PERCEIVING STRUCTURE FROM MOTION: FAILURE OF SHAPE CONSTANCY

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ABSTRACT

Four experiments are reported on human perception of structure from motion that show systematic failures of shape constancy as a function of object motion. Two of these experiments show that the perceived depth of an object undergoing sinusoidal oscillation about a vertical axis increases monotonically with the amplitude of oscillation. The other two experiments show how the shape of an object varies in the neighborhood of a degeneracy within the stimulus domain, at which shape information is absent. These systematic failures of shape constancy are considered for their implications about the process underlying human perception of structure from motion.

Introduction

In recent years, there has been growing interest in what continuous optical flow or discrete optical change affords in the way of information about 3-D structure and motion that is otherwise lost in the projective transformation from the 3-D scene to the 2-D imagery. The resulting theoretical work has shown that, subject to various assumptions, an intelligent image processing system can use optical motion to recover much or, in some cases, all of the information about 3-D structure and motion lost in the 3-D to 2-D projection [e.g., 1-7]. Besides suggesting algorithms for the recovery of structure by intelligent machines, this work hints at possible mechanisms [e.g., 8-10] that might account for the ability of humans to perceive the 3-D shapes of moving objects on the basis of a succession of 2-D projections [11-17]. However, it is important to recognize that the perception of structure from motion is often non-veridical as indicated by its dependence upon a multitude of stimulus factors such as the type of object motion, number of points defining the surface, number of frames in the image sequence, and signal-to-noise ratio [e.g., 18-23]. At the least, these failures of veridical perception imply that understanding the perception of structure from motion in humans involves more than just an analysis of the optical motion. We believe that instances of non-veridical perception have far greater significance, for a systematic investigation of how perception fails ought to provide more powerful constraints on theorizing about the underlying perceptual process than any comparable investigation of veridical perception. The work reported here is concerned with the variation of perceived shape as a function of object motion.

General method

The structure-from-motion sequences were created on an IBM PS/2 Model 80 computer using 3-D graphics software developed for the 320x200 resolution mode of the IBM color/graphics adapter [24]; sequences were displayed on an IBM 8513 video display and, except where stated, were viewed monocularly from the proper viewing point (resulting in correct retinal projections for the simulated objects and motion.) All objects were defined by 256 white points randomly positioned on their surfaces. The image points were seen against a black background with the room lights extinguished. The animation sequence was displayed at the rate of 60 frames/sec. During each trial, subjects were permitted to view the animation sequence for as many times as needed to make a response. No feedback of any kind was given until the end of the experiment. Subjects were maintained in a state of moderate light adaptation either by illuminating the video screen or the room lights between trials.

Experiment 1

In this experiment, subjects viewed simulated cylinders of radius 1.5 cm differing in length: 9, 12, 15, and 18 cm. Each cylinder underwent sinusoidal oscillation about the vertical rotation axis; the orientation of the cylinder at the oscillation midpoint (0 deg phase) was such that its axis was normal to the screen surface (thus coinciding with the primary line of sight). The independent variable of greatest interest was the range of oscillation in degrees, varying from 23 to 180 deg. (As defined, range represents the peak-to-peak value or twice the sinusoidal amplitude). In the 180 deg case, a cylinder oscillated in depth between two extreme orientations, each of which coincided with the frontoparallel plane. The

animation sequences were displayed at two different speeds. In the slower condition, all 170 frames were displayed cyclically at 60 frames/sec resulting in a full oscillation every 2.8 sec; in the faster condition, every other frame was skipped, resulting in a full oscillation every 1.4 sec. The last independent variable was viewing distance (50 cm and 100 cm), manipulated to assess the effects, if any, of angular size and of depth of field associated with accommodation. Simulated object distance and projection plane distance remained constant at 50 cm; thus, only for the 50 cm viewing distance were the retinal projections of the objects correct. Subjects judged the apparent length of the cylinder presented on each trial.

The average responses of 6 naive subjects are given in Figure 1. The important result is that the reported length of a given cylinder increases with the range of oscillation up to about 90 deg. This indicates a significant failure of shape constancy from one condition to the next. It is important to mention, however, that there is no salient perception of shape change (non-rigidity) while a given cylinder is undergoing oscillation.

The minimal effects of oscillation speed and viewing distance are of some interest. It might be thought that doubling the range of oscillation, say from 45 deg to 90 deg, exerts its influence on perceived length by way of a doubling of the instantaneous rotational velocity. However, keeping range of oscillation constant and doubling rotational velocity directly has little effect on perceived distance. The negligible effect of viewing distance suggests that the greatly reduced depth of field associated with the nearer viewing distance accounts for relatively little of the depth foreshortening of the cylinders obtained with the small oscillation ranges.

Relevant to an understanding of the effect of range of oscillation is the idea that perceiving the shape of rotating objects, the so-called *kinetic depth effect* [17], entails two mechanisms. We suppose that one of these involves the direct elicitation of perceived depth by relative motion parallax; such apparent depth is particularly salient in displays of moving smooth surfaces covered with randomly positioned dots [9, 14, 15]. The other hypothesized mechanism relies not upon
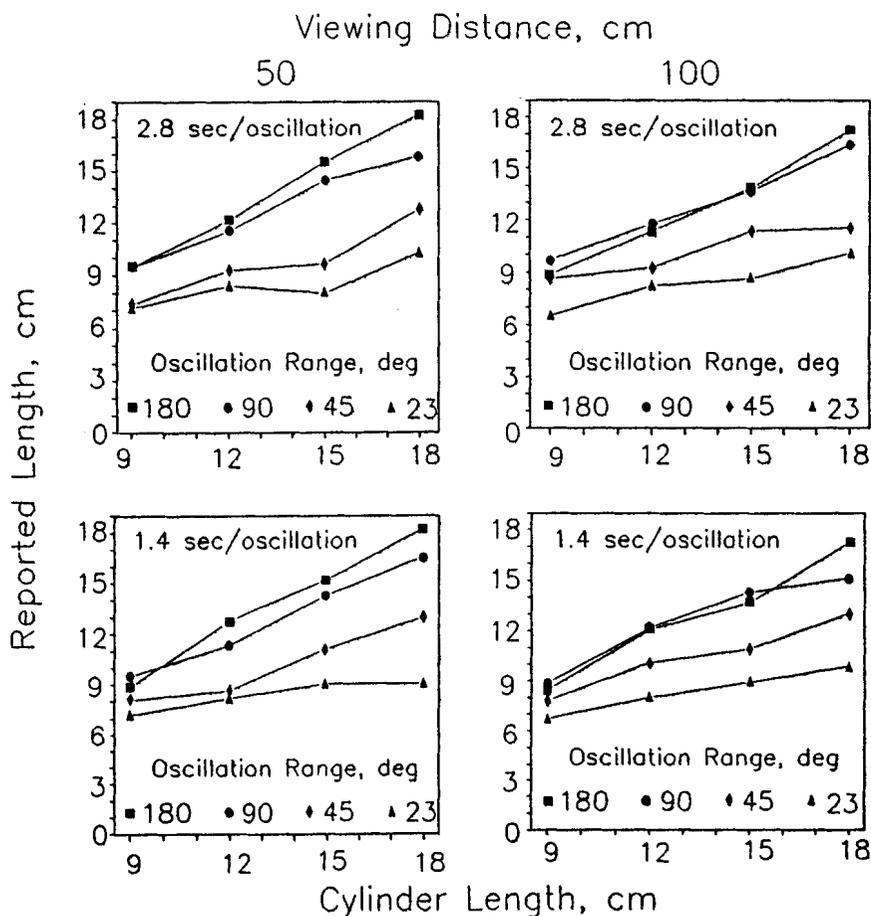

Figure 1: The results of Experiment 1.

local dot structure but upon the projected outlines of objects or object features as they rotate. In the most extreme case, it has been demonstrated that observers can sometimes perceive the shapes of solid objects by their changing silhouettes or shadows [16, 17, 25]. Clearly, for an observer to perceive the shape of such a rotating object requires some form of temporal integration of the changing perspective views, perhaps in the spirit of Ullman's *incremental rigidity scheme* [10]. If the hypothesis of two mechanisms is tenable, it might account in part for the basic result of Experiment 1. When the cylinders undergo the smallest sinusoidal oscillations, the subject must judge their lengths largely on the basis of the perceived depth. If the relative motion parallax fails to generate depth signals of sufficient strength to overcome the *equidistance tendency* [26] and whatever flatness cues remain in the display, then the cylinders will appear foreshortened. The greater the range of oscillation, however, the less the judgment of length depends upon the perception of depth, for the observer has access to larger and larger frontal projections of the object. While advancing the hypothesis of two mechanisms, we do not believe it accounts fully for the effect of increasing the range of oscillation, for as the results of Experiment 2 indicate, the effect is still present even for small oscillations for which the projected size varies only slightly.

Experiment 2

In this experiment, the objects were half-ellipsoids simulated to be concave with respect to the observer. The range of sinusoidal oscillation about a vertical axis took on just three values: 20, 40, and 60 deg. Two size ranges of half-ellipsoids were used. For a base radius of 2.5 cm, lengths of 5, 7.1, and 10 cm were used, while for a base radius of 5 cm, lengths of 10, 14.1, and 20 cm were used. The smaller objects were viewed from 100 cm while the larger ones were viewed from 200 cm; the simulated object distances (and projection plane distances) were set equal to the viewing distances. The effect of this arrangement was to present the observer with the same retinal images in the two viewing conditions, except for the quantizing effects of our display. Our purpose in including this manipulation was to look for any effects of quantization noise produced by the limited spatial resolution of our 320 x 200 graphics board; when expressed as a proportion of the intended dot motion, quantization noise is half as large at the larger distance. In addition, doubling viewing distance without changing retinal image size reduced the accommodative distance by half, which in turn reduced whatever flatness cues were provided by accommodation.

Subjects judged the depth of the recessed half-ellipsoids in cm. The average depth values of the 5 naive subjects are given in Fig. 2. The most important result is that increasing oscillation range had a large effect on reported depth. This dramatic failure of shape constancy is much like that found in Experiment 1; it differs, however, in that the projected envelopes of the ellipsoids

increased only slightly with range. Expressed as ratios of the diameter of the envelope of the mid-sized ellipsoid in its central position, the horizontal extents of the projected envelopes in the extreme orientations were only 1.01, 1.07, and 1.20 times the base value for the 20, 40, and 60 deg ranges of oscillation; thus the projected size increased by a mere 19% in going from the smallest to largest oscillation, while the perceived depth increases by 65%. This suggests that the relative motion parallax that produces the perception of depth is increasing in its effectiveness with range of oscillation. The manipulation of viewing distance had little effect as judged by the shapes of the functions. This implies that quantization noise and flatness cues provided by accommodation play little role in this failure of shape constancy.
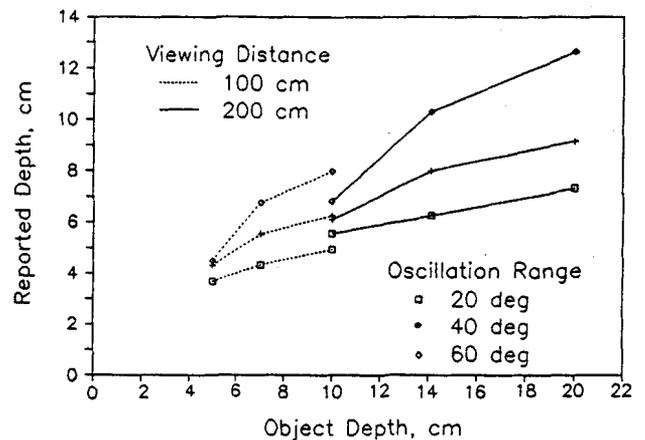

Figure 2: The results of Experiment 2.

Optical motion degeneracies

One thing that distinguishes optical motion from retinal disparity as a cue to object shape is the existence of degeneracies within the stimulus domain at which shape information disappears. One such degeneracy occurs whenever the eye lies on the axis about which a 3-D object is rotating [27, 28]. The retinal optical motion of such a rotating object is one of pure curl [1, 28], which conveys no information about relative depth. Thus, in the absence of textural or other static information, different objects rotating about an observer's line of sight cannot be identified.

Consider an object rotating about a fixed axis through its center. The effect of varying the inclination of this fixed axis with respect to the observer's line of sight through the object center is of considerable interest. From a theoretical standpoint, whenever the axis of rotation fails to coincide with the line of sight through its center, information is available to fully recover the shape of the object provided that the stimulus sequence consists of at least 3 views of 4 non-coplanar

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object points under the assumption of rigidity [6]. Naturally, measurement noise in any real image-processing system would present problems for recovery of structure when the axis inclination is very small. For the human observer, however, even large axis inclinations do not result in the correct perception of shape [29]. The following experiment illustrates the basic effect of varying axis inclination.

Experiment 3

Three ellipsoids varying in eccentricity were used as stimuli. The circular cross-section of each had a diameter of 6 cm. The variation in eccentricity was produced by using lengths for the orthogonal axis that had the following ratios with respect to 6 cm: .75 (shortened sphere), 1.0 (sphere), and 1.25 (elongated sphere). The simulated object and projection plane distances of 175 cm matched the viewing distance. Each ellipsoid underwent sinusoidal rotary oscillation about an axis that varied in inclination with respect to the line of sight that passed through the ellipsoid center; inclination angle increased toward the vertical, so that the full 90 deg inclination resulted in oscillation about a vertical axis through the object center. The oscillatory motion was arranged so that midway

through the oscillation (0 deg phase), the ellipsoid axis of varying length coincided with the line of sight (normal to the screen); thus, for example, the "elongated sphere" was oriented with its long axis collinear with the observer's line of sight midway through its oscillation. Two ranges of oscillation were used: 40 and 80 deg. Subjects were asked to judge the depth of the object at its mid-position; this is tantamount to their judging the length of the ellipsoid axis that varied.

In the lower panels of Fig. 3, the average depth judgments of 7 naive subjects have been expressed as proportions of the lengths of the varying axis. As expected, subjects see virtually no depth in the 0 deg inclination conditions, presumably because the equidistance tendency [26] combined with residual flatness cues determine the perception in the absence of any information signaling depth. More importantly, subjects' judgments of depth have not reached asymptote even with axis inclinations of 60 deg signifying that the effective optical motion is not sufficient to outweigh the determinants of flatness. The effects of the two other independent variables are also of interest. The degree of depth underestimation is greater the more elongated the object. Also, as was found in Experiments 1 and 2, perceived depth is smaller for the shorter range of oscillation.
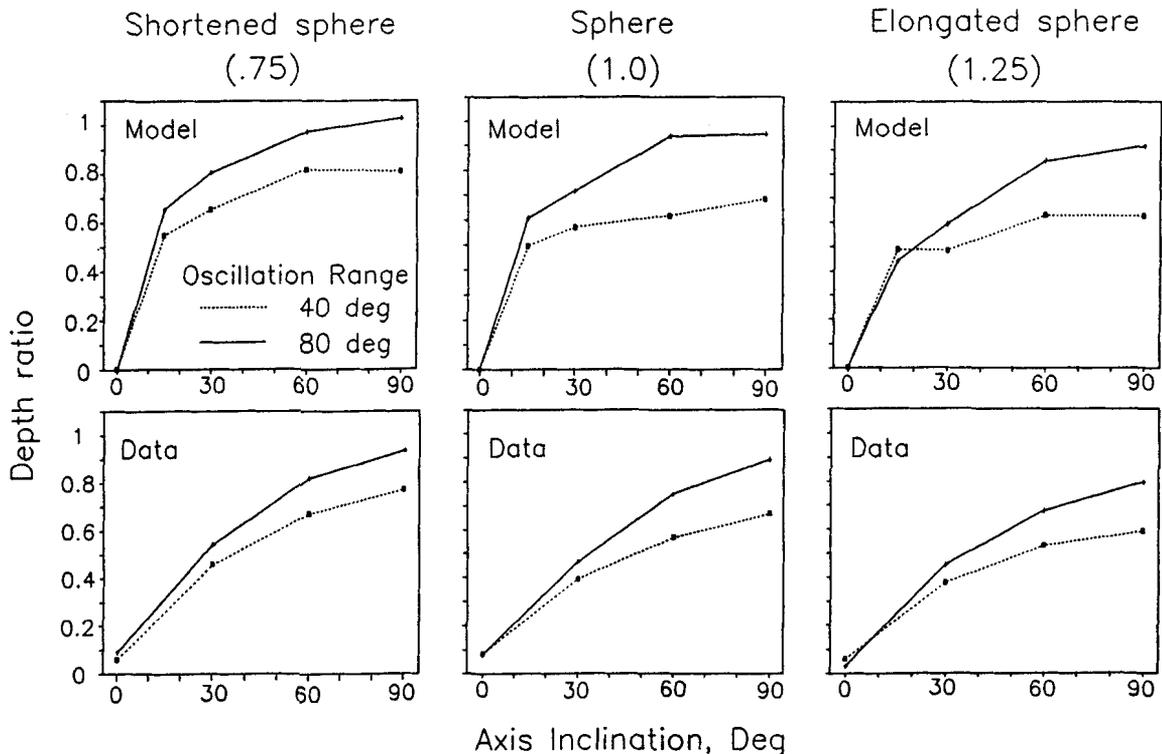

Figure 3: The lower panels give the results of Experiment 3. The upper panels give the results of our simulations of the incremental rigidity scheme [10]. The recovered or perceived depth of each ellipsoid is expressed as a proportion of the actual depth value (simulated for the observer or provided as input to the incremental rigidity scheme).

The top panels of Fig. 3 give the results of some simulations using our implementation of the incremental rigidity scheme of Ullman [10]. There are some differences between the conditions of our experiment and the simulations. Our simulations made use of ellipsoids with only 27 points distributed more or less uniformly over their surfaces while our experiments made use of 256 points distributed randomly. The simulations involved orthographic projection while our experiments involved a modest degree of perspective. Most importantly, subjects made their judgments within several oscillations of the ellipsoid, by which time perceived depth had stabilized, while our simulations involved 80 oscillations (in 320 frames) of each object, by which time the asymptotic values of recovered depth were only being approached. This very slow convergence of the internal model has already been noted by Ullman [10]. In spite of these methodological differences between experiment and simulation, the perceptual and simulated effects of oscillation range and ellipsoid shape are surprisingly similar. The scheme even exhibits the gradual increase of depth with axis inclination, although the function is considerably steeper for the smaller inclination values. These similarities alone warrant further investigation of the role played by incremental rigidity in the perception of structure from motion.

Experiment 4

Another degeneracy in the motion domain arises when an object that is revolving about a line of sight also undergoes an appropriate simultaneous rotation about an axis that is parallel to the axis of revolution. Fig. 4 depicts the observer's view of a radially symmetric object, such as a cone or half-ellipsoid, the axis of which is normal to the plane of the figure (or video screen). The object is simultaneously rotating about its long axis and revolving about an axis normal to the plane of the figure and positioned at the center. The degenerate case results when the object rotates 1 degree (ω) for each degree of revolution (α), provided that this rotation is in the "positive" direction, as indicated in the figure. In this case, one can consider that the object is really part of a larger imaginary object that is rotating about the original object's axis of revolution (along which the observer's viewing point is positioned.) As we saw earlier, this produces an optical flow pattern of pure curl, which conveys no information about 3-D structure [28]. As before, the interesting question is whether 3-D structure is perceptually recovered for combinations of object rotation and translation that do not produce pure curl.

Three simulated recessed cones of base radius 3.5 cm were used; their altitudes were 50, 100, and 200 cm. Their bases were simulated to be 50 cm from the observer, the same value used for the simulated projection plane distance and the physical viewing distance to the video screen. The independent variable of primary interest was the number of degrees of rotation per degree of

revolution, with the rotation going in either the positive or negative direction. The third independent variable was the number of frames shown to the subject. Because the simulated revolution was 1 deg/frame and the animation rate was 60 frames/sec, the degrees of revolution correspond to the exposure durations shown in Fig. 5. For a given condition, the subject was able to judge the animation sequence for as long as desired, but a dark interval of 0.5 sec was interposed between each repetition.
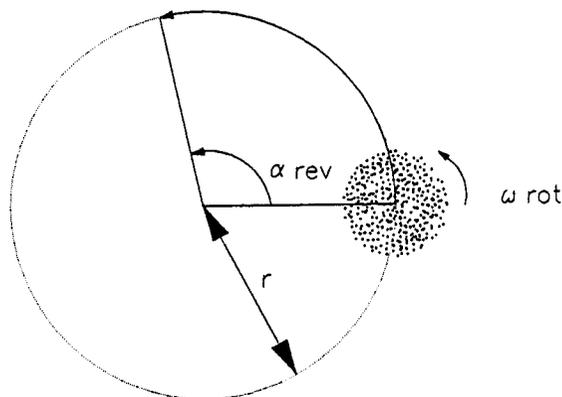

Figure 4: Schematic representation of the motion of a recessed cone from the viewpoint of the observer (Experiment 4). The cone is shown revolving in a counter-clockwise direction with radius of revolution equal to r. The observer's eye is collinear with the axis of revolution. The object is shown rotating in a positive direction with angular velocity ω degrees/sec as it undergoes a revolution of α degree/sec.

There are some definite virtues of this paradigm for studying the perception of depth from motion. First, the static textural cues are held virtually constant as the number of rotations per revolution is varied, allowing one to conclude that variations in perceived depth are due solely to the optical motion. Second, an object that is revolving about screen center presents virtually constant motion parallax information throughout its trajectory, unlike an object that is undergoing linear translation or oscillatory rotation. This makes it ideal for studying the buildup of apparent depth with exposure duration. The only drawback of using translation, including revolution, is that the observer must track the moving object with pursuit eye movements. Undoubtedly, part of the reason that perceived depth remains low for the shortest exposures is due to imperfect tracking of the revolving object. Our use of multiple presentations minimizes the problem, for the subject learns to synchronize eye movements with the onset of motion.
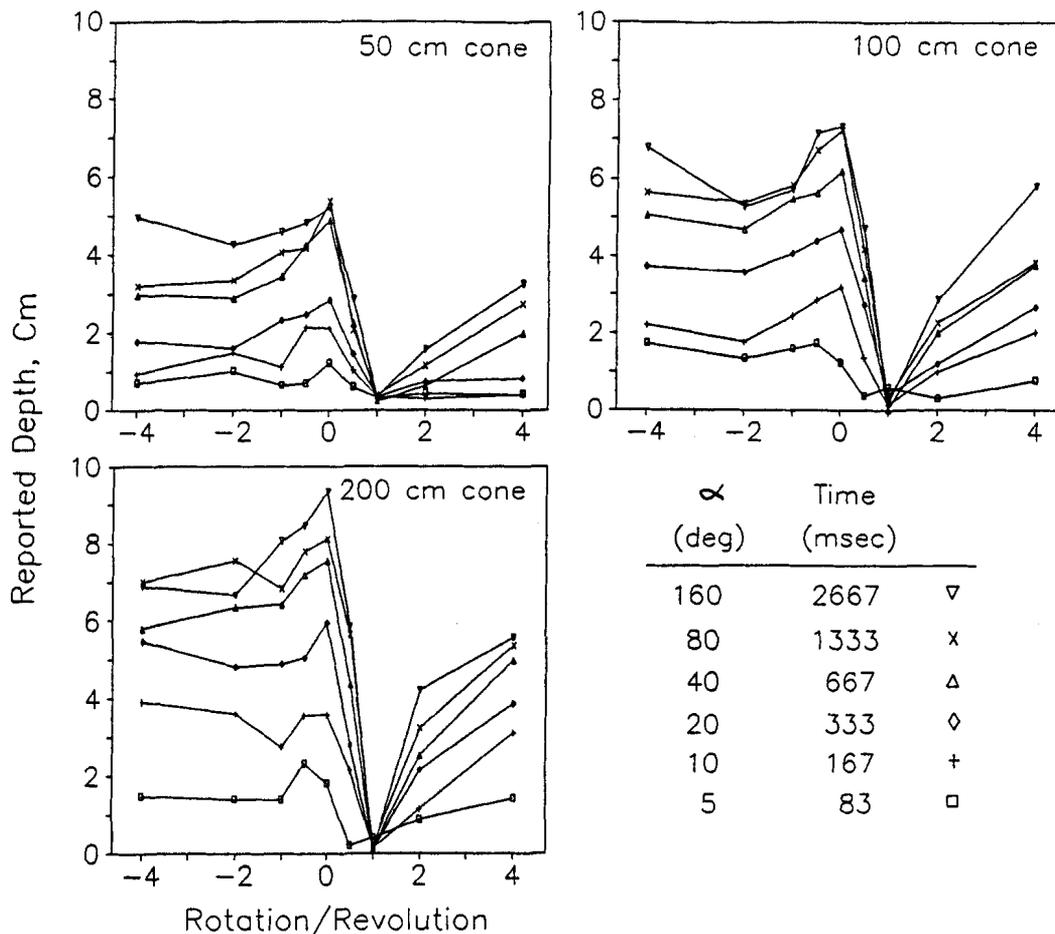
Figure 5: The results of Experiment 4.

Fig. 5 gives the average results of three experienced observers, the two authors and a third, who was naive about the purpose of the experiment. The most important result is the variation in perceived depth with the added rotation. As expected, the degeneracy at +1 rotations/revolution manifests itself with virtually zero perceived depth. On either side of the degeneracy, the perceived depth increases gradually rather than abruptly as would be expected of an ideal observer. The effect of duration is also important, for it shows that perceived depth reaches asymptotic values within about 0.6 sec. In addition, there is no obvious interaction between duration and simulated motion. Finally, it is important to note that the maximum reported depth values are at least an order of magnitude smaller than the corresponding simulated depth values. In addition, reported cone depth is not proportional to simulated cone depth, but increases according to some compressive non-linearity.

As a preliminary attempt at linking the effect of rotation with properties of the optical flow, we computed some summary measures of the flow patterns (using the images of frames 1 and 2 of the animation sequences). In Fig. 6, these summary measures are plotted as a function of the rotation variable and cone altitude. Velocity magnitude is simply the average velocity magnitude of an image point going from frame 1 to 2; averaging was performed over all 256 points. The common translatory component of image motion was partialled out of this calculation on the supposition that subjects nulled this component with pursuit eye movements. Not surprisingly, the condition involving pure translation (zero rotation) produced a minimum value of average image velocity.

The second measure was obtained by computing the magnitude of the vector velocity difference for every pair of image points and averaging over all 32640 such pairs. As a summary measure, it indicates the amount of velocity variation within the pattern. Like velocity magnitude, its minimum value occurs for the case of pure translation.

The computation of our third measure, shear, was prompted by the work of Koenderink [28] and Koenderink and van Doorn [1, 2]. They have observed that local instantaneous flow, when
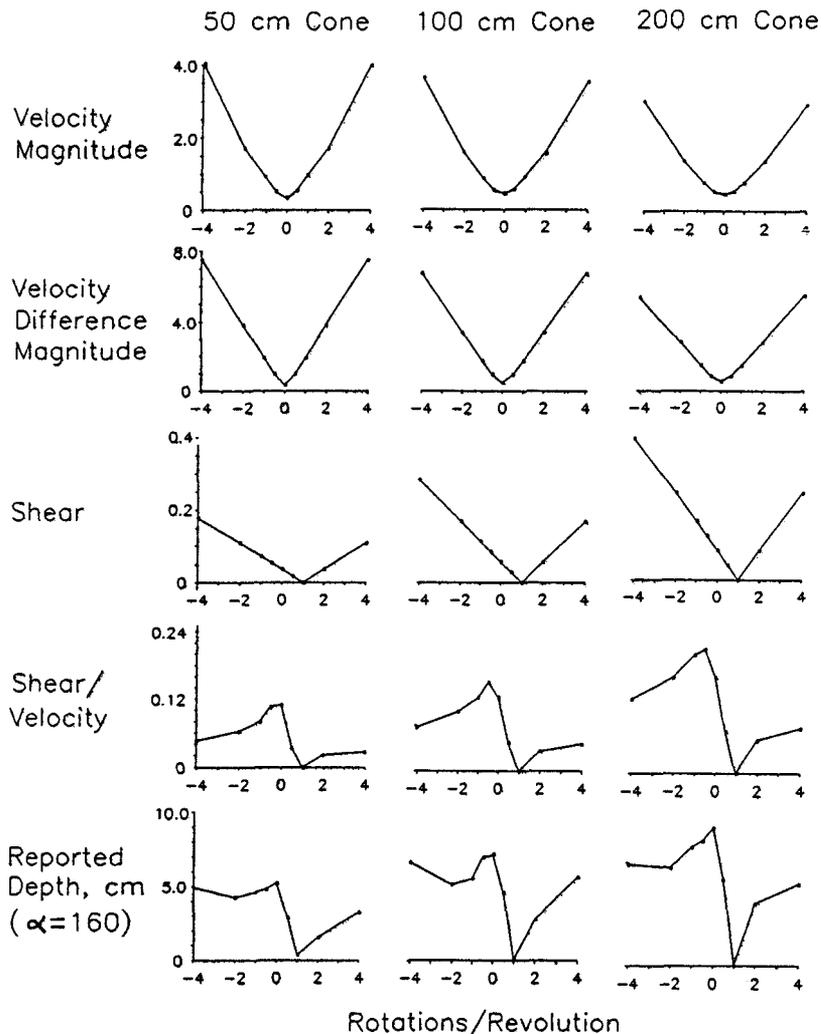
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50 cm Cone   100 cm Cone   200 cm Cone

Velocity Magnitude

Velocity Difference Magnitude

Shear

Shear/ Velocity

Reported Depth, cm (α=160)

Rotations/Revolution

Figure 6: Some summary measures of optical flow for the stimuli used in Experiment 4 together with the reported depth values (bottom panels) for the $\alpha = 160$ conditions.

differentiable, can be decomposed into a translational component and the differential invariants of curl (pure rotation), divergence (pure size change), and deformation (pure shape change or shear). They have noted that only deformation carries information about relative depth; as an alternative to computing the deformation of the velocity field, they have suggested computing some measure of shear from the changing positions of image triplets [2, 28]. An important implication of their work is that any meaningful measure of relative motion parallax must be computed on a minimum of three image points; defining relative motion parallax in terms of just two points, as is commonly done, does not distinguish between the motions associated with curl, divergence, and deformation. Accordingly, we have chosen to work with the measure of shear given in Fig. 7; the global summary measure is shear averaged over all triplets of image points. Fig. 6 shows that, as expected, shear goes to zero in the

case of +1 rotation/revolution, for the optical flow is pure curl. (Note that velocity difference magnitude is non-zero for the same condition. This means that there is considerable relative optical motion within such a rotary pattern even though there is no change in the relative positions of the image points). Surprisingly, shear is observed to increase linearly with rotation difference on either side of the degeneracy.

The bottom panels of Fig. 6 summarize the perceptual results of the maximum duration ($\alpha = 160$) conditions. Clearly, perceived depth is not proportional to average shear, nor would one expect it to be. The problem with time rate of shear, were it to be a local determinant of local depth, is that it would vary with the speed of rotation of an object rotating about an axis in the frontal plane. For shape constancy to occur with variations of rotation speed, as was observed in Experiment 1, it would be necessary for the local

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rate of shear to be divided by the rotational rate of the object motion. Because object rotational rate, like the 3-D shape, needs to be recovered from the optical flow, it is unlikely to be taken into account in early computations within the visual system. This being the case, might there be some aspect of optical flow that can be used in place of object rotational rate? We tentatively suggest that local image velocity might be used to rescale shear in order to permit some degree of constancy, at least for objects rotating about an axis lying in the frontal plane. When average image shear is divided by average image velocity magnitude (top panels), the resulting measure, shear/velocity, exhibits many of the qualitative trends evident in the perceptual results.

Frame 1   Frame 2
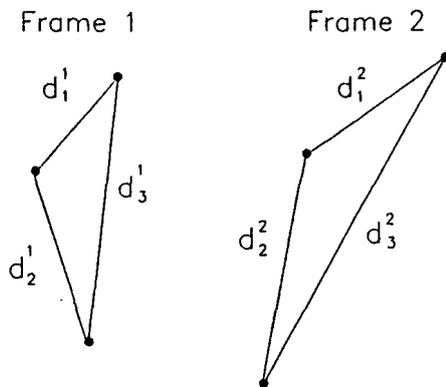
$$\text{Shear} = \left| \log \left(\frac{d_2^2 / d_1^2}{d_2^1 / d_1^1} \right) \right| + \left| \log \left(\frac{d_3^2 / d_1^2}{d_3^1 / d_1^1} \right) \right|$$

Figure 7: The definition of shear used in the computations of Fig. 6. At the left are shown the images of a triplet of object points during Frame 1. At the right are shown the image points corresponding to the same triplet of object points during frame 2. Besides the translation from left to right, the image points have undergone dilation (divergence), rotation (curl), and deformation (shear).

Obviously, no summary measure like shear/velocity computed over the entire images can predict the variations of perceived shape obtained with different objects (cones, ellipsoids, etc.). Nevertheless, the qualitative similarity of average shear/velocity with perceived depth for objects of constant shape suggests that local shear/velocity might possibly give rise to depth signals in the presence of smoothly varying optical flow patterns such as these. There would seem to be some additional merit in this proposal, for as the inclination of the axis of rotation is increased with respect to the line of sight, average shear increased while average image velocity decreases; this would predict increasing depth with axis

inclination, as was found in Experiment 3. However, analyses we have performed indicate that neither shear nor shear/velocity correctly predict the function relating perceived depth to axis inclination. Which property of optical flow underlies the perception of depth elicited by optical motion requires considerably more investigation.

The experiments we have reported show that when the shape of an object is conveyed primarily by optical motion, the recovered shape depends strongly upon the manner in which the object moves. This failure of shape constancy can be traced, in the case of motion degeneracies, to the geometrical properties of optical flow [28]. In the majority of the cases we have examined, however, the reasons must be sought in the processing of the optical flow. We have seen that some of this variation has promise of being explained by properties of the momentary optical flow without the necessity of auxiliary internal assumptions (e.g., rigidity). In addition, this approach is appealing, for perceived depth variation begins to appear almost immediately in these motion displays. On the other hand, the buildup to full depth does take some time, as indicated in Fig. 5, while other observations from Experiment 1 suggest that one does not see variations of shape as an object rotates through large angles, certainly not within epochs on the order of 0.5 sec. Thus, because these facts and others relating to the time-varying changes in projected envelope [16, 17, 25] point to some form of temporal integration, one needs to look beyond the momentary optical flow and consider some mechanism for integration, such as the incremental rigidity scheme proposed by Ullman [10]. An interesting possibility is that motion shear, properly scaled, generates instantaneous depth signals which simply constitute an additional source of information for the incremental rigidity scheme or some similar model [e.g., 9]. This joining of the two approaches would be consistent with the evident temporal integration of perspective information while explaining how the buildup of perceived depth is far more rapid than is apparent in simulations of this type of model [Experiment 3; 8, 10].

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