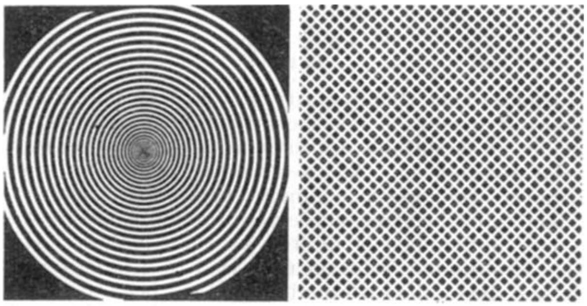
PSYCHOLOGY

Dynamic Distortions of Perceived Form

It is becoming increasingly clear that the visual perception of form involves the interaction of a number of parallel computing or 'image-processing' operations. Stimuli which preferentially excite one or other of the neural computing networks concerned can give rise to visual anomalies indicative of the way in which the optical information is normally broken down.

I have recently observed a striking anomaly in this class. When two regular patterns such as Figs. 1*a* and *b* are superimposed, they form the 'Moiré pattern' of Fig. 2. If now the second is moved relative to the first, say along the vertical axis of the figure, the lobes of Fig. 2 undergo a continual displacement in the general direction of the arrows, and so long as motion persists the form seen presents an asymmetrical appearance. As soon as motion ceases, however, symmetry returns, the lobes appearing to 'snap back' in a fraction of a second to their positions in Fig. 2. The transition from dynamic to static form is sufficiently striking to evoke incredulity in some subjects. Since the static form is seldom exactly symmetrical, it is necessary (and proves convincing) to control for residual asymmetry by arranging that the figure returns to exactly the same rest-position from opposite directions alternately.

The problem here is that identical figures of excitation on the same retinal area lead successively to different perceptions of form. In general terms the explanation is obvious, that the form seen under dynamic conditions is some kind of running average over the moving figures presented; but since this 'average' is as sharp as the static form, and since it is asymmetrical, it must be computed from a logically dissected representation of the stimulus, in which the velocity-components of the retinal image can apparently contribute to the computation of contour position. With translation of a rigid figure, this 'abstract averaging' need give rise to no anomalies of form perception, though it may be responsible for some illusions of displacement[1]. When, however, the figure is a Moiré pattern the components of which have a different 'phase velocity' from the velocity of the group as a whole, the contributions from contour velocity can grossly bias the perceptual estimate.



a *b*

Fig. 1

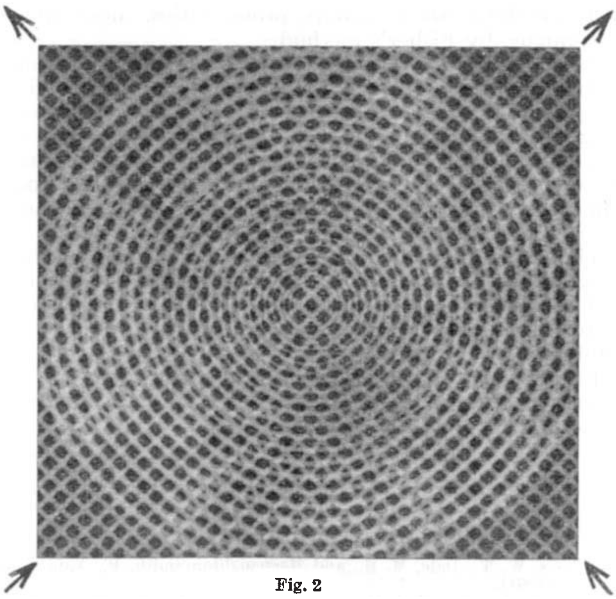


Fig. 2

The phenomenon seems to be in line with other recent evidence[2–4] suggesting that different neural networks are concerned with the signalling of contour direction, contour velocity and contour location.

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STATISTICS

Some Properties of Random Variables

In text-books on probability the following (perhaps curious) properties of random variables do not appear to have been noticed so far.

Consider a bi-variate distribution (Z, U) where both Z and U are positive finite random variables. There will be occasion to refer to Z as the numerator variate. Then one property of positive random variables is that:

$$\mathrm{Cov}\left(U, \frac{Z}{U}\right) \leqq V(\sqrt{Z}) \qquad (1)$$

To prove this inequality we have, by definition:

$$\mathrm{Cov}\left(U, \frac{Z}{U}\right) = E(Z) - E(U)\, E\left(\frac{Z}{U}\right) \qquad (2)$$

Applying Cauchy's inequality, we find:

$$E(U)\, E\left(\frac{Z}{U}\right) \geqq \left[E\left(\sqrt{U}\sqrt{\frac{Z}{U}}\right)\right]^2 \qquad (3)$$

or

$$E(U)\, E\left(\frac{Z}{U}\right) \geqq [E(\sqrt{Z})]^2 \qquad (3\cdot1)$$

In equation (3), equality is attained if, and only if, $\sqrt{\frac{Z}{U}}/\sqrt{U}$ is equal to a constant, say $\sqrt{c}$, for all pairs (Z, U), leading to the condition

$$Z = cU^2 \qquad (4)$$

On multiplying each side of (3·1) by -1 and reversing the sign of inequality, and then adding $E(Z)$ to each side, the result given by equation (1) will be obtained. When equation (4) holds, $\mathrm{Cov}\,(U, cU) = V(\sqrt{c}\,U) = cV(U)$, an identity which is to be expected.

Another property of positive random variables is that the correlation with their respective reciprocals, that is, $\rho\left[U, \frac{1}{U}\right] \leqq 0$. The proof of this proposition follows from equations (2) and (3·1) by putting $Z = 1$. Condition equation (4) shows that this correlation is zero only when all the U's are equal.