THE PERCEPTION OF FORCE[1]

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I. Introduction

(*a*) *Statement of the Problem.*—If an individual is asked to pull a spring to a point where the tension will amount to twenty pounds he will very probably make a large error. If he is permitted to repeat the pull after being told the amount and direction of his error he will probably either reverse the direction of the error or approach nearer to the required twenty pounds. Succeeding corrections and trials will enable him to reduce the size of his errors; and they will, if tabulated, give an approximation to the normal probability curve with the twenty pounds as a mean.[2] To delineate the mechanism which makes such learning possible and to describe the modifications involved would be no simple task; evidence for which is given in the antagonistic results of previous investigations on this subject.[3]

Disagreement in the findings of science is due either to faulty experimental control, to incorrect interpretation of results, or to both; hence when one meets contradiction in the results of different experiments it is first necessary to see whether the conclusions as stated by the investigators necessarily follow from their experimental data, and if the trouble

[1] The experiment of which this article is a report was made possible through a Cutting Travelling Fellowship granted the writer by the trustees of Columbia University in the spring of 1916. The apparatus was made and the experimenting largely done in the psychological laboratory of the Johns Hopkins University under the supervision and with the generous assistance of Professor John B. Watson. The experimenting was completed in the Princeton psychological laboratory and the data put in shape while the writer was instructor in psychology in Princeton University.

[2] Fullerton, G. S., and Cattell, J. McK., 'On the Perception of Small Differences,' Publ. of the Univ. of Penna. Philos. Series, 1892, No. 2.

[3] Sherrington, C. S., 'The Muscular Sense,' Shäfer's 'Text-book of Physiology,' 1002–1025. James, Wm., 'Principles of Psychology,' Vol. II., 486–520. Woodworth, R. S., 'Le Mouvement.'

cannot be located there to repeat and modify the experimental conditions in order to obtain supplementary information on the subject. Our problem will therefore be to investigate the nature of the conflicts in previous experiments on the perception of force and then after we have clearly formulated the point at issue to attempt to supply data based on experimental modifications of such a nature as to give them strong evidential character.

(*b*) *Some of the Points at Issue.*—In connection with this problem one fact that previous investigations have demonstrated is that the 'sense of tension,' 'sense of resistance,' or 'sense of force' is a very complex affair. If there were a unitary sensation of innervation accompanying the discharge of motor impulses; if there were a simple sensation of tension, of energy expended, or of muscle change accompanying movements of the body; if extent or time were the fundamental thing in the perception of movement;—if any one of these were the irreducible element, no such discordant results as have been obtained would have confused investigators and no doubt the problem would long ago have been settled. Different investigators have as the result of different experiments argued for the importance of each of these factors in the production and perception of movement, and each new experiment seems to bring to light some new phenomenon which contradicts what some previous investigator has found. One factor among those mentioned that seems to have been discredited by recent experiments is the sense of innervation.[1] At least there are such strong arguments against it that we need not consider it here. If then kinesthesis depends wholly upon afferent impulses, the question is left as to whether it is determined principally by the quality or intensity of these impulses or by their spatial or temporal relations, or by all these combined. Under certain conditions it has been shown that one judges force by movement speeds,[2] in other conditions extent has been shown to be important, while in others the criterion is the latent time required to overcome the

[1] *Op. cit.*

[2] Müller and Schumann, *Arch. f. d. ges. Physiol.*, 1889, 45.

opposing resistance.[1] Clinical experiments on the effect of anesthetizing the superficial areas on the perception of lifted weights have given discordant results.[2]

Fullerton and Cattell give three arguments against the contention of Müller and Schumann that force is judged by speed. (1) The force of a movement can be judged better than its time. (2) The judgment of time follows Weber's law more nearly than the judgment of force. (3) When the rate is altered so that the one is lifted four times as rapidly as the other, either by being lifted higher in the same time or the same distance more quickly, the probable error is not increased. They state that this latter unexpected result proves conclusively that we do not judge of difference in weights by the rate at which they are lifted. They are of the opinion that lifted weights are judged by a combination of skin, pressure and muscle sensations.[3]

(*c*) *Previous Work Leading to the Present Paper.*—The thesis which led to the experiments to be reported in this paper was that the perception of force is very crude and its seeming accuracy in certain instances depends upon the adoption of secondary criteria. We will first refer very briefly to the facts which led to this thesis, show how the use of secondary criteria could have entered into Fullerton and Cattell's experiments on the perception of force, and this will bring us to an explanation of the method and plan of our experiments.

It has been found that when a person is told to raise a weight with all the force he can, if the weight is changed he will tend to pull varying masses with the same speed, which means that the force of the muscular contraction must vary with the different weights.[4] The time required for this adjustment of force has been found to be fifty sigma or less, in no case more than 100 sigma. It must therefore be either a reflex or a local muscular phenomenon. Further, when the

[1] See discussion of the experiments of Jacobi in Woodworth's 'Le Mouvement.'

[2] Sherrington, *op. cit.*

[3] *Op. cit.*

[4] Morgan, J. J. B., 'The Overcoming of Distraction and Other Resistances,' *Arch. of Psychol.*, No. 35, 1916.

subject is told to pull several weights with the same force he can make but crudely the time adjustment that is necessary.[1]

These facts show that if one has any elementary sensation of force it is not the same thing we have to deal with in physics. In physics, if we have a certain force acting against a certain mass we will get a definite acceleration, the force being equal to the mass multiplied by the acceleration. If the mass is changed and the force kept the same the acceleration can be determined from the equation and is found to be borne out by experiment. If we tell a subject to set his own force in pulling a certain weight we can measure the acceleration and can determine the amount of physical force used. If we change the mass and tell him to use the same force as before we find that acceleration is but little changed but that the resultant physical force is. Certainly this shows that the subject has no unitary sensation of physical force; or, if he has, he is grossly ignorant of its relation to acceleration. The force of a spring, of an explosion, or of gravity is vastly more accurate in its adjustments than the human muscle.

Over against these facts stand the experiments of Fullerton and Cattell which showed that force can be judged more accurately than time, although somewhat less accurately than extent. In all their experiments, except in one with extent, they used the following procedure: The subject was given a practice series in which to learn the standard magnitude, whether extent, time, or force. After the practice series the movements were made in pairs. The first of each pair was an attempt to approximate the standard magnitude, the second of the pair was an attempt to equal the first. In this way the subject made his own standard which he used for comparison. The average errors were taken from the differences between the two movements of the pairs. The subject was told at the end of each ten pairs how much he was above or below the standard magnitude, and thus could

[1] *Ibid.*, 'The Speed and Accuracy of Motor Adjustments,' J. of Exper. Psychol., June, 1917.

attempt to make the necessary correction. In the experiments on extent they used standard magnitudes of 100, 300, 500 and 700 mm.; in the experiments on time they had the subjects make a 50 cm. movement in minimum time, in 250 sigma, in 500 sigma and 1,000 sigma; in the experiments on force the subjects endeavored to pull the handle of a spring dynamometer with a force of 2, 4, 8 and 16 kg. In the experiments on extent no record was taken of the time of the movements, in the experiments on force no record was taken of the time, and in all except the 16 kg. pull the force was a direct function of the extent of the pull. For these reasons we believe that a comparison of the relative accuracy of the perception of time, force and extent cannot be derived from their experiments.

Since we are mainly interested in the force of movements we will study a little more closely their experiments on this phase of the subject. The dynamometer they used moved 6.4 mm. for every kg. up to 10 kg., for pulls of 10 kg. or more they changed the apparatus so that movement began at 10 kg. and for each kg. above 10 the handle moved 6.4 mm. This means that in pulling a standard of 2,000 grams the subject had a standard extent of 12.8 mm. to strive for; in pulling a standard of 4,000 grams he had an extent standard of 25.6 mm.; in pulling a standard of 8,000 grams he had an extent standard of 51.2 mm.; and, in pulling a standard of 16,000 grams he had an extent standard of 38.4 mm.

We have no way of telling what the variable errors of Fullerton and Cattell's subjects would have been had they been given extent standards of these values. If they had experimented with such extent standards we could compare them with the extent errors when they used 2, 4, 8 and 16 kilograms as standards and might thus ascertain whether the force pulls were influenced by the extent of the movements made.

It might nevertheless be interesting to determine what the extent errors for the 12.8 mm., 25.6 mm., 51.2 mm. and 38.4 mm. standards would have been on the basis of their extent experiments with 100, 300, 500 and 700 mm. standards, if calculated by Weber's or Cattell's psycho-physical laws.[1] The ratio of error for the 100 mm. movement was for their subject F. one to 18.9.[2] If this ratio were to hold according to Weber's law this subject would have scored a variable error of .676 mm. with the standard of 51.2 mm., 1.42 mm. with the standard of 25.6 mm., 2.84 mm. with the standard of 51.2 mm. and 2.13 mm. with the standard of 38.4 mm. If on the other hand Cattell's square root law held good this subject's variable errors for the several linear magnitudes would have been respectively 1.90, 2.69, 3.79 and 3.29 mm. Now in their force experiments a pull of one mm. on the dynamometer which they used equalled nearly 156 grams. If then we transmute the linear errors into gram errors we will get the result shown in Table I. By comparing these

TABLE I

THE RELATION BETWEEN THE VARIABLE ERRORS OBTAINED FROM SUBJECT *F* IN FULLERTON AND CATTELL'S FORCE EXPERIMENTS AND THE VARIABLE ERRORS AS COMPUTED FOR MOVEMENTS OF SIMILAR EXTENT FROM THEIR EXPERIMENTS ON EXTENT

	Force Standards Extent Standards	2,000 12.8	4,000 25.6	8,000 51.2	16,000 Grams 38.4 Mm.
V. E.	Subject *F* in force exp	183	280	373	434 grams
V. E.	Weber's law from extent exp	105	221	443	332 grams
V. E.	Cattell's law from extent exp	296	420	591	513 grams

two sets of computed variable errors with the actual variable errors made by this same subject in the force experiment it will be seen that for the most part the actual force errors fall between the force errors computed from the 100 mm. standard by the two methods. This is evidence enough at least to suggest to one the hypothesis that the force pulls were in large part guided by extent, and Fullerton and Cattell could have secured the results they did if their subjects possessed only a very crude perception of force as such. Theoretically this removes the disparity between the

[1] Cattell, J. McK., 'On Errors of Observation,' *Am. Jour. of Psychol.*, 1893, *5*, 285–293.

[2] Fullerton and Cattell, 'On the Perception of Small Differences,' Univ. of Penna. Publ. Philos. Series, 1892, p. 48.

results of our experiments and those of Fullerton and Cattell. We found that the force a person exerted was determined by the resistance encountered and that the subjects could not consciously adjust the speed of their movements so as to use the same force in moving different masses. Fullerton and Cattell found that force could be judged better than time; but, as their subjects may have been judging by extent, any comparison of force with time or extent is ruled out. It remains for experiment to show whether this hypothesis, that the perception of force is largely dependent on other factors, can be proven and it is this we have attempted to do in the experiment we are about to report.

An observation made by Professor Woodworth has a bearing on the problem.[1] He has shown that if the several factors that enter into a perception are perfectly correlated the variable error will increase in direct proportion to the stimulus (Weber's law), while if the factors are not at all correlated but are operative in a purely chance way the variable error will increase in proportion to the square root of the stimulus (Cattell's law). Where there is some correlation the error of observation will fall between the error required by the two laws. Now if all the causal factors of any perception are directly correlated, by the very nature of the case these factors must be relatively few, for by chance we mean a number of factors working indiscriminately, hence the larger the number of factors the greater the likelihood of a pure chance series. In Fullerton and Cattell's experiments the variable error in time followed Weber's law very closely, the variable error in extent followed Cattell's square root law, and the variable error of force fell between. This would indicate that the perception of time is relatively simple compared with that of extent or force regardless of their relative accuracy, and we believe our experiments will show the complex nature of the perception of force and extent. This lack of correlation between the factors controlling the force of movements may account for the conflicting results

[1] Professor Cattell's Psychophysical Contributions, in the Psychological Researches of James McKeen Cattell, *Arch. of Psychol.*, 1914, No. 30, pp. 70–72.

obtained in experiments on the perception of force. Müller and Schumann found that under certain conditions the perception of lifted weights correlated with the speed with which they were lifted. Cattell varied the speed and found perception as accurate as before. If weights are judged by a number of non-correlated factors, a subject could readily shift from one basis of judgment to another. We feel that in the study of the subject this fact should receive strong emphasis. If we are studying a form of perception which depends upon, let us say, five correlated factors and we experimentally interfere with one factor, the total perception will be changed more radically than would be the case if we were to interfere with one element in a perception that depended on five factors operating in a purely chance manner. Translated into the terms of our problem this would mean that, if the perception of force depends on several non-correlated factors, under normal conditions the subject will judge the force of his movements; or, in objective terms, the error in his movements will be determined by all, several or perhaps only one of these factors. Suppose his force movements are determined largely by time; then, if time is varied he might shift to extent. If extent were varied or eliminated he might revert back to time unless it were still controlled. If both time and extent were eliminated, skin and muscle sensations might be called upon to bear the larger part in the force control and judgment. If, therefore, force depends upon one factor the control of this would perfectly control the error of a force movement. If it depends on several partially correlated factors the relative importance of the different factors could be determined experimentally. If it depends on several chance factors the only way to change the error and judgment of force would be to have adequate control of all the factors. We believe our results will show that Woodworth was right in his theory, and that force depends on a number of partially correlated factors. We may also be able to show the relative importance of some of them.

If it is possible to show that this is the case it may shed some light on what we mean by adaptation. It is possibly nothing more than an evidence of the multiplicity of causes underlying activity of any sort. If the causes of an act are few or closely correlated, adaptation will be less complete than if the causes are numerous and related only in a random way.

II. General Plan of the Experiment

To arrange an experimental procedure which would show what determines the control and judgment of the force of movements was our problem. Three major experimental variations were used, the same general procedure being used in all. The general procedure was to inform the subject of the task; that is, whether to attempt to make a movement of a certain length or to pull with a certain force. Having received his instructions he was given twenty-five practice trials, the amount and direction of his error being given after each trial. After this practice series the movements were made in pairs. In the first of each pair the subject tried to produce the standard, while in the second he tried to reproduce the first. This is the procedure devised by Cattell and Fullerton and it permits the subject to make his own standard for each movement and gives a much more accurate record of the subject's perception and control than if the arbitrary standard was used as a base from which to compute the errors. After the second movement of each pair the the subject gave a judgment as to the direction of the difference between the two. After he had given this report the experimenter told him the direction and amount of error of the second of the pair when compared with the arbitrary standard given at the beginning. He was thus enabled to make an intelligent effort to correct the first movement of the next pair which we will call the standard. He was not told whether his judgment as to the direction of the error between the two pulls was correct or not. Fifty pairs in addition to the twenty-five practice movements constituted one experimental sitting.

The experimental variations were as follows:

1. The subject was instructed to pull with a certain force

and corrections were given him in terms of the number of grams too heavy or too light. In all these experiments the extent of the pulls was the same for each standard force from 2 to 16 kg. Time records were taken for each pull, the chronoscope starting when the pull began and stopping the instant the return stroke was initiated.

2. With a change in the arrangement of springs on the dynamometer between experimental sittings the subject was given instructions to pull a certain distance and corrections were given in terms of millimeters too long or short. Time records were taken for each pull, as in the previous procedure.

3. The subject held his arm as nearly stationary as possible and the experimenter increased the tension, the subject calling out when he judged that the tension had reached the required amount.

An experimental sitting consisted of 25 practice pulls followed by 100 paired pulls with one set of springs. A set of experiments included experiments with 2, 4, 6, 8, 10, 12, 14 and 16 springs. Including the 3,200 practice pulls the experiments to be reported are based upon 16,000 pulls and 6,400 judgments.

The sequence of the experiments was varied with the different subjects so that when averaged together practice effect was eliminated in the average scores. Subjects *A*, *B*, *C* and *D* had an entirely different order from *E*, *F*, *G* and *H*. In addition the sequence for *A* and *B* was exactly the reverse of that for *C* and *D*, and that for *E* and *H* the exact reverse of that for *F* and *G*.

The subjects in the experiment ranged from those who were highly trained in experimental psychology and laboratory procedure to those distinctly untrained. We could find no tendency for the untrained to differ specifically from the trained.

III. Description of Apparatus

The dynamometer consisted of a handle connected to a rod which ran on roller bearings so as to minimize friction. From this rod projected a smaller rod at right angles which moved an indicator before it as it made the forward stroke.

On the return stroke this rod left the indicator, thus giving the experimenter an opportunity to read the extent of the movement from a millimeter scale which was attached to the

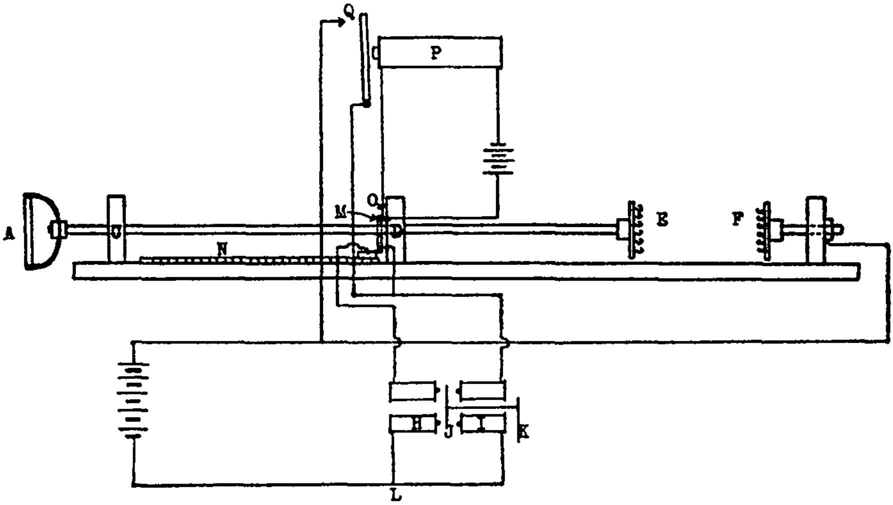

FIG. 1. Outline of Dynamometer and Dunlap Chronoscope with Electrical Connections. *A* is the handle of the dynamometer which connects with the rod *B* which in responding to pulls on the handle works on roller bearings at *C* and *D*. *E* and *F* are plates each equipped with 16 hooks upon which may be fastened 16 springs in parallel. *H* and *I* are the two sets of magnets in the Dunlap chronoscope. The magnets *H* are connected to the armature of a synchronous motor which is in motion throughout the experiment. When everything is set for a pull the rod at *M* is connected both through the magnets *I* by a contact which breaks as soon as the handle moves forward and through the magnets *H* by a contact with the marker which rides on the scale *N*. At *L* is inserted a Dunlap key (omitted in the drawing for the sake of clearness) which closes the contact through *I* an instant before it closes that through *H*, thus making certain that the armature *J* is against the stationary magnets *I*. The breaking of the contact through the magnet *I* at the beginning of a pull permits the armature to be pulled away from the stationary magnets *I* to the revolving magnets *H* and the indicator *K* begins to revolve. The beginning of a pull likewise breaks a contact at *O* which breaks an independent circuit through the relay *P*. The armature of the relay being released falls back and makes a contact at *Q* which reconnects the circuit through the magnets *I* of the chronoscope. The armature of the relay is so adjusted that the remaking of the current would not be strong enough to pull it up and so the breaking of the contact at *Q* is not accomplished until the experimenter pushes it up with his hand. This prevents any inadvertent starting of the chronoscope. The forward movement of the handle moves the indicator along the scale *N* and as soon as the return stroke is initiated the contact through it and the rod *M* through the revolving magnets of the chronoscope *H* is broken, thus allowing the magnets *I* to pull the armature *J* forward and stop the chronoscope.

dynamometer. After reading the indicator was returned to *O*. An electrical contact was broken when the arm left its back stop at the beginning of a pull and a second contact broken when the rod left the indicator at the beginning of the return stroke. These two break contacts operated the magnets of a Dunlap chronoscope. The details of the chronoscope connections and operation are shown in Fig. 1. The rod attached to the handle of the dynamometer had at its other end a plate upon which were 16 hooks. This plate faced another similar plate which was fixed to the other end of the dynamometer. Between each of these 16 pairs of hooks springs could be placed or removed as desired. They were however always used in pairs so as to prevent lateral torsion. This was accomplished by using together two springs on a line with the center of the rod and equidistant from it. The springs were as nearly alike as possible but were carefully calibrated and the variations that were found were taken into consideration in the records. An extension of 175 mm. required a pull of one kilogram on a single spring. By arranging, in different experiments, 2, 4, 6, 8, 10, 12, 14 and 16 springs in parallel we were enabled to use standards of 2, 4, 6, 8, 10, 12, 14 and 16 kilograms and in each case keep the extent of the movement the same.

In the experiment where the subject held his arm stationary a second handle was attached by a wire to the rear end of the dynamometer, which in this case was suspended from two standards and the movable end connected to a windlass which the experimenter operated to tighten the springs. The subject held the handle as near to an index point as possible throughout the experiment and consequently the dynamometer and the handle he held only moved with the waverings of his hand. In every case the dynamometer was screened from the subject, and when he made arm movements a screen was placed between his arm and body so that he could not observe his movements.

IV. Results

In the first experiment the subjects were required to pull 2, 4, 6, 8, 10, 12, 14 and 16 kilograms, a different force being asked for at each experimental sitting, the sequence of the different forces being determined by chance. Records were taken of the extent error and the time of each pull. The force errors were later computed from the extent error. In this series four subjects *A*, *B*, *C* and *D* were used and their

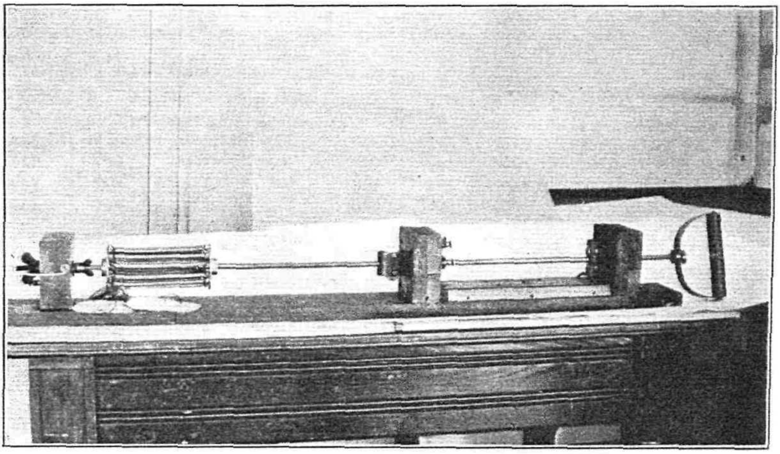

Fig. 2.

force, extent and time errors are given in Tables II., III. and IV. In each of the tables four records are given for each subject for each set of springs used. The first is the average force, time or extent of the first pull of each pair, called the standard. The second score is the average error, that is, the average difference between the first and second pulls. The third score is the variable error showing the variability in the size of the error between the first and second pulls. The last record is the ratio of the variable error to the standard.

In this experiment the extent factor was the same with all the various forces used. The subject could pull the handle different distances, as there was nothing on the apparatus to prevent this (except that a stop was arranged so that he could not pull hard enough to damage the springs), but if he pulled the exact force standard the extent of the movement would be the same for each force standard. The time factor was not controlled in any way, the subject being left free to pull as quickly or as slowly as he pleased.

TABLE II

FORCE RECORDS IN MM. OF 1,600 EXPERIMENTS IN WHICH THE SUBJECTS ATTEMPTED TO PRODUCE TWO MOVEMENTS OF EQUAL FORCE, AND IN WHICH CORRECTIONS WERE GIVEN IN GRAMS.

Set Standard ...	2	4	6	8	10	12	14	16 Kg.
Subject A:								
Av. of standard pull ..	1,980	3,910	5,766	7,970	9,893	11,906	13,923	15,744
A. D.	*107*	*162*	*244*	*562*	*304*	*360*	*546*	*618*
Average error.	102	246	264	304	325	354	469	584
Variable error	64	138	162	188	210	234	245	344
Ratio V. E. to stand .	*30.9*	*29.3*	*35.6*	*42.4*	*47.1*	*50.9*	*56.9*	*45.8*
Subject B:								
Av. of standard pull	2,020	4,126	6,012	8,008	9,885	11,460	14,091	15,984
A. D..	*46*	*190*	*315*	*365*	*458*	*456*	*582*	*736*
Average error	134	232	330	388	650	666	560	824
Variable error	85	156	189	192	230	324	329	496
Ratio V. E. to stand. ..	*23.8*	*26.4*	*31.8*	*42.1*	*43.0*	*35.4*	*42 8*	*32.2*
Subject C:								
Av. of standard pull.	1,943	3,946	5,902	7,972	10,099	11,974	13,780	15,887
A. D.	*89*	*191*	*250*	*358*	*434*	*446*	*476*	*429*
Average error	117	251	395	310	479	492	549	507
Variable error	65	138	167	181	334	278	311	288
Ratio V. E. to stand...	*29.9*	*28 6*	*25.3*	*44.1*	*30.0*	*43*	*44.3*	*55.1*
Subject D:								
Av. of standard pull	2,008	4,093	5,980	8,094	9,789	12,026	13,789	16,025
A. D. .	*73*	*154*	*197*	*237*	*360*	*367*	*591*	*464*
Average error	86	126	237	274	418	516	600	507
Variable error	60	78	145	156	266	396	358	294
Ratio V. E. to stand.....	*33.4*	*63.3*	*41.2*	*51.8*	*36.8*	*30.4*	*38.4*	*54.5*
Averages:								
Standard pull	1,987.7	4,018.7	5,915	8,011	9,916	11,841	13,896	15,910
A. D.	*78.7*	*174.2*	*251.5*	*380.5*	*389*	*407.3*	*548.7*	*561.7*
Average error	109.7	213.7	306.5	319	468	507	544.5	605.5
Variable error	68.5	127.5	165 7	179.2	260	308	310.7	355.5
Ratio. ..	*29.0*	*36.9*	*33.5*	*45.1*	*39.2*	*39.9*	*45.6*	*46.9*

Before we examine the experimental results in detail it may be well to consider the relation of some of the factors involved and what certain results would mean. This will give a point to the presentation of the results, and give their exposition greater clearness. If the subject used extent as a basis of control the extent errors should have been nearly the same in all cases, which would of course mean that the force errors would vary in conformity with Weber's law. If the time errors appear the same with all forces, it would indicate that time might help the subject to control force.

Table III

Extent Records in Grams of 1,600 Experiments in which the Subjects Attempted to Produce Two Movements of Equal Force, and in which Corrections Were Given Them in Grams.

Set Standard......	2	4	6	8	10	12	14	16
Subject A:								
Av. of standard pull...	171.92	168.54	164.2	171.26	169.86	174.44	173.92	172.8
A. D.	*10.68*	*8.08*	*8.12*	*14.04*	*6.08*	*6.0*	*7.8*	*7.72*
Average error	10.2	12.3	8.8	7.6	6.5	5.9	6.7	7.3
Variable error	6.4	6.9	5.4	4.7	4.2	3.9	3.5	4.3
Ratio V. E. to stand...	*26.9*	*24.4*	*30.4*	*36.4*	*40.4*	*44.7*	*49.6*	*40.2*
Subject B:								
Av. of standard pull...	176.04	179.28	172.4	172.2	169.7	167.62	176.3	175.8
A. D.	*4.6*	*9.52*	*10.52*	*9.12*	*9.16*	*7.6*	*8.32*	*9.2*
Average error	13.4	11.6	11.0	9.7	13.0	11.1	8.0	10.3
Variable error	8.5	7.8	6.3	4.8	4.6	5.4	4.7	6.2
Ratio V. E. to stand...	*20.7*	*23.0*	*27.4*	*35 9*	*36.9*	*31.0*	*37.5*	*28.3*
Subject C:								
Av. of standard pull...	168.28	170.28	168.74	171.3	173.98	175.56	171.86	174.54
A. D.	*8.92*	*9.56*	*8.32*	*8.96*	*8.68*	*7.44*	*6.80*	*5.36*
Average error	11.66	12.56	13.16	7.74	9.58	8.2	7.84	6.34
Variable error	6.52	6.92	5.56	4.52	6.68	4.64	4.44	3.6
Ratio V. E. to stand...	*25.8*	*24.7*	*30.3*	*38.0*	*25.9*	*37.8*	*38.7*	*48.5*
Subject D:								
Av. of standard pull	174.8	177.64	171.34	174.36	167.78	176.44	171.98	176.32
A. D.	*7.27*	*7.72*	*6.56*	*5.92*	*7.2*	*6.12*	*8.45*	*5.8*
Average error	8.64	6.3	7.9	6.86	8.36	8.6	8.56	6.34
Variable error	5.97	3.88	4.84	3.9	5.32	6.6	5.12	3.68
Ratio V. E. to stand.	*29.3*	*45.8*	*35.5*	*44.7*	*31.50*	*26.7*	*33.6*	*47.6*
Averages:								
Standard pull	172.6	173.93	169 17	172.28	170.33	173.51	173.51	174.86
A. D.	*7.87*	*8.72*	*8.38*	*9.51*	*7.78*	*6 79*	*7.84*	*7.02*
Average error	10 97	10.69	10.21	7.97	9.36	8.45	7.77	7.57
Variable error	6.85	6.37	5.52	4.48	5.20	5.13	4.44	4.44
Ratio	*25.68*	*29.47*	*30.9*	*38.75*	*33.67*	*35.05*	*39.85*	*41.15*

If the subject were ignorant of the relation of force and extent and could only judge time, the force and extent errors might show an interrelation and the time errors be the same.

In the actual experimental results the force errors (Table II.) increase with the size of the stimulus, not as rapidly as would be required by Weber's law and more rapidly than would be required by Cattell's square root law (see Fig. 3).

Table IV

Time Records in Sigma of 1,600 Experiments in which the Subjects Attempted to Produce Two Movements of Equal Force, and in which the Corrections Were Given in Grams.

Set Standard ..	2	4	6	8	10	12	14	16
Subject A:								
Av. of standard pull.....	1,393	1,235	1,485	1,725	1,425	1,224	1,163	967
A. D.........	*174*	*97*	*108*	*266*	*99*	*127*	*85*	*124*
Average error.	117	116	118	201	113	91	68	81
Variable error.	83	64	84	112	79	52	42	55
Ratio V. E. to stand.... ...	*16.8*	*19.3*	*17.7*	*15.4*	*18.1*	*23.6*	*27.7*	*17.6*
Subject B:								
Av. of standard pull.........	516	430	462	527	562	432	492	601
A. D....... .	*57*	*25*	*40*	*53*	*37*	*30*	*43*	*47*
Average error	51	33	39	52	41	42	52	56
Variable error	31	22	24	31	28	27	31	42
Ratio V E. to stand. .. .	*16.6*	*19.6*	*19.3*	*17.0*	*20.1*	*16.0*	*15.9*	*14.3*
Subject C:								
Av. of standard pull	918	945	1,105	827	891	916	1,012	782
A. D....	*77*	*84*	*97*	*46*	*44*	*61*	*65*	*42*
Average error..	71	86	86	47	61	75	76	43
Variable error..	44	50	47	30	41	40	48	29
Ratio V. E. to stand.... ...	*20.9*	*18.9*	*23.5*	*27.6*	*21.7*	*22.9*	*21.1*	*27.0*
Subject D:								
Av. of standard pull.......	441	447	442	438	439	413	433	455
A. D...... .. .	*20*	*19*	*17*	*13*	*15*	*19*	*16*	*24*
Average error..	19	24	24	22	19	22	18	27
Variable error.	11	14	15	12	11	15	11	14
Ratio V. E. to stand.	*40.0*	*31.8*	*29.4*	*36.5*	*39.9*	*27.6*	*39.4*	*32.5*
Averages:								
Standard pull	817	764	873	879	829	746	775	701
A. D... ..	*82*	*56*	*65*	*94*	*49*	*59*	*52*	*59*
Average error..	64	65	67	80	58	57	53	52
Variable error .	42	37	42	46	40	33	33	35
Ratio.. . .	*23.6*	*22.4*	*22.5*	*24.1*	*25.0*	*22.5*	*26.0*	*22.8*

According to Woodworth's interpretation referred to above this would mean that force is not controlled by factors perfectly correlated nor operating by pure chance. Time seems to have some part to play (Table IV.), since the ratio of the errors to the standard time (that is, the time of the first pull) is about the same with all forces. It cannot, however, be the controlling factor, since the relative error is greater than either force or extent.

An examination of the average and variable error records (Table III.) shows that as the resistance to a movement is increased the ability to make two movements of the same extent is increased while on the other hand the ability to reproduce a standard pull (shown by the average deviation of the standard) is the same regardless of the resistance opposed to the movement, which means that the production of a standard force increases in direct proportion to the magnitude of the force (Weber's law). This together with evidence we will presently adduce points to the importance of extent in judging force.

Table V

Extent Records in Mm. of 1,600 Experiments in which the Subjects Attempted to Produce Two Movements of Equal Extent, and in which the Corrections Were Given in Mm.

Number of Springs	2	4	6	8	10	12	4	16
Subject A:								
Av. of standard pull	172.18	168.78	173.02	168.22	171.94	173.4	171.62	174.06
A. D.	*6.16*	*6.2*	*8.72*	*6.72*	*6.64*	*4.03*	*6.24*	*6.24*
Average error	10 0	7.1	9.4	10.4	6.9	5.0	5.26	6.7
Variable error	5.8	2.1	5.68	5.7	4.2	2.9	3.24	3.9
Ratio V. E. to stand.	*29.7*	*80.3*	*30.5*	*29.7*	*40.9*	*59.9*	*53*	*44.7*
Subject B:								
Av. of standard pull	182.58	177.3	175.4	179.2	178.72	172.86	172.72	171.78
A. D.	*13.12*	*11.86*	*7.34*	*7.02*	*7.21*	*8.88*	*5.56*	*5.36*
Average error	15.7	7.92	10.0	9.2	7.1	8.9	6.3	5.1
Variable error	7.1	4.4	5.6	5.0	3.3	2.3	4.4	2.7
Ratio V. E. to stand.	*25.7*	*40.3*	*31.3*	*35.8*	*54.1*	*75.0*	*39.2*	*63.6*
Subject C:								
Av. of standard pull	167.68	163.56	160.62	174.88	173.52	166.48	168.06	177.12
A. D.	*11.96*	*9.4*	*11.76*	*6.76*	*8.08*	*8.48*	*8.2*	*8.88*
Average error	13.04	12.62	10.52	5.8	9.14	9.28	9.7	9.34
Variable error	6.6	7.48	6.55	3.8	5.56	5.24	6.75	5.0
Ratio V. E. to stand.	*25.4*	*21.9*	*24.5*	*46.0*	*31.3*	*31.8*	*24.9*	*35.4*
Subject D:								
Av. of standard pull	174.24	175.46	181.9	173.88	178.04	174.68	175.02	169.44
A. D.	*7.9*	*8.32*	*7.32*	*6.98*	*8.04*	*8.1*	*5.1*	*7.56*
Average error	9.58	10.44	9.72	10.04	8.7	8.34	8.94	9.28
Variable error	5.56	5.85	5.03	6.55	5.28	5.12	5.36	6.32
Ratio V. E. to stand.	*31.4*	*30.0*	*36.1*	*26.5*	*33.8*	*34.1*	*32.7*	*26.9*
Averages:								
Standard pull	174.17	171.27	172.73	174.04	175.55	171.85	171.85	173.1
A. D.	*9.78*	*8.94*	*8.78*	*6.87*	*7.49*	*7.37*	*6.27*	*7.01*
Average error	12.08	9.52	9.91	8.86	7.96	7.88	7.55	7.61
Variable error	6.26	4.96	5.71	5.26	4.58	3.89	4.94	4.48
Ratio	*28.05*	*43.12*	*30.6*	*34 5*	*40.02*	*50.2*	*37.45*	*42.65*

With the same subjects another experiment was tried in which they were told to pull the handle a certain distance and corrections made in terms of millimeters. A sitting was given with each group of springs used in the force standard experiments. Here an error of 100 grams with two springs

Table VI

Time Records in Sigma of 1600 Experiments in which the Subjects Attempted to Produce Two Movements of Equal Extent, and in which the Corrections Were Given in Mm.

Number of Springs..	2	4	6	8	10	12	14	16
Subject A:								
Av. of standard pull....	1,015	1,149	1,400	1,374	1,212	1,031	1,345	1,352
A. D.........	*68*	*106*	*171*	*147*	*92*	*48*	*79*	*100*
Average error.	60.2	87.4	138	125	85.6	52.5	99.8	96.8
Variable error.	39.1	25.8	92.5	65.4	44.2	37.2	51.7	58.3
Ratio V. E. to stand.......	*26.0*	*44.5*	*15.2*	*21.0*	*27.4*	*27.7*	*26.1*	*23.2*
Subject B:								
Av. of standard pull....	503	488	432	476	616	513	497	550
A. D.........	*39*	*37*	*24.5*	*41.2*	*21.6*	*28.2*	*26.5*	*33.1*
Average error	44.8	38	33.6	33.4	40.3	40.3	28.6	26.6
Variable error.	26.7	20.3	23.3	20.3	25.5	23.8	16.7	20.5
Ratio V. E. to stand.......	*18.9*	*24*	*18.6*	*23.4*	*24.1*	*21.6*	*29.8*	*26.8*
Subject C:								
Av. of standard pull.. .	955	942	545	1,155	1,045	924	762	1,035
A. D.......	*84.4*	*70*	*84.7*	*103.5*	*70.9*	*71.7*	*58.6*	*66.7*
Average error.	70.1	64.2	53.3	37.8	61.7	72.3	57.3	89.0
Variable error.	48	37.3	31	56.8	37.3	41.8	38.5	51.7
Ratio V. E. to stand.......	*19.9*	*25.3*	*17.6*	*20.4*	*28*	*22.1*	*19.8*	*20*
Subject D:								
Av. of standard pull....	479	428	512	441	418	504	470	448
A. D........	*25.4*	*27.9*	*34.1*	*20.5*	*19.6*	*27.9*	*47.2*	*16.8*
Average error.	31.4	30.7	34.1	32.5	30.0	25.0	45.7	29.7
Variable error	16.4	20.6	22.9	19.4	19.7	19.4	30	18.9
Ratio V. E. to stand......	*29.2*	*20.8*	*22.4*	*22.7*	*21.2*	*26.0*	*15.7*	*23.7*
Averages:								
Standard pull.	738	752	722	861	823	743	768	846
A. D........	*54.2*	*60.2*	*78.6*	*78*	*51*	*44*	*52.8*	*54*
Average error.	51.6	55	64.7	57.2	54.4	47.5	57.8	60.5
Variable error.	32.5	26	42.4	40.5	31.7	30.5	34.2	37.3
Ratio. . . .	*23.5*	*28.6*	*18.4*	*21.9*	*25.1*	*24.3*	*22.8*	*23.4*

would involve the same extent error as an error of 800 grams with 16 springs. The records of this experiment are however not materially different from the records when a force standard was used (Tables V. and VI.). When identical conditions exist efforts to reproduce a standard force or a standard extent produce identical results (see Fig. 3). When stated in terms of force we have found that as the magnitude of the stimulus increases the variable error increases more slowly than in direct proportion to the stimulus, but more rapidly than in proportion to the square root of the stimulus. When

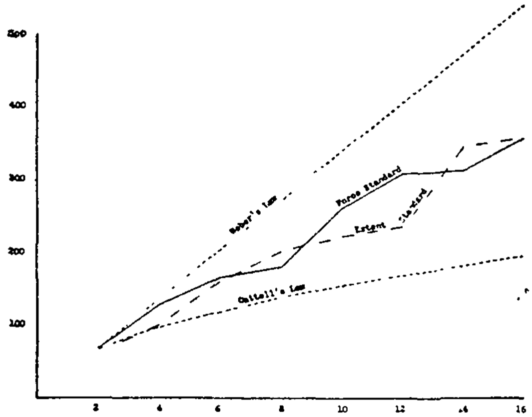

FIG. 3. This graph shows the force records of subjects *A*, *B*, *C* and *D*, when they were guided by both a force and an extent standard, compared with the records that would be expected from Weber's and Cattell's psycho-physical laws.

stated in terms of extent we have found that as the resistance offered to a movement of the same extent increases the accuracy with which the movement can be reproduced increases. We cannot however make too much of this point because, as we shall show later, our other four subjects do not show this reaction. We may suggest as an explanation that our later four subjects may have used extent as a basis of judgment to a greater extent than did our first four.

With the end in view of examining further this relation between force and extent, it was planned to eliminate extent altogether and to see what effect this would have upon the subject's judgment and control of force. The dynamometer was suspended so that it would swing freely, the subject grasped a handle fastened to the rear of the dynamometer, and the experimenter increased or decreased the tension by means of a windlass attached by a cord to the handle. In this experiment, as before, twenty-five practice trials were given, after which the pulls were made in pairs. The spring was tightened until the subject gave a vocal signal that what he deemed was the standard had been reached. All the tension was then removed and again the spring tightened until the subject gave the signal. The spring was tightened with approximately the same speed that the subjects used in making the movement themselves. Of course with such a procedure a greater variability would appear in the results due to the introduction of the reaction time of the experimenter. However, this increase in the error records would be the same regardless of the amount of the force standard and so would not prevent a study of the relations of the different force standards.

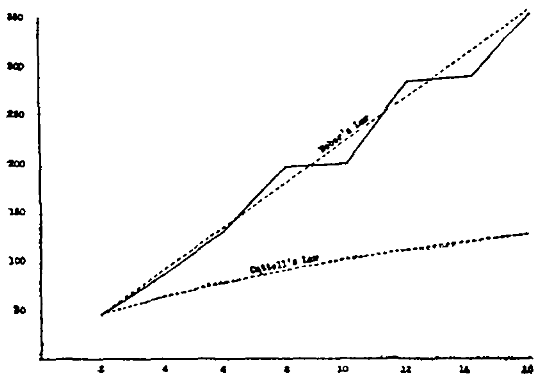

FIG. 4. This graph shows the force records of subjects *E*, *F*, *G* and *H* when they were guided by a force standard in pulling the dynamometer compared with the records that would be expected from Weber's and Cattell's laws.

As new subjects (*E*, *F*, *G* and *H*) were used in this experiment, additional experiments were made with them using the same procedure as with the subjects *A*, *B*, *C* and *D*, with the exception that no time records were taken. Contrary to the former results, the variable errors (Table VII.) followed very closely Weber's law (Fig. 4). This means that these subjects made errors of approximately the same extent when pulling with a standard force of 16 kilograms as with a force standard of 2 kilograms, indicating that they were largely influenced by extent. The records where extent was

Table VII

Force Records in Grams of 1600 Experiments in which the Subjects, attempted to Produce Two Movements of Equal Force, and in which Corrections Were Given Them in Grams.

Set Standard. .	2	4	6	8	10	12	14	16
Subject E:								
Av. of standard pull....	1,981	4,027	5,910	7,634	9.718	12,520	13,427	15,819
A. D.........	*46*	*122*	*199*	*351*	*470*	*478*	*636*	*598*
Average error	49	142	263	546	649	439	593	544
Variable error	28	72	150	290	298	249	291	326
Ratio V. E. to stand.....	*70.7*	*29*	*39.4*	*26.3*	*32.6*	*50.3*	*46.2*	*48.5*
Subject F:								
Av. of standard pull ...	2,033	3,992	6,134	8,162	9,900	12,275	13,744	16,495
A. D.........	*135*	*144*	*297*	*349*	*382*	*508*	*683*	*605*
Average error	85	109	180	246	296	374	424	366
Variable error	51	73	80	104	109	170	272	221
Ratio V. E. to stand......	*39.8*	*54.6*	*76.6*	*78.5*	*90.8*	*72.2*	*50.6*	*74.6*
Subject G:								
Av. of standard pull....	2,095	3,972	5,771	7,700	9,643	11,506	13,581	15,086
A. D.	*107*	*160*	*288*	*372*	*362*	*577*	*518*	*828*
Average error	108	236	431	510	467	1,120	794	1,374
Variable error	70	133	204	254	263	552	369	544
Ratio V. E. to stand......	*42.2*	*29.7*	*28*	*30.3*	*40.8*	*20.9*	*36.8*	*27.7*
Subject H:								
Av. of standard pull....	1,999	4,090	5,969	7,992	10,275	12,149	14,270	16,210
A. D.	*80*	*93*	*175*	*262*	*354*	*316*	*515*	*528*
Average error	71	107	159	259	286	253	335	466
Variable error	34	73	100	141	163	168	231	330
Ratio V. E. to stand.....	*58.8*	*56.1*	*56.7*	*56.7*	*63.4*	*72.3*	*61.8*	*49.2*
Averages:								
Standard pull	2,027	4,020	5,946	7,872	9,884	12,112	13,755	15,902
A. D...	*92*	*130*	*240*	*333*	*392*	*470*	*588*	*640*
Average error	78	148	258	390	424	546	536	687
Variable error	46	88	133	197	201	285	291	355
Ratio . .	*52.9*	*42.3*	*50.2*	*48.0*	*56.9*	*53.9*	*48.8*	*50.0*

eliminated do not follow Weber's law but fall between Weber's law and Cattell's law (Table VIII. and Fig. 5). This result is not merely a result of the difference in method, but a result of the elimination of extent. Evidence for this is given by a study of the judgment thresholds. As we noted above after each pair of pulls the subject gave his judgment as to whether the second was heavier or lighter than the first. From these judgments thresholds were calculated and are

Table VIII

Extent Records in Mm. of 1,600 Experiments in which the Subjects Attempted to Produce Two Movements of Equal Force, and in which Corrections Were Given in Grams.

Set Standard.	2	4	6	8	10	12	14	16
Subject E:								
Av. of standard pull	172.1	174.34	169.04	162.86	166.36	184.66	166.82	173.74
A. D.	*4.64*	*6.12*	*6.64*	*8.76*	*9.4*	*7.96*	*9.08*	*7.48*
Average error	4.92	7.1	8.78	13.66	12.98	7.32	8.46	6.8
Variable error	2.8	3.6	5.0	7.24	5.96	4.16	4.16	4.08
Ratio V. E. to stand.	*61.5*	*48.5*	*33.8*	*22.5*	*27.9*	*44.4*	*40.2*	*42.6*
Subject F:								
Av. of standard pull	177.28	172.56	176.46	176.04	170.00	180.58	171.34	182.18
A. D.	*13.52*	*7.2*	*9.92*	*8.72*	*7.64*	*8.64*	*9.76*	*7.56*
Average error	8.52	5.44	5.98	6.14	5.92	6.24	6.06	4.58
Variable error	5.14	3.64	2.68	2.6	2.18	2.84	3.88	2.76
Ratio V. E. to stand.	*34.5*	*47.4*	*65.5*	*67.6*	*78.0*	*63.6*	*44.2*	*66.0*
Subject G:								
Av. of standard pull	183.54	171.62	164.38	164.5	164.86	167.76	169.02	164.58
A. D.	*10.72*	*8.02*	*9.6*	*9.32*	*7.24*	*9.64*	*7.4*	*10.36*
Average error	10.84	11.8	14.34	12.76	9.36	18.66	11.36	17.18
Variable error	7.04	6.68	6.8	6.4	4.72	9.2	5.28	6.92
Ratio V. E. to stand.	*26.1*	*25.7*	*24.2*	*25.7*	*34.8*	*18.2*	*32.0*	*23.8*
Subject H:								
Av. of standard pull	173.9	177.52	170.98	171.8	177.7	178.48	178.86	178.62
A. D.	*8.02*	*4.64*	*5.84*	*6.56*	*7.08*	*5.28*	*7.36*	*6.6*
Average error	7.06	5.36	5.3	6.34	5.72	4.22	4.78	5.84
Variable error	3.4	3.64	3.32	3.52	3.26	2.8	3.3	4.13
Ratio V. E. to stand.	*51.1*	*48.8*	*51.8*	*48.8*	*54.5*	*63.7*	*54.2*	*43.2*
Averages:								
Standard pull	176.6	174.01	170.09	168.8	169.73	177.87	171.51	174.78
A. D.	*9.22*	*6.49*	*8.0*	*8.34*	*7.84*	*7.88*	*8.4*	*8.0*
Average error	7.83	7.42	8.6	9.72	8.5	9.11	7.66	8.6
Variable error	4.6	4.39	4.45	4.94	4.03	4.75	4.15	4.47
Ratio	*43.3*	*42.6*	*43.82*	*41.15*	*48.8*	*47.48*	*42 65*	*43.9*

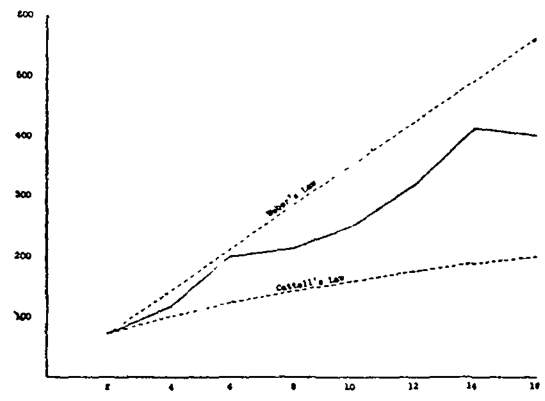

Fig. 5. This graph shows the force records of subjects *E*, *F*, *G* and *H* when they held the arm stationary in judging the tension of the spring compared with the records that would be expected from Weber's and Cattell's laws.

given in Table IX. In the experiments where the subjects made arm movements in pulling the dynamometer the threshold follows Weber's law (Fig. 6); where the subject held his arm stationary it falls between Weber's law and

TABLE IX

FORCE RECORDS IN GRAMS OF 1,600 EXPERIMENTS IN WHICH THE SUBJECTS ATTEMPTED TO JUDGE THE POINTS IN TWO SUCCESSIVE TRIALS WHEN THE TENSION ON A HANDLE, WHICH WAS HELD STATIONARY, WAS EQUAL.

Set Standard....	2	4	6	8	10	12	14	16
Subject E:								
Av. of standard tension	1,992	3,986	6,006	7,623	9,973	11,069	13,846	16,470
A. D........	*151*	*164*	*298*	*302*	*318*	*482*	*358*	*396*
Average error	157	188	350	543	337	605	690	400
Variable error	80	110	228	254	212	341	448	202
Ratio V. E. to stand......	*24.9*	*36.2*	*26.3*	*30.0*	*47.0*	*32.4*	*31.0*	*81.5*
Subject F:								
Av. of standard tension.	2,141	4,413	6,464	8,422	10,224	12,814	14,384	16,357
A. D..	*140*	*234*	*257*	*422*	*452*	*396*	*617*	*697*
Average error	109	201	361	343	581	433	686	694
Variable error	*56*	*105*	*190*	*192*	286	*199*	*350*	358
Ratio V. E. to stand.....	*38.2*	*42.0*	*34.0*	*43.8*	*35.8*	*64.4*	*41.0*	*45.7*
Subject G:								
Av. of standard tension	2,184	4,000	5,996	8,280	10,542	12,245	14,475	16,906
A D.	*96*	*182*	*280*	*363*	*338*	*451*	*484*	*739*
Average error	172	190	401	354	498	458	663	1,133
Variable error	102	123	208	181	306	214	624	560
Ratio V. E. to stand......	*21.4*	*32.5*	*28.8*	*45.7*	*32.8*	*57.2*	*23.2*	*30.2*
Subject H:								
Av. of standard tension.	2,125	4,133	6,278	8,268	10,451	12,632	14,437	16,741
A. D........	*113*	*185*	*256*	*321*	*324*	*482*	*392*	*595*
Average error	109	199	416	487	309	812	434	940
Variable error	54	131	224	218	202	518	235	509
Ratio V. E. to stand......	*39.4*	*31.5*	*28.0*	*37.9*	*51.8*	*24.4*	*61.4*	*33.0*
Averages:								
Standard tension.. ...	2,111	4,133	6,186	8,148	10,297	12,190	14,285	16,618
A. D.........	*125*	*191*	*273*	*352*	*358*	*453*	*463*	*607*
Average error	137	194	382	432	431	577	618	792
Variable error	73	117	202	211	251	318	414	407
Ratio.. . .	*31.0*	*35.5*	*29.3*	*39.3*	*41.8*	*44.6*	*39.1*	*47.6*

Cattell's law (Fig. 7). Referring again to Woodworth's interpretation this means that when the subject makes the arm movement the judgment of force depends on a factor or factors closely correlated with force. When extent is ruled out the judgment of force is determined by factors related more by chance. This indicates that with some subjects at least the judgment of force is closely correlated with judgment of extent.

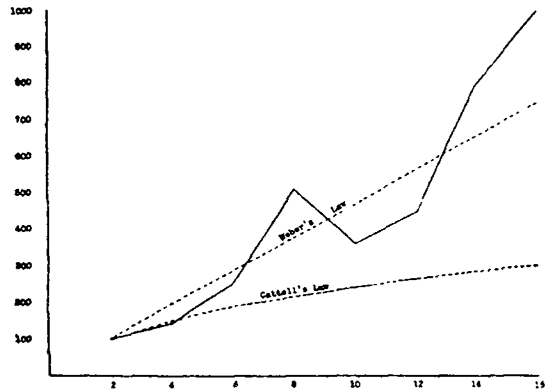

Fig. 6. This graph shows the judgment thresholds of subjects *E*, *F*, *G* and *H* when they had a force standard in pulling the dynamometer compared with what would be expected from Weber's and Cattell's laws.

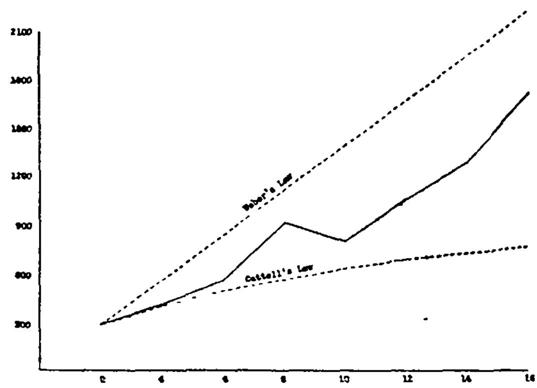

Fig. 7. This graph shows the judgment thresholds of subjects *E*, *F*, *G* and *H* when they held their arm stationary while the experimenter changed the tension of the dynamometer spring compared with what would be expected from Weber's and Cattell's laws.

In Table X. we have assembled the judgment thresholds derived from the different experiments. In part *A* are given the judgment thresholds when the subjects attempted to

Table X

A. Extent Thresholds Based on the Judgments of the Subjects in 1,600 Experiments in which the Attempt Was Made to Produce Two Movements of Equal Force. Unit 1 mm.

Set Standard	2	4	6	8	10	12	14	16
Subjects								
A	9	11	8	14	10	11	8	8
B	11	13	13	12	16	9	10	16
C	7	7	11	9	16	24	14	19
D	24	22	30	6	8	19	27	17
Average	12.75	13.25	13	10.25	12.5	15.75	14.75	15

B. Extent Thresholds Based on the Judgments of the Subjects in 1,600 Experiments in which the Attempt Was Made to Produce Two Movements of Equal Extent. Unit 1 mm.

Set Standard	2	4	6	8	10	12	14	16
Subjects								
A	13	11	11	11	13	7	9	12
B	18	10	12	13	15	7	7	8
C	13	14	15	8	7	12	17	10
D	13	17	12	11	10	10	14	15
Average	14.25	13	12.5	10.75	11.25	9	11.75	11.25

C. Force Thresholds Based on the Judgments of the Subjects in 1,600 Experiments in which the Attempt Was Made to Produce Two Movements of Equal Force. Unit 1 gram.

Set Standard	2	4	6	8	10	12	14	16
Subjects								
E	50	100	270	840	500	420	630	1,040
F	180	100	210	320	300	720	910	1,040
G	90	200	330	240	300	360	560	880
H	110	180	300	680	350	300	1,120	1,120
Average	107.5	145	277.5	520	362 5	450	805	1,020

D. Force Thresholds Based on the Judgments of the Subjects in 1,600 Experiments in which the Attempt Was Made to Indicate the Points in Two Successive Trials When the Tension on a Handle, which Was Held Stationary, Was Equal. Unit 1 gram.

Set Standard	2	4	6	8	10	12	14	16
Subjects								
E	250	560	900	1,240	500	1,440	2,170	1,520
F	210	280	270	680	750	780	980	1,520
G	520	340	480	720	1,200	840	1,050	1,520
H	140	420	570	1,040	500	1,200	980	2,240
Average	280	400	555	920	737.5	1,065	1,295	1,700

pull with a given force and were corrected in terms of grams; in part *B* are the records from the experiments where the subjects attempted to pull a certain extent and were given corrections in millimeters. When they had the force standard the errors in judgment increased as the size of the standard increased. When they had the extent standard the errors in judgment became smaller as the resistance to be overcome increased. This is interesting when we consider that in each of these cases the subject had the same sensation complex. If he was told to pull 16 kilograms and had to make a movement 175 millimeters long to do so, or if he was told to pull the handle 175 millimeters and had to pull 16 kilograms to do this, the only factor changed was the difference in instructions. In some way the difference in the two standards striven for must have changed his interpretation of the same sensation complex. As we have shown above the variable errors in the two experiments were not materially different (Tables II., III., V. and Fig. 3). There must therefore have been something in the experiment that rendered their judgments different in the two cases while it did not change their accuracy of movement. In all probability this was the nature of the corrections given. When a subject made an extent error of 1 millimeter and was told that he had made an error of 80 grams in one case (with a 16 kilogram standard), and 10 grams (2 kilogram standard) in another, the effect would be different from that induced if with two similar pulls with different forces he was told that in each case he had made an error of 1 millimeter. It must be remembered that in no case was the subject told whether he had made a correct or false judgment, but after he had made two pulls he was told how the latter pull differed from the arbitrary standard of the experiment. When a subject had made two pulls which he thought were both near the standard and was told that in one case the latter pull was 80 grams heavy and in the other 1 millimeter long he would in the first instance be led to make a larger correction, or at least have his confidence in his accuracy shaken, while in the latter case he would think he had done well and attempt to do the same the next time.

Part *D* of Table X. is a tabulation of the judgment thresholds when the subject merely held his arm stationary and the tension on the handle was changed by the experimenter. Part *C* is a tabulation of the judgments of these same subjects when they made arm movements. We have said above that the reaction time of the experimenter in the former situation probably made the variable errors larger, but there is no reason why this should have made the judgment thresholds differ. Yet we find that when the extent factor is eliminated the thresholds are from 1.6 to 2.75 (average 2.1) times as large as when the extent element was present. It seems unquestionable that when a subject could he used sensations of extent to help him judge the force of a movement just made by him, when extent was eliminated he could judge with less accuracy the force of his movements although he could judge them with some degree of accuracy.

We have seen that whether an extent or force standard were used the subjects kept their time fairly uniform. While the ratio of the variable error in time to the total time of the pull shows that taken by itself it is more variable than either extent or force, it may nevertheless play a large part in the control and judgment of the other two factors. We therefore tried a short supplementary experiment to ascertain the effect of radical variations in time.

The procedure last described was used, namely that of having the subject hold the handle while the experimenter changed the tension of the springs; and, as these subjects had no previous knowledge of the experiments, they were given a series of trials with the arm movement procedure for comparison. The standard used was 10,000 grams. When the subjects made arm movements the judgment thresholds of the two subjects were (*I*) 650 grams and (*J*) 350 grams. When they held the handle in a fixed position and the speed with which the springs were tightened was radically different in the two trials of each pair according to a prearranged schedule, the thresholds were (*I*) 1,050 and (*J*) 1450. When the speed of release was changed as well as the speed of tightening, that is, when the spring was tightened quickly, care was taken to release it quickly and vice versa; the thresholds were respectively 1,250 and 2,200 grams. Here where extent was eliminated and time so varied as to eliminate it as a help the judgment threshold showed a ratio of 1 to 8 and 1 to 4.5. Where extent was eliminated and no attempt was made to change radically the time the judgment thresholds averaged 1 to 10.8, and when the subject made the movement and so could have the benefit of extent and time the average judgment ratio was 1 to 21.3.

V. Conclusions

An outstanding feature of these experiments is the adaptation of the subjects to changed conditions. Adaptation in *fairly complex situations is generally recognized under the* name of learning and an individual's intelligence is judged by the extent to which and the speed with which he makes a selection of the most favorable reaction toward any given situation. If he makes the most intelligent reaction he will respond in such a way as to produce a certain result in the most efficient way. If after he has learned this most expeditious form of reaction some block is put in his way or this mode of action prevented in any way, he does not then and there give up all effort to attain the end in view but tries some new method of attack. All this is perfectly obvious and well recognized. That the same thing holds in more elementary fields has likewise been recognized by biologists and psychologists, but it has not received the emphasis which it deserves.

It is clear from our experiments that to execute a movement of a certain force is a learned act and that to make a judgment of the force of a movement that has been made is also a learned act. Like every learned act it requires practice for one to become at all skilled in it and like any complex process it is easily interfered with by a change in the circumstances surrounding the act. The subject when asked to pull a handle 175 millimeters, when asked to pull with a force of 8 kilograms, or when asked to hold his arm stationary and signal when the tension has reached 8 kilograms, must in the first instance make a random reaction just as one does in learning a strange puzzle or a maze. This random reaction produces a certain complex of sensations, and when he is told that he has made an error he tries to change his next movement so as to make the correction and is guided in this attempted correction by a comparison of the second sensation complex with the memory of the first. There may be various elements that he uses in successive trials, just as one makes various efforts to solve any new situation, and the final result of the elimination process is to select that element which will give him the best results.

It is important that the complex nature of the perception of force be clearly recognized. Investigators have spoken of it as though it were some simple sensation and statements have been made that weights were judged not by force but by the distance to which they were raised, by the speed of the movement, or by the latent time between the initiation of effort and the actual movement of the weight. Others have contended that weights are judged by sensations of force. The parties on both sides of the argument seemed to consider force as an elementary process, and through failure to analyze it as a complex process, could not agree.

We have shown that one is best able to learn to produce a movement of a certain force when extent and time are both involved. The elimination of extent greatly interferes with the act, as does any radical variation in the time. When these modifications are introduced, however, one can learn to produce movements approximating the desired force, but with less efficiency than when the extent and time factors are present.

We believe that the evidence herein adduced is sufficient to prove that the force of a movement is controlled by a number of factors; that extent is for most individuals a dominant factor and, though with less certainty, that time is an important factor. Besides these two there are a number of less closely correlated factors that an individual uses when he is prevented from using extent and time. That there is a simple sensation of force seems out of the question.