displacement. We also present another possible explanation for the modest displacements: the observers' inability to maintain fixation on the fixation stimulus. Observers have the difficult task of maintaining fixation on the stimulus while monitoring the Nonius stimulus (contrast the *Method* sections of Ex-

[3] A *horopter* is defined as loci in space that provides single vision for a particular binocular eye position. Theoretically, it is the circle passing through the optic centers of the eyes and the intersection of the two visual axes.

Table 1
Displacements (Min Arc) of the Monocular Line and the Occluding Edge for Observers in Experiment 2

	Displacement			
	Monocular line		Occluding edge	
Observer	*M*	*SD*	*M*	*SD*
Experienced	1.70	1.15	−1.04	1.56
Inexperienced	0.93	1.51	−0.58	1.46

Note. Positive numbers indicate displacement outward, and negative numbers indicate displacement inward.

periments 2 and 3) and simultaneously attending to the relative direction of the standard and comparison stimuli. While adjusting the position of the movable comparison line to appear aligned with the standard stimulus, one's natural tendency is to move the eyes to, or to fixate, the comparison stimulus. Moreover, our use of the Nonius stimulus was designed to indicate the difference between conditions of far and near fixation but not to indicate precise eye position. Therefore, the required eye position for each condition might not have been met. The methodology of Experiment 3 attempts to overcome this possible shortcoming. Whether or not the methodology of Experiment 2 underestimated the amount of displacement, we can conclude that there is a perceptual displacement in the direction required to overcome Leonardo's constraint (i.e., outward for the monocular element on the background when fixation is on the occluder and inward for the edges of the occluder when fixation is on the background).

Experiment 3

The direction of the displacements required to meet Leonardo's constraint, which we considered in Experiments 1 and 2, is not the only factor to be considered. As mentioned in the introduction, some stimulus elements seen from two vantage points cannot be represented in their correct direction by a painter or by the visual system. There is another geometric fact: The visual angle subtended by two points (the relative direction of two points) cannot be represented correctly in certain situations. The diagram by Leonardo shown in Figure 4 illustrates this fact. It shows the situation where all of the background, except a point (b), is seen monocularly. In this particular situation, the sum of the two monocularly seen areas is twice as large as each individual monocularly seen area. Consider representing the visual angle subtended by Points a and b from a station point between the two eyes. If this visual angle is represented correctly, there is no room remaining to represent the visual angle subtended by Points b and c.[4] Given this geometric fact, the issue we considered in Experiment 3 was how the visual system constructs a cyclopean view from what each eye sees when viewing the space between two occluders.

In theory, one way to construct the cyclopean view is to suppress perceptually one half of the monocularly seen area, but the studies on da Vinci stereopsis cited in the introduction show that ecologically valid monocular zones are not suppressed.[5] Moreover, the results of Experiments 1 and 2 indicate that when an element of the occluder is fixated, the monocularly seen areas are displaced away from the occluder and are included in the cyclopean view. Also, when a background element is fixated, the edges of the occluder are displaced, and the monocularly seen areas are included in the cyclopean view. To address the fact that the cyclopean view cannot correctly include the entire visual angle subtended by surfaces, Ohtsuka (1995a, 1995b) and his colleagues (Ohtsuka & Ono, 2002; Ohtsuka & Yano, 1994; Ono, Ohtsuka, & Lillakas, 1998) hypothesized that there is a perceptual compression of some part of the visual field accompanying the displacement. For example, when both edges of the occluder are displaced inward, compression must occur toward the midportion of the occluder, as in the stimulus situation considered in Experiment 2.

Experiment 3 examined the hypothesized perceptual compression that is expected to accompany the perceptual displacement. The compression of the background would be inferred from displacement of a monocular stimulus relative to a binocular stimulus on its surface. Compression of an occluder would be inferred from the displacement of its edges. The predicted value of displacement depends on the accuracy with which observers fixate the target stimulus. Therefore, we also measured fixation disparities for each observer (see Appendix B).

Method

Observers. Six observers from the York University community participated. All had normal or corrected-to-normal vision and normal depth perception. Four observers were naive to the purpose of the experiment and had little or no experience in psychophysical experiments.

Apparatus and stimuli. The stereoscope consisted of a 19-in. color computer monitor (Model STD 9752, Supermac Technology, Sunnyvale, California), two pairs of mirrors, and two small circular apertures. Stereoscopic images were presented side by side on the monitor. The optical distance was 90 cm, and each pixel subtended 1.1-min arc. The apertures were placed close to the eyes, which restricted each eye's field of view to 5.5°. A pair of mirrors and one of the apertures were oriented in front of each eye such that the left eye saw only the left half of the computer screen and the right eye saw only the right half.

Figure 5 shows a schematic illustration of the experimental conditions. Two black rectangles (88-min × 165-min arc) were joined at the top by a narrow black bridge (264-min × 22-min arc). Below the bridge and

[4] Wade, Ono, and Lillakas (2001) failed to note this diagram and two others when they compiled Leonardo's diagrams concerned with binocular vision. B. Gillam (personal communication, December 7, 2001) brought this omission to our attention. We have conducted an experiment comparable with that of Experiment 1 of this article using a stimulus similar to the diagram and have obtained comparable results. Namely, we found that (a) when the letters on a computer screen were viewed through an aperture and fixated, all the letters were discernable through suppressed parts of the cardboard adjacent to the aperture, and (b) when a point near the aperture was fixated and the letters were made larger, the larger letters appeared superimposed or rivaled.

[5] An ecologically valid monocular stimulus is that created in natural three-dimensional space (Shimojo & Nakayama, 1990). The monocularly seen stimuli in this study are valid because they can occur in a natural setting. Examples of invalid monocular stimuli are (a) those stimuli presented in binocular rivalry studies and (b) a monocular stimulus that is usually seen by the right eye is presented to the left eye and a monocular stimulus that is usually seen by the left eye is presented to the right eye. One can create the stimuli discussed in (b) by crossing the pair of stimuli made for uncrossing or by uncrossing the pair of stimuli made for crossing in the demonstration of da Vinci stereopsis in Appendix A.

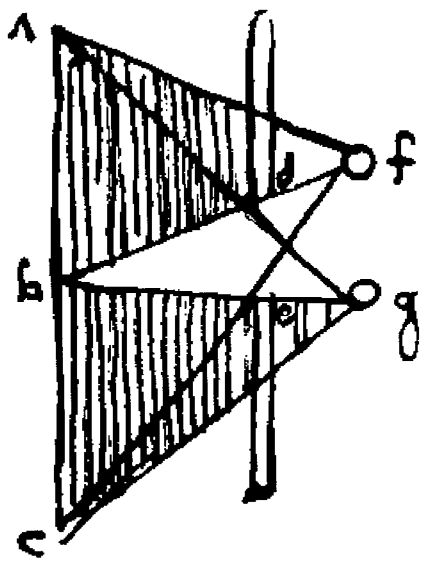

Figure 4. An illustration in one of Leonardo's notes described in Strong (1979) with the heading "What part of the field can the eye see which looks through a small hole." The note states, "Let two eyes look through the hole *d e* at the field *a c*. I say that the two eyes will not see any more of this field than the space *b* and the remainder of the space *a b* on the right will be seen by the left eye in *g* and the remaining space *b c* on the left will only be seen by the right eye *f*" (p. 82). In the illustration, we have "corrected" the mirror image lettering of Leonardo with a computer graphics program.

between the two rectangles were four thin lines, which were four of the six standard stimuli. Although each eye's image contained only three stimulus lines between the two rectangles, when the middle two fused, observers perceived four lines through the space between the two rectangles. The lines appeared farther than the occluders and the bridge, with a disparity of 11-min arc. Above the bridge, there was a binocular comparison stimulus, consisting of a single vertical line. Directly above the comparison stimulus was a Nonius stimulus, which the observers used to subjectively monitor the accuracy of their fixation. This Nonius stimulus consisted of a binocular horizontal bar flanked top and bottom by monocular dots. The right eye's view consisted of the horizontal bar and a dot directly below it. The left eye's view consisted of the horizontal bar and a dot directly above it.

On any given trial, the comparison line was randomly positioned (in one of seven locations) above one of the six standard stimuli (two monocular lines, two binocular lines, and two edges of the occluders). One should note that the monocular line in Experiment 3 was placed in the middle of the monocular area, whereas in Experiment 2 it was placed at the outer edge of the monocular area.

Procedure. We measured the visual direction of different elements in the visual field using the method of constant stimuli instead of the method of adjustment used in Experiment 2. We also used the comparison stimulus as the fixation stimulus. With these changes, we hoped to reduce the possible fixation error mentioned in the *Results and Discussion* section of Experiment 2. There were six experimental conditions (defined by the two monocular lines, the two binocular lines, and the two inside edges of the occluders), each with a control condition. Each observer served in all 12 conditions.

Observers sat in a dark room with their chin in a chin rest. On a given trial, observers pressed a mouse button to initiate the presentation of a stereogram on the screen. While maintaining fixation on the comparison stimulus, observers first checked that the Nonius stimulus was aligned. Then they judged whether the comparison stimulus was to the left or to the right of the standard stimulus, using a two-alternative-forced-choice procedure. For the experimental condition, the standard stimulus and the comparison stimulus were in different perceptual planes (one near and one far), whereas for the control condition they were in the same plane (both near or both far). See Figure 5 for examples of these conditions. When ready to respond, observers clicked the mouse button, extinguishing the stimulus and producing the words *Left?* and *Right?* on the screen. Observers moved the computer mouse toward one of the words until their choice was highlighted and then clicked the mouse button. The stimulus reappeared with the comparison stimulus in a new location, and the observer repeated the above procedure. The comparison stimulus was randomly positioned above one of the stimulus lines and was located in one of seven positions relative to each standard stimulus line (0-, ±1.1-, ±2.2-, or ±3.3-min arc). Observers made 20 judgments for each comparison stimulus location for a total of 840 trials completed in 30-min sessions over several days (7 locations × 4 stimulus lines and 2 edges × 20 observations each).

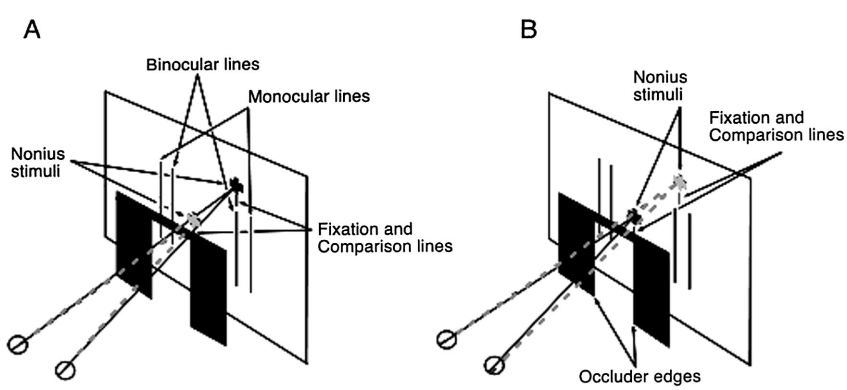

Figure 5. Schematic illustration of two pairs of conditions to measure displacements in Experiment 3. A: Measuring the displacement of the monocular or binocular line. A shows the judgment of whether the comparison line appears left or right of the monocular or binocular line in the experimental condition (near comparison line) and the control condition (far comparison line). B: Measuring the displacement of the edge of the occluder. B shows the judgment of whether the comparison line appears left or right of the edge of the occluder in the experimental condition (far comparison line) and the control condition (near comparison line). The relative dimensions of the stimulus used are not drawn to scale.

During the experiment, observers reported that the top and the bottom of the Nonius stimulus did not always appear aligned when they fixated on the comparison line. Instead of adjusting the apparatus each time such misalignments were reported, we measured the fixation disparity (the error of fixation specified in terms of angular units) that was responsible for this misalignment. This measurement is described in Appendix B.

Results and Discussion

We defined the actual location of each standard stimulus as zero, the deviation outward from this point as positive, and the deviation inward as negative. We then used Probit analysis (Finney, 1971) to determine the position of the comparison stimulus that was judged to appear in the same direction (the point of subjective equality; PSE). This analysis was performed for each of the six standard stimuli for the experimental and control conditions. For each observer, the data from the right side and the data from the left side were combined, leaving three conditions for the purpose of analysis. We then computed the displacement score (the PSE value of the experimental condition minus that of the control condition) for the three conditions (monocular, binocular, and the edge).

For the monocular line condition, we also computed displacement scores that were corrected for fixation disparity, which we calculated using the fixation disparity values reported in Appendix B. The corrected values were computed on the basis of Wells's (1792) and Hering's (1879/1942) laws of visual direction and represent displacement without fixation disparity. According to these laws, if there is no fixation disparity when fixating the plane of the occluder, the perceived angular displacement of a monocular line is half the angular disparity between the near and far stimuli (in our case, 5.5-min arc). If, however, an observer has an uncrossed fixation disparity (i.e., the intersection of the two visual axes is located beyond the target), then the perceptual displacement of a monocular feature will be smaller than if the observer were accurately fixating the target. (In Figure 2A, Angle β will become smaller if the intersection of the visual axis is farther away from the observer. The apparent outward displacement of the chimney will become smaller if Angle β is smaller in Figure 2B.) Conversely, if the observer had crossed fixation disparity (i.e., the intersection of the two visual axes is located in front of the desired target) the displacement would be larger. In our experiment, however, all observers had uncrossed fixation disparity. To correct the measured displacement responses for fixation disparity, we added the amount predicted by the laws, namely, half of the measured fixation disparity, to the value of the perceived displacement of the monocular lines. Also, we computed the minimum displacement value required for the monocular line (placed 3.3-min arc from the butted end of the monocularly seen area) to be seen, assuming that there is no fixation disparity. The computed value was 3.9-min arc (11-min arc minus 3.3-min arc divided by 2).

There is no provision in Wells's (1792) and Hering's (1879/1942) laws to predict the displacement of the fused binocular line or the binocularly fused edge of the occluder. In fact, the laws, as stated, predict that a fused stimulus would be seen in the correct visual direction with respect to the cyclopean eye. This prediction is made on the basis that two different visual-line values would average to produce a correct visual direction value. Accordingly, fixation disparity should not affect the visual direction of a binocular object. Assuming that the monocular line is seen where it is (when fixating the background), we computed the minimum displacement of the occluder edge required for the monocular line to be seen. This value is 3.9-min arc, the same as the minimum displacement value required for it to be seen when there is no fixation disparity.

The displacement scores for each observer served as the basic units of analysis for a one-way ANOVA. The analysis showed that the differences among the three conditions were statistically significant, $F(2, 10) = 65.90$, $MSE = 1.89$, $p < .005$, $\eta^2 = 0.93$. (Substituting the uncorrected displacement scores for the corrected ones in the monocular line condition also yielded statistical significance, $F[2, 5] = 102.21$, $MSE = 0.75$, $p < .005$, $\eta^2 = 0.95$.)

Given that the conditions were statistically different, we now consider the magnitude of the obtained displacements. The mean displacement scores across observers for the edge and the binocular line, in addition to that of the corrected displacement for the monocular line, are shown in Table 2. Also shown in Table 2 is the 95% confidence interval for each mean. (The mean displacement value for the monocular line before correction is not shown in the table; it was −3.20-min arc with a 95% confidence interval of −2.11 to −4.29.) The 95% confidence interval for the mean of the corrected displacement of the monocular line contains the predicted value of −5.5-min arc and that of the minimum required displacement for it to be seen. The 95% confidence interval for the mean displacement of the binocular line contains the predicted value of zero displacement. The 95% confidence interval for the mean displacement of the edge did not include either the value of 5.5-min arc or zero, as would be predicted by the laws of visual direction, but it did include the required minimum displacement of 3.9-min arc. These obtained means, with their respective confidence intervals, highlight the inadequacy or incompleteness of these laws when dealing with structured surfaces.[6]

The obtained mean magnitudes of the displacement for the monocular stimulus and that of the edge were much larger than those of Experiment 2 (compare the values shown in Table 1 to those shown in Table 2). The smallness of the mean displacements of the monocular line in Experiment 2 is likely due to our not controlling for fixation disparity. But it could also be partly due to the location of the monocular line within the monocular zone, because one cannot expect a uniform displacement of elements in this zone. The smallness of the mean displacements of the edge of the occluder in Experiment 2 is more difficult to explain, because we are uncertain about the role played by the fixation disparity. If we assume, however, that the extent of the obtained displacement is reliable, we can speculate that there is compression of the fixated plane as well as of the nonfixated one. In any event, the displacement of the edge and the consequent compression of the inner area

[6] In another experiment, we measured the visual direction of the outer edge of a partially occluded rectangle. When the occluded rectangle was seen behind a central occluder, its outer edges were displaced outward with respect to the mean of the monocular visual lines, as predicted by Wells's (1792) and Hering's (1879/1942) laws of visual direction. The visual directions of the surface edges seen binocularly were more eccentric than those of isolated lines in the same location, underscoring our claim that Wells's and Hering's laws of visual direction account for isolated points in space but fail to account for the direction of features belonging to surfaces in depth.

Table 2
The Means and 95% Confidence Intervals (CI) of Displacements (Min Arc) for Each Standard Stimulus of Experiment 3

	Standard stimulus		
Displacement	Monocular line	Binocular line	Occluding edge
M	−5.17	−0.18	3.93
95% CI	−7.24 to −3.10	−0.84 to 0.47	3.29 to 4.56

Note. Positive numbers indicate displacement outward, and negative numbers indicate displacement inward.

of the occluder highlight the fact that visual direction of surface features needs further exploration.

As the discussion above implies, the displacement of the monocular line found in Experiment 3 and in Experiment 2 is not new, although discussing it in terms of Leonardo's constraint is. This displacement has the same basis as that shown in Experiment 1, in which the binocular stimulus with sufficiently large disparity was displaced in two different directions (i.e., seen double). The displacement of a nonfixated and nonfused stimulus, or a nonfixated monocular stimulus, has been known for over two millennia (see Howard & Wade, 1996). For example, see Hering's (1879/1942) classical demonstration illustrated in Figure 2. Studies by van Ee, Banks, and Bachus (1999), Nakamizo, Shimono, Kondo, and Ono (1994), Takeichi and Nakazawa (1994), and Ono, Mapp, and Howard (2002) confirm Hering's observation.

What is new in our findings is the displacement of a binocular stimulus. Usually, a binocularly fused stimulus is assumed to be seen in its correct direction, because the visual direction is created from the average of two visual-line values that pass through the stimulus (Mapp et al., 2002; Ono & Mapp, 1995). The results of Experiments 2 and 3, however, clearly indicate that displacement of a binocularly fused stimulus occurs on the surface of the nonfixated plane. Also new is the idea about the compression of a portion of the nonfixated visual field as indicated by the results of Experiment 3. The inward displacements of the background seen between the two occluders in Experiment 3 and the inward displacements of the edges of the occluder in Experiments 2 and 3 require the concept of compression of the nonfixated plane.

General Discussion

The results of the three experiments show that the displacement and compression of part of the nonfixated plane allows the visual system to meet Leonardo's constraint. The compression that we found reveals an inadequacy within Wells's (1792) and Hering's (1879/1942) laws of visual direction and the need for a revision. Moreover, displacement and compression produce a horizontal directional distortion of visual space, and a horizontal distortion without a vertical one has implications for perception of shape and alignment of oblique lines. Therefore, the possible consequences of the distortion must also be addressed. We discuss below the two issues raised by the results of the experiments as well as further issues of geometric illusions and the possibility that visual direction and shape alignment are processed by different mechanisms.

Wells's (1792) and Hering's (1879/1942) laws are not merely a summary of existing experimental results. Predictions that were deduced from these laws regarding double images have been empirically confirmed using different stimuli (e.g., Le Conte, 1881; Towne, 1870; Wells, 1792). Evidence confirming predictions deduced from the laws for monocular stimuli, including some nonintuitive predictions, has also been reported. For example, the visual direction of a stimulus seen with one eye is predictable from the angular position of the nonviewing eye (Ono & Gonda, 1978, 1997; Ono, Mapp, & Howard, 2002; Ono & Weber, 1981). The observations we reported from the three experiments in this study further support the perceptual displacement predicted from the classic Wells's and Hering's laws of visual direction, but these laws have no provision for the compression we found in Experiment 3. The obtained compression violates the portion of the laws that state that a visual angle subtended by two points in space to either eye is seen correctly. That is, the laws predict that if a visual angle subtended by two points is 5°, the relative visual direction of the two points is also 5° with respect to the cyclopean eye (although their absolute directions are not always correct). For an example, see Angle α or β in Figure 2. However, this prediction cannot hold when there is a perceptual compression between two points, as in Experiment 3.

Although the laws were thought to adequately explain the experimental stimuli considered previously, the results of this study indicate that at least one more modification is needed. From our experimental results, we can specify the stimulus situations in which compression occurs. From our findings alone, however, a quantitative prediction is not yet possible. The results have clearly shown that compression occurs, but they have not determined the detailed nature or the extent of the compression. Thus, more empirical work is required to specify whether the compression is uniform or nonuniform between two points. Moreover, the extent of compression for a given area may depend on the type of stimulus (see Footnote 6).

In addition to challenging the existing laws of visual direction, the data reported here predict certain distortions in the view of some three-dimensional stimuli. Two such distortions are illustrated in Figure 6B. Specifically, given the displacements reported in the present study when an occluder is fixated, one would expect that (a) the square, which is occluded in its midportion, would appear rectangular because of the outward displacement of its two vertical sides (i.e., horizontally elongated) and (b) the two oblique lines would not appear collinear because of the outward displacements of their abutted ends. Studies (Ohtsuka & Ono, 2002; Ono et al., 1998; van Ee & Erkelens, 2000) that measured both shape perception and visual direction, however, show that these two predicted distortions do not occur in three-dimensional perception. Taken together, these results suggest that visual-direction information and shape and alignment information are processed by two separate mechanisms and that the shape and alignment mechanism corrects for the expected distortions illustrated in Figure 6B. This correction mechanism is also suggested by the directions of the Kanizsa (1979) and Poggendorff (see Robinson, 1972) illusions, which are opposite to what is expected from the displacement (see Figures 6C and 6D). That is, although there is no requirement for displacement and compression to meet Leonardo's constraint when viewing drawings, the correction mechanism is nonetheless applied, thereby causing the illusions.

The extent to which this idea can account for the two illusions described here has yet to be determined. Nonetheless, the idea

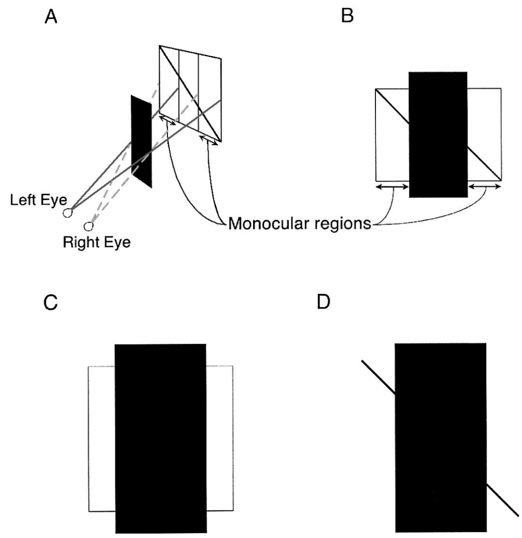

Figure 6. Schematic illustration of the expected consequences of the displacement of portions of the visual field and two geometric illusions. A: Bird's eye view of the stimulus. B: Expected front view. C: Kanizsa illusion. D: Poggendorff illusion. One should note that the two illusions in Panels C and D are opposite to what is expected from the displacement, as shown in Panel B. This figure illustrates the stimulus situation that is said to produce da Vinci stereopsis and differs from those previously presented (Ono, Wade, & Lillakas, 2002; Ono, Lillakas, & Mapp, 2003). For a demonstration of depth seen with da Vinci stereopsis, see Appendix A.

provides a different level of explanation for the illusions than what has been offered to date. The other explanations are in terms of some feature of the stimulus or a cognitive process: for example, an error in angular perception (Hering, 1879/1942), underestimation of the extent of the occluder (Zanuttini, 1976), an optical blur at the intersection of contours (Coren, 1969), or a different spread of attention across the occluded and nonoccluded surfaces (Linnell & Humphreys, 1999). The present hypothesis invokes the idea of an inappropriate correction mechanism that evolved for veridical perception in a three-dimensional world. It is similar to Gregory's (1963) hypothesis about the inappropriate constancy scaling as an explanation for the Mueller–Lyer illusion (see Day & Knuth, 1982; Robinson, 1972). Moreover, the ideas presented here address theoretical puzzles, namely, (a) why shape perception remains veridical in three-dimensional perceptual space despite visual direction being nonveridical (Ohtsuka & Ono, 2002; van Ee & Erkelens, 2000); (b) why in three-dimensional perceptual space, a square behind an occluder is seen as a square (Ohtsuka, 1995a; Ono et al., 1998) and an oblique line behind an occluder is seen aligned (Drobnis & Lawson, 1976; Gyoba, 1978; Liu & Kennedy, 1995; Ohtsuka, 1995b); and (c) why the end of an oblique line that abuts an occluder in a drawing appears more misaligned than does its far end (Wenderoth, 1980).

To propose that the underlying mechanism for visual direction examined in this study is different from that for shape and alignment is plausible but ad hoc. The idea that the visual system has different mechanisms for processing different attributes, however, is well documented. For example, it was once proposed, and evidence was provided, that there are separate mechanisms for processing distance and direction (Wells, 1792); more recently, it was proposed that there are separate mechanisms for processing location and shape (Loomis, Philbeck, & Zahorik, 2002). These proposals are consistent with the idea of the existence of separate pathways for processing brightness, color, depth, texture, and relative motion (see, e.g., Cavanagh, 1992; Regan, 2000). In this light, our proposal that different mechanisms process visual direction and shape alignment is a reasonable one worthy of further exploration.

Finally, these results support the claim made elsewhere (Ono, Wade, & Lillakas, 2002) that the perception of the stimulus situation that concerned Leonardo can be better understood as a constraint for a painter and for the visual system rather than as a paradox, as claimed by Boring (1942). The results reinforce our suggestion to use the designation *Leonardo's constraint* instead of *Leonardo's paradox*. This designation honors Leonardo for his insight that the two views from the two eyes together cannot be represented correctly on a canvas. At the same time it indicates an imperative for a painter and the visual system: Two opaque objects cannot be represented or seen in the same direction.

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(*Appendixes follow*)

Appendix A

Demonstrations Using Figure A1: Wheatstone (Disparity) Stereopsis and da Vinci Stereopsis

Instructions to Readers

It is possible to achieve a three dimensional perception from the images in the panels labeled *L* and *R* in Figure A1. (See Howard & Rogers, 2002, for a comprehensive review on autostereograms.) To do this, you must direct your left eye at the panel labeled *L* and your right eye at the panel labeled *R.* This can be done in two ways. One way is to gaze at a point beyond the plane of the figure. This is called *uncrossed fusion.* The other way is to direct your gaze at a point in front of the figure. This is called *crossed fusion.* We describe both methods in turn below. Most people find they prefer one of the methods. One should note that 2–4% of the population is stereo blind and 10–15% have some stereo deficiency (Julesz, 1971; Richards, 1970). With the stimulus in Figure A1A, readers who are stereo blind would not see depth between the fused virtual bars, and readers with stereo deficiency may have some difficulty. Whether these subpopulations can experience da Vinci stereopsis is not yet known.

Uncrossed Fusion

The idea in uncrossed fusion is to direct the left and right eyes at the left and center panels in Figure A1A, respectively, by looking straight ahead as illustrated on the left side of Figure A1B. To do this, bring Figure A1A close to your face with your nose between the left and center panels (labeled *L* and *R,* respectively). The left panel (with the *L* below it) should be in front of your left eye, and the right panel (with the *R* below it) should be in front of the right eye. Next, pretend to keep looking at something far away. Hold your gaze in that position and slowly move the figure away from your face until the page is approximately 40 cm in front of your nose. With this procedure your left eye will remain directed at the left panel, and your right eye will remain directed at the right eye's panel. Hold this position until a fused three-dimensional virtual image appears directly in front of your nose. (You might notice three other panels: one on each side of the fused virtual image and another farther to the right. These are to be

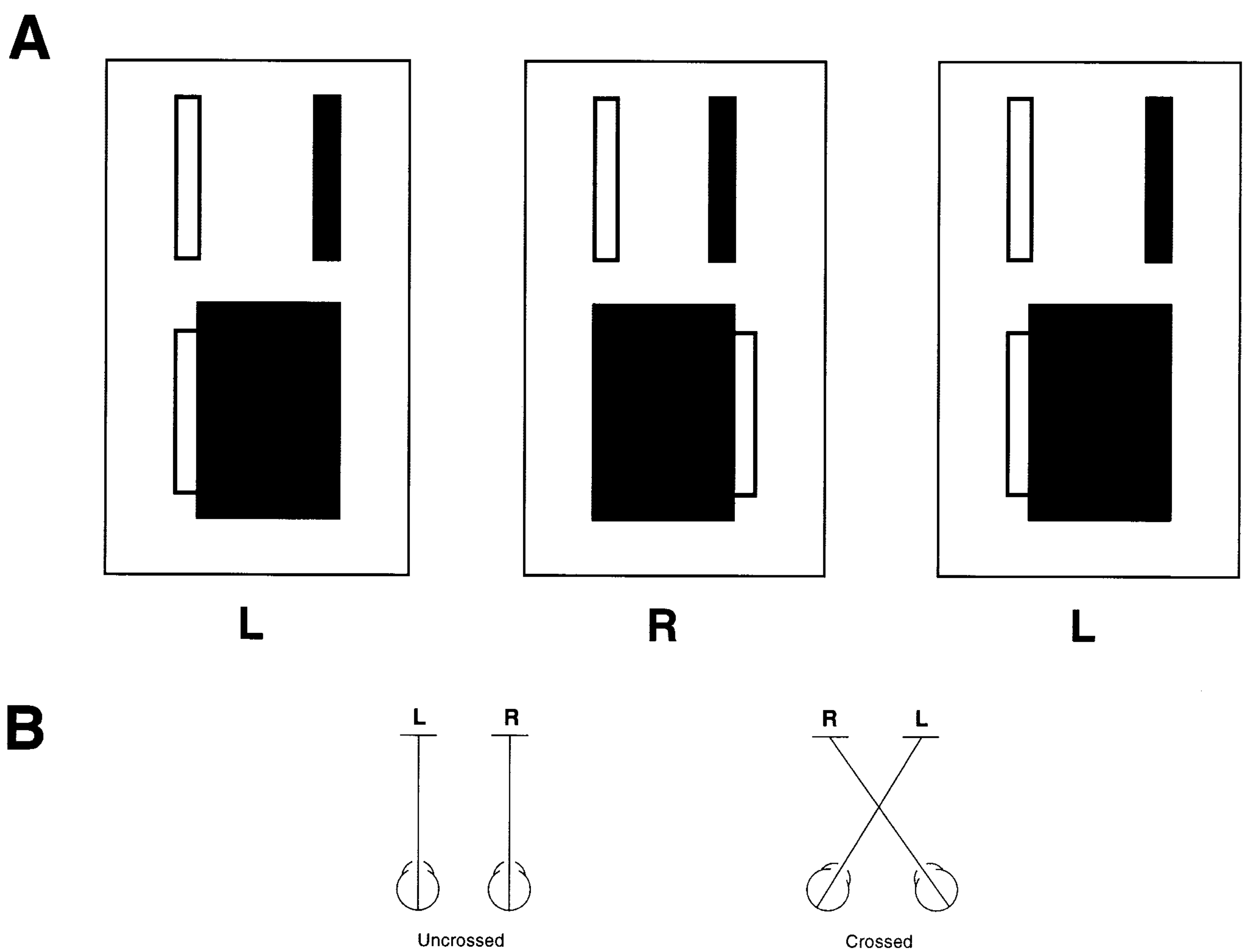

Figure A1. Demonstration of Wheatstone (disparity) stereopsis (Wheatstone, 1838) and da Vinci stereopsis described in the introduction. A: Autostereogram for the demonstration. B: Illustration of two possible eye positions for viewing the demonstration. Seeing the black bar closer than the white bar demonstrates Wheatstone stereopsis, whereas seeing the black occluder in front of the white square demonstrates da Vinci stereopsis. L = left eye; R = right eye.

ignored for the purpose of this demonstration.) Attend only to the fused virtual image directly in front of your nose.

Another way to fuse the left–center pair is to place a sheet of plastic or glass over the figure and look at the reflection of your face. Then, attend to the fused three-dimensional virtual image directly in front of your nose. While looking at your face in the reflection, you are directing your gaze beyond the plane of the figure. The reflection of your face is optically twice as far away as the figure, and at this distance your two lines of sight are not parallel. They will approach being parallel by increasing the distance between your face and the figure with the sheet of glass–plastic over it.

Crossed Fusion

The idea in crossed fusion is to direct the left and right eyes at the right and center panels in Figure A1A, respectively, by crossing your eyes as illustrated on the right side of Figure A1B. To do this, place Figure A1A approximately 40 cm in front of your face. Your nose should point between the center and right panels. Place a pencil or your fingertip approximately half way between the figure and your face (approximately 20 cm in front of your face). Wink your eyes back and forth to make sure your fingertip is lined up with both the *R* and your right eye and the *L* and your left eye. When you are satisfied that this is the case, look at your fingertip with both eyes. A fused 3-D virtual image will appear above your fingertip. (You might notice three other panels: one on each side of the fused virtual image and another farther to the left. These are to be ignored for the purpose of this demonstration.) Attend only to the fused virtual image directly in front of your nose.

What You See and a Brief Explanation

When the stimulus pairs are fused, either by crossed or uncrossed fusion, you see a white bar and a black bar at the top of the figure. The black bar appears closer than the white bar. Depth is seen between the bars because the separation between the images of the bars on the right retina is smaller than on the left retina. The disparity in the separation of the two bars between the left eye's retinal image and the right eye's retinal image is a cue to depth (Wade, 1983; Wheatstone, 1838). At the bottom of the figure you see a black rectangle occluding a white square. Depth is seen between the black rectangle and the occluded square in the bottom part of the figure despite the fact that there are no positional disparities as defined by Wheatstone. Only the left and right eyes view the left and right edges of the square, respectively, as though each edge was occluded to one eye. The depth perceived from this occlusion cue is called *da Vinci stereopsis* (Nakayama & Shimojo, 1990).

Appendix B

Measuring Fixation Disparity

It was impractical in Experiment 3 to adjust the apparatus each time the observers reported a misalignment of the Nonius stimulus. Instead, we determined the magnitude of each observer's fixation error. We used the results to estimate the perceptual displacements if observers had accurately fixated the targets in Experiment 3.

A fixation error is not unique to this study; when people fixate an object, there is usually a slight misconvergence, known as fixation disparity. For more details, see Howard (2002). A fixation error in which the observer is fixating a point in space nearer to them than the target is termed *crossed fixation disparity,* because what is seen with the right eye is to the left of what is seen by the left eye. A fixation error in which the observer is fixating a point in space farther than the target is termed *uncrossed fixation disparity,* because what is seen with the right eye is to the right of what is seen by the left eye.

Method

Observers

The same six observers from Experiment 3 participated.

Apparatus and Stimuli

The apparatus was the same as used in Experiment 3. The stimuli were similar but differed in what constituted the standard and comparison stimuli. The standard stimulus was the bottom Nonius line. The top Nonius line served as the comparison stimulus and was positioned at one of seven locations relative to the bottom Nonius line (0-, ±2.2-, ±4.4-, ±6.6-min arc). On a given trial, the single line and Nonius stimulus were randomly positioned above one of the six stimuli, which were the standard stimuli in Experiment 3 (the two monocular lines, the two binocular lines, and the two edges of the occluder).

Procedure

The procedure was similar to that of Experiment 3. Observers fixated the single line that was also the standard stimulus in Experiment 3 and judged whether the top Nonius line was to the left or to the right of the bottom Nonius line. Their judgments were recorded in the same way as in Experiment 3. Observers completed 28 judgments for each location of the top Nonius line in each of two depth planes. The order of depth planes was randomized across observers. Observers completed 392 trials in this experiment (7 top Nonius line positions × 2 fixation conditions × 28 trials).

Table B1
Each Observer's Fixation Disparity (Min Arc) in Each Depth Plane

	Depth plane	
Observer	Near	Far
PG	−2.37	−2.60
LL	−1.60	−0.70
NV	−0.82	−0.21
VB	−4.17	−3.35
EL	−2.88	−2.55
OE	−1.57	−0.74

Note. Negative signs indicate uncrossed fixation disparity, which means the observers converged their eyes slightly farther than the desired fixation plane.

Results

We first defined the actual location of the bottom Nonius line as zero, crossed disparity from this point as positive, and uncrossed disparity as negative. We then used Probit analysis (Finney, 1971) to determine the position of the top Nonius line that was judged to be collinear (point of subjective equality) with the bottom Nonius line. The mean fixation disparity values of each observer for two fixation distances are shown in Table B1. All observers had uncrossed fixation disparity for both fixation distances. We combined these data with the data from Experiment 3, as described in the *Results* section of that experiment, to estimate the perceived displacement of monocular features in the absence of fixation disparity.

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