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Large systematic deviations in the haptic perception of parallelity

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Abstract. Recently, Kappers and Koenderink (1999 *Perception* **28** 781 – 795) showed that what subjects haptically perceive as parallel often deviates greatly from what is actually physically parallel. In their experiment, subjects had to rotate a test bar in such a way that it felt as though it was parallel to a reference bar. Their data were obtained with the right hand on a table plane to the right side of the median plane of the subject. The present study extends that work in a number of ways: (1) the locations of the stimuli cover the total reachable table plane; (2) distances between stimuli can also be large (more than 1 m); (3) experiments are done both unimanually (with the right and left hand) and bimanually. Like in the previous study, the results show large systematic deviations that correlate significantly with horizontal (left – right) distance between the two bars but not with vertical (forward – backward) distance. Thus we have established that a description of the results in terms of a horizontal gradient in the deviations is valid over a large part of haptic space, over large distances, and in both unimanual and bimanual conditions. The subject-dependent horizontal gradients ranged from $-12^\circ\ \mathrm{m}^{-1}$ to $-27^\circ\ \mathrm{m}^{-1}$ in the present experiment. In all conditions a significant haptic oblique effect can be demonstrated.

1 Introduction

There is increasing evidence that haptic space is not Euclidean, that is haptic perception of, for example, distances or parallelity does not always conform to physical reality. One of the first to report on this phenomenon was Blumenfeld (1937). By transferring his earlier experiments on the perception of visual space into the haptic domain, he investigated the ability of subjects to construct (haptically) lines parallel to the median plane. Since these lines turned out to be far from straight, Blumenfeld termed them the "alley curves". Although he theorised about the possible causes of these deformations of haptic space, he did not come up with a model to describe his data.

In the vision literature, quite a number of papers can be found which are inspired by Blumenfeld's ideas (eg Battro et al 1976; Indow and Watanabe 1984; Luneburg 1950; Yamazaki 1987) but this is not the case in the haptic literature. The study of Brambring (1976) is one of the few papers investigating the metric of haptic space. In his experiments, he focuses on an aspect of haptic space quite different from that of Blumenfeld, namely distance estimates. He reports that, as the length of the detour distance increases, the estimate of the shortest distance deteriorates from the Euclidean metric to the city-block metric. Using a different experimental paradigm, Lederman et al (1985) also found that haptic distance estimates are far from veridical. They report that both blind and blindfolded sighted subjects increasingly overestimate the distance between the endpoints of a path as the length of the explored pathway increases.

In a recent study, we (Kappers and Koenderink 1999) report on a series of experiments which have a somewhat closer resemblance to the original experimental ideas of Blumenfeld. Blindfolded subjects were asked to perform three tasks with their right hand: (i) a reference bar was presented under four different orientations and subjects were asked to rotate a test bar to such an orientation that it felt as though it was parallel to the reference bar; (ii) subjects had to rotate two test bars in such a way that they felt collinear; (iii) subjects had to point a test bar in the direction of a marker. Bars and marker could appear at nine different locations in a table plane at waist height to the right side of the median plane of the subject. In all experiments large systematic deviations (up to 40°) were made. The results of all three experiments can be summarised as follows: the size of the deviation depends on the horizontal (right – left) distance between the two stimuli and not on the vertical (forward – backward) distance; the larger the horizontal distance, the larger the deviation. In the first two tasks (parallelity and collinearity), the right bar always deviates clockwise with respect to the left bar; similarly, in the third task, the test bar deviates clockwise if the marker is located to the left of it. Thus the deviations can be described efficiently by means of a horizontal gradient (determined by taking the slope of the line fitted through the deviations plotted against horizontal distance) in the subjects' orientation settings with respect to veridical. Subjects showed similar trends but the size of the gradient was strongly subject dependent and ranged from $-2^\circ\ m^{-1}$ to $-34^\circ\ m^{-1}$. In addition, in the parallelity experiment a significant oblique effect was found: deviations of the test bar were larger if the reference bar had an orientation of 45° or 135° than if the orientation was 0° or 90°. These results provide strong evidence that haptic space is non-Euclidean.

The aim of the present study is to extend the above experiments along various directions in order to find an answer to the following research questions. (i) Do we find similar deformations of haptic space if we shift the area of investigation from the right side of the median plane as in the previous study towards symmetrically around the median plane? (ii) Do we find similar deformations of haptic space if we increase the distances over which decisions have to be made? (iii) Is performance with the left hand identical to that of the right hand? (iv) Is bimanual performance identical to that of unimanual performance? To answer these questions, the first task of the previous study (the parallelity task) will be used.

The first two questions focus on the validity of a description of the results in terms of a horizontal gradient. Although such a description was of use in the previous study, it is of interest to investigate whether such a description is also applicable here where distances are larger and the median plane is crossed or whether vertical deviations also come to play a role. Moreover, such data are necessary before one could make a start at describing and understanding the deformations of haptic space.

There are several motivations for the interest in the comparison of right-handed and left-handed performance. In the previous study (Kappers and Koenderink 1999) it was argued that the horizontal gradient in the settings could not simply be explained by rotations of the arm about the shoulder or the elbow, or of the hand about the wrist. For such an explanation to be plausible, much larger deviations from veridical are needed. However, at present alternative explanations do not exist and it is probable that the limitations in possible movements somehow influence the perception of haptic space. One way to gain insight into this issue is to compare directly settings made with the right and with the left hand. If the right hand moves from one location to another the movements involved are (in general) completely different from those the left hand has to make when moving between the same two locations. Thus, if the deviations from veridical are mainly determined by the kind of movements that are made or that are possible, one would expect differences between the right-handed and left-handed settings in such cases.

The possible differences in right-handed and left-handed performance could appear in various ways. A first possibility is the size of the horizontal gradient, which may not necessarily be identical for the right and the left hand. Second, the variable errors (standard deviations) in the settings could be different. Third, the settings made with the left hand might deviate systematically from veridical but in a different manner from the settings made with the right hand. In such a case a description of the deviations in terms of a horizontal gradient is not a suitable one for the left hand. Last, the deviations from veridical might be not systematic at all. The latter two possibilities should be considered as rather unlikely.

Our final question deals with a possible difference between unimanual and bimanual performance. An important difference between these two conditions is that in unimanual cases the stimuli were necessarily touched successively whereas they are touched simultaneously in bimanual conditions. Exploration time should not be considered as a factor benefiting bimanual performance since in our experiments subjects are allowed to use as much time as they wish. Still, the simultaneous availability of both stimuli suggests an advantage for bimanual exploration. On the other hand, on the basis of previous research on shape and curvature discrimination (eg Kappers and Koenderink 1996; Kappers et al 1994) one might expect a disadvantage for the bimanual conditions. It is, of course, unclear whether the findings from such different experiments can be used as a predictor for the outcome of our current experiments on spatial relations. Appelle and Countryman (1986) report results from an experiment where subjects had to match the orientations of two rods, and they also find a disadvantage for the bimanual conditions. In their case, however, both bimanual and unimanual stimuli were touched successively.

Possible differences between unimanual and bimanual performance can appear in the same way as summarised above for the comparison of the right-hand and left-hand conditions. Owing to the inherent differences between unimanual and bimanual strategies (eg the need for arm movements or not) it is hard to predict which of the possibilities is the most likely but none of them can a priori be ruled out as improbable.

2 Methods

2.1 *Apparatus*

The same setup as in the previous study (Kappers and Koenderink 1999) was used (see figure 1). The setup consisted of a large table on which rested an iron plate. This plate was covered with a plastic layer on which fifteen protractors were printed (the circles and disks in the figure). In the present experiment, the protractors indicated by the open circles were used. Two aluminium bars of length 20 cm and diameter 1.1 cm were used as reference and test bars. In the middle of the bars a small pin was attached which fitted exactly into holes in the centres of the protractors. In this way, the bars could be rotated without being displaced. Small magnets fastened under the bars increased the resistance against movement (hence the use of the iron plate).

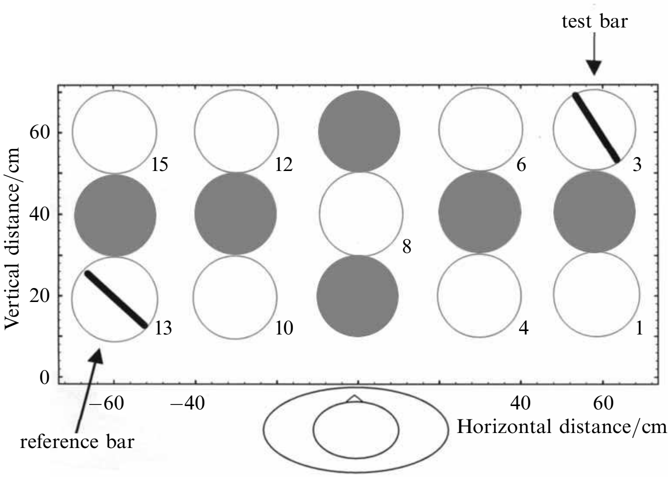

Figure 1. Top view of the setup and a subject sitting in front of it. The disks and circles, numbered between 1 and 15, all represent protractors allowing the experimenter an accurate reading of the orientation of the test and reference bars. The centres of the two bars can be positioned on the centre of any of the circles. The open circles indicate protractors used in the current experiment. The navel of the subjects is always positioned at the coordinates (0, 0).

On both ends of the bars small needles were fixed, allowing the experimenter an accurate reading of the orientation of the bars (uncertainty of about 0.5°).

2.2 *Stimuli*
All nine locations indicated by the open circles were used as positions for the reference and test bars. In an experimental session, either the protractors numbered 1, 3, 8, 13, and 15 (subset 1) or the protractors 4, 6, 8, 10, and 12 (subset 2) were used. The choice of positions was motivated by a symmetrical distribution of the number of positions that is maximally possible if one considers measuring-time limitations. Given a position for the reference bar, the remaining positions within the same subset were used for the test bar. The reference bar could appear under four possible orientations: 0°, 45°, 90°, and 135° (0° is parallel to the long side of the table; increasing orientation values signify a rotation in counterclockwise direction). Thus the total number of combinations is 2 (subsets) × 5 (positions of reference bar) × 4 (positions of test bar) × 4 (number of reference orientations) = 160. For each of the three conditions (right hand, left hand, and two hands), all 160 combinations were presented three times in random order. These orders were different for each of the subjects.

2.3 *Subjects*
Three paid subjects (undergraduates of Utrecht University) voluntarily participated in the experiment. They were naive in all aspects, that is, they had never seen the setup, knew nothing about the reference orientations or locations, were unaware of the experimental purposes, and were never given any feedback. All three were strongly right-handed as assessed by means of a standard questionnaire devised by Coren (1993). Subject NK had already participated in our previous study (Kappers and Koenderink 1999).

2.4 *Procedure*
A blindfolded subject was seated in front of the table with his/her navel at coordinates (0, 0). The experimenter told the subject which of the conditions would be measured: right hand, left hand, or two hands. Next, the experimenter positioned the reference and test bars, the reference bar in the predetermined orientation and the test bar in a random orientation. In the unilateral conditions the experimenter took the required hand of the subject and placed it first on the reference bar and subsequently on the test bar. The task of the subject was to rotate the test bar in such a way that it felt as being parallel to the reference bar. The subject was allowed to go back and forth between the two bars as often as he/she preferred. When the subject was satisfied with the setting of the test bar, the experimenter noted down the orientation and positioned the two bars for the next trial. Subjects were free in their choice of strategy as long as they did not touch the edges of the table with their hands and remained seated at the correct position. The subjects were not given any feedback.

In the bimanual condition, the experimenter first positioned one of the hands on the reference bar and next placed the other hand on the test bar. The decision of which hand was to be placed on the reference bar and which on the test bar was determined uniquely by the rule that the hands should never cross each other. Subjects were not allowed to move their hands from one bar to the other. The remainder of the procedure was identical to that of the unilateral conditions.

The duration of an experimental session was always limited to 1 h to avoid weariness of the subjects. Different sessions took place on different days over a period of a number of weeks. Trials from different subsets never occurred during the same experimental session. Blocks consisted of 40 trials of one of the three conditions; the blocks were presented in random order. ML needed a total of 31 h to complete all trials, NK needed 24 h, and RH 26 h. The total time was divided about equally over the three conditions.

3 Results

A representative example of the results can be seen in figure 2. This figure shows the average settings (over 3 trials) of subject ML in the right-hand condition measured with subset 1. The upper rectangles give a graphical representation of the results whereas in the lower ones a numerical representation is given. Each rectangular box gives a top view of the setup; the small dot indicates the position of the subject [coordinates (0, 0)], the thick line indicates the reference bar, and the thin lines the test bars. In the horizontal direction the orientation of the reference bar varies systematically whereas in the vertical direction the position of the reference bar is varied. The numbers give the deviation from veridical in degrees. If the subject had responded veridically all lines within a rectangle would have been parallel and all numbers would equal zero. Clearly this is not the case; deviations as large as 43° can be seen (for subject RH in the bimanual condition a deviation of 62° occurred!).

Although the subject's settings are far from veridical, the settings are not random. A number of trends can be distinguished in the results of all subjects under all conditions. (i) If the test bar is on the left side of the reference bar, the deviations are mostly positive, which means that they are rotated in counterclockwise direction. The opposite is true for test bars to the right of the reference bar. (ii) The deviations of test bars above or below the reference bar are often small [especially as compared with the deviations mentioned in (i)]. (iii) The size of the deviations is dependent on the orientation of the reference bar. The average absolute deviations resulting from the 90° reference orientation are almost always (eight out of nine cases) smallest, followed by those resulting from the 0° reference. (iv) The larger the horizontal distance between reference and test bar, the larger the deviation.

In order to be able to compare the results of the three subjects in the three conditions, the mean absolute deviations and the average standard deviations are given in table 1. The averages are taken over all 160 different combinations of reference and test bar within a condition. In all cases, that is, for each subject and each condition, the average standard deviations are significantly smaller (paired t-test, $p < 0.0001$) than the mean absolute deviations, showing that the deviations from veridical are indeed systematic. There are some differences between the deviations of the three subjects, but they do not seem to be systematic.

Table 1. Mean absolute deviation and average standard deviation for all three subjects. The averages are taken over all 160 different combinations of reference and test bar within a condition. For all subjects and all conditions, the mean absolute deviation is significantly larger than the average standard deviation (paired t-test, $p < 0.0001$).

Subject	Mean absolute deviation/°			Average standard deviation/°		
	right hand	left hand	two hands	right hand	left hand	two hands
ML	12.0	11.0	13.4	6.5	6.0	6.3
NK	9.6	8.1	9.4	4.0	3.4	5.3
RH	12.7	9.9	14.7	5.5	6.1	6.0

3.1 *Dependence on distance*

In order to analyse the data and the trends mentioned above in more detail, the deviations are plotted against the horizontal and the vertical distance between the reference and the test bar (see figures 3 and 4). A positive horizontal distance indicates that the test bar is located to the right of the reference bar, and a negative distance, to the left; a positive vertical distance indicates that the test bar is located further away (in vertical direction) from the subject than the reference bar. The three rows of graphs show data from the three subjects; the four columns show data for the four different reference orientations.

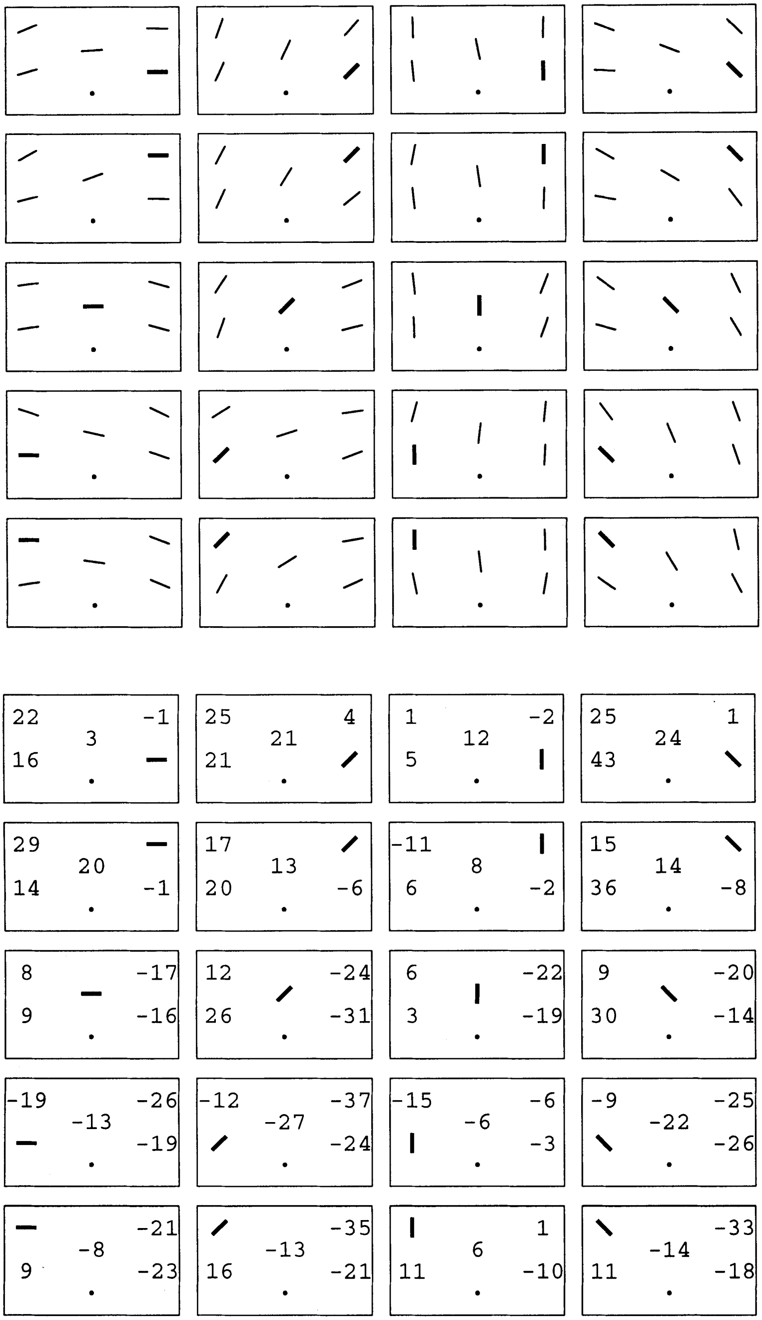

Figure 2. Results of subject ML in the condition where the task had to be performed with the right hand. The five upper rows give a graphical representation of the results whereas the five lower rows give a numerical representation of the same results. Each rectangle symbolises a top view of the setup. The small dot indicates the position of the subject, the thick bar shows the location and orientation of the reference bar, the thin bars show the settings of the test bars averaged over the three trials, and the numbers give the deviation from veridical in degrees. Positive numbers indicate counterclockwise deviations, negative numbers clockwise deviations. From left to right the orientation of the reference bar varies, and from top to bottom the location of the reference bar. If the subject responded veridically, all thin lines within a rectangle would be parallel to the thick line and all deviations would be zero.

The solid line in each graph is the line of best fit (a least-squares fit). Slopes of these lines and r^2 values of the data points in a graph are given in table 2. In this table, the columns labelled 'all' give the values for all reference orientations taken together.

In figure 3 and table 2 it can clearly be seen that for all subjects, all conditions, and almost all reference orientations there is a highly significant linear dependence of the deviation on the horizontal distance between reference and test bars (the exceptions are a few cases of the 90° reference orientation). On the other hand, figure 4 and table 2 show that there is no such dependence of the deviations on the vertical distance between reference and test bar. In the few occasions where the slope is significantly different from 0, the r^2 values belonging to the data points are at most 0.17 but often close to 0. In this respect, it should be noted that the solid lines in figure 4 mean little.

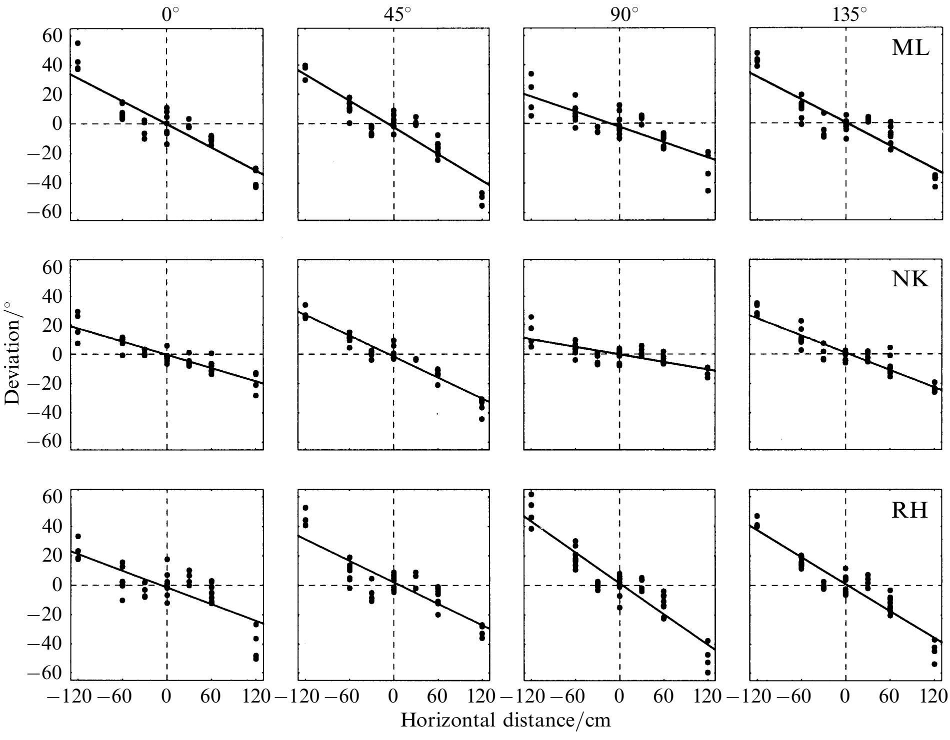

Figure 3. Deviations as a function of the horizontal (right – left) distance between reference and test bar. A negative distance indicates that the reference bar is located to the right of the test bar, and a positive distance, to the left. From left to right, the columns give the data for the four reference orientations 0°, 45°, 90°, and 135°, respectively. The rows show data of the three subjects. The solid line indicates the best fit (least mean squares) through the data points. Slopes, r^2, and significance levels are given in table 2. As can be seen in this figure and table 2, most data points are highly correlated.

3.2 *Dependence on reference orientation*

Both veridicality (deviation from veridical) and accuracy (standard deviation) might depend on the reference orientation. Indeed, in figure 3 and table 2 it can easily be seen that the deviations depend on reference orientation. Closer inspection of table 2 also shows an oblique effect; that is, the deviations are smaller for the horizontal (0°) and vertical (90°) reference orientations than for the oblique (45° and 135°) reference orientations.

In the analysis it does not make sense to look at the signed deviations, since the sign depends on the relative positions of reference and test bars; I therefore used the absolute values of the deviations. A one-way ANOVA shows that for all three subjects

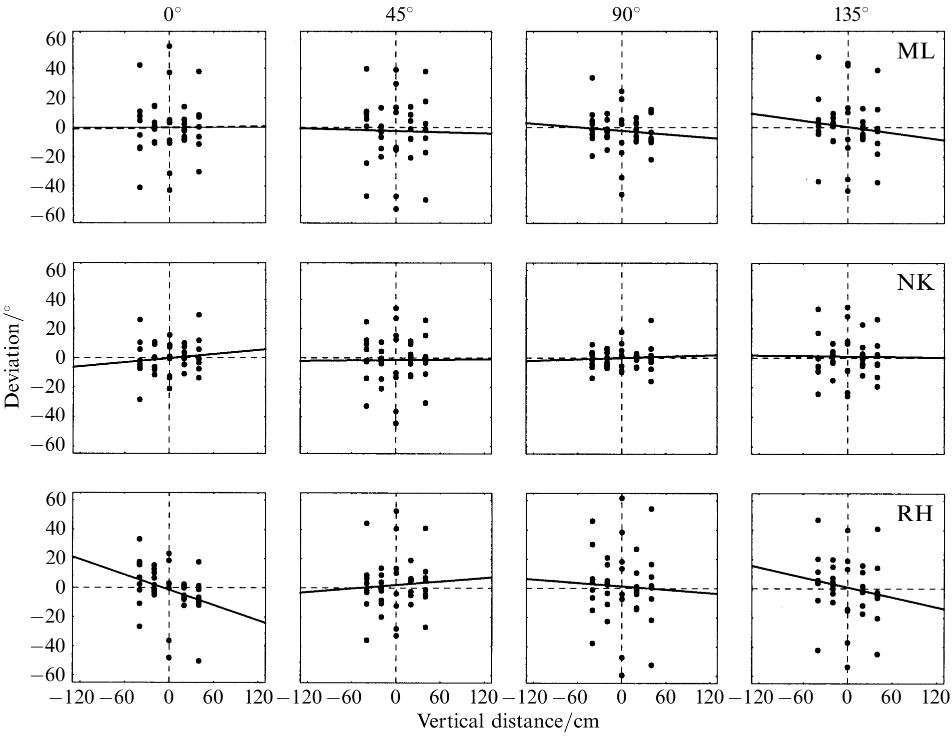

Figure 4. Deviations as a function of the vertical (forward – backward) distance between reference and test bar. A negative distance indicates that the reference bar is located further away from the subject in vertical direction than the test bar and a positive distance nearer. From left to right, the columns give the data for the four reference orientations 0°, 45°, 90°, and 135°, respectively. The rows show data of the three subjects. The solid line indicates the best fit (least mean squares) through the data points. Slopes, r^2, and significance levels are given in table 2. From both this figure and the table, it is clear that the data points are hardly correlated. As a consequence, the solid lines in the figures mean little.

the influence of reference orientation on absolute deviation is significant (for ML, $F_{3,476} = 13.4$, $p < 0.0001$; for NK, $F_{3,476} = 17.6$, $p < 0.0001$; for RH, $F_{3,476} = 3.3$, $p < 0.02$). In order to investigate whether this is indeed an oblique effect, the data were collapsed into two categories: oblique and horizontal – vertical. It turns out that for all three subjects the oblique effect is highly significant (for ML, $F_{1,478} = 30.9$, $p < 0.0001$; for NK, $F_{1,478} = 43.0$, $p < 0.0001$; for RH, $F_{1,478} = 7.4$, $p < 0.007$).

Similarly, I performed a one-way ANOVA on the standard deviations to investigate the possible dependence on reference orientation. For two subjects, the influence of reference orientation is significant (for ML, $F_{3,476} = 2.7$, $p < 0.04$; for NK, $F_{3,476} = 1.6$, $p > 0.21$; for RH, $F_{3,476} = 3.6$, $p < 0.02$). If the data are once more collapsed into the two categories oblique and horizontal – vertical, only for subject RH a significant effect remains (for ML, $F_{1,478} = 2.8$, $p > 0.09$; for NK, $F_{1,478} = 1.3$, $p > 0.2$; for RH, $F_{1,478} = 10.3$, $p < 0.001$). It should be remarked that this is due to higher standard deviations for the 0° and 90° reference orientations, which is just the opposite from what would be expected for an oblique effect.

3.3 *Dependence on condition*

One way to investigate the possible dependence of the results on the condition (right hand, left hand, or two hands) is to perform an ANOVA on the absolute deviations and the standard deviations (such as shown in table 1). A one-way ANOVA reveals that for only one subject condition had a significant influence on the absolute deviations

Table 2. Slopes and r^2 values for the deviations (in °) as a function of horizontal (right – left) distance and vertical (forward – backward) distance and as shown in figures 3 and 4. 0°, 45°, 90°, and 135° indicate the orientation of the reference bar. In columns labelled 'all', the data points obtained for all four reference orientations are taken together. The asterisks indicate the significance level of the slopes (*$p < 0.05$; **$p < 0.0001$); R right hand, L left hand, RL right and left hand.

Subject	Condition	Slope/° m^{-1}					r^2				
		0°	45°	90°	135°	all	0°	45°	90°	135°	all
Horizontal distance											
ML	R	−18**	−25**	−4	−24**	−18**	0.82	0.83	0.08	0.84	0.62
	L	−15**	−24**	3	−21**	−14**	0.77	0.89	0.05	0.70	0.48
	RL	−26**	−30**	−17**	−26**	−25**	0.80	0.85	0.67	0.79	0.76
NK	R	−14**	−16**	−9**	−20**	−15**	0.79	0.80	0.58	0.84	0.73
	L	−10**	−17**	−4*	−15**	−12**	0.73	0.83	0.30	0.78	0.64
	RL	−15**	−24**	−9**	−20**	−17**	0.80	0.91	0.57	0.84	0.75
RH	R	−16**	−20**	−14**	−23**	−18**	0.63	0.74	0.48	0.79	0.64
	L	−12**	−20**	−2	−20**	−14**	0.64	0.80	0.04	0.82	0.53
	RL	−19**	−24**	−35**	−30**	−27**	0.58	0.74	0.86	0.87	0.75
Vertical distance											
ML	R	−4	−7	−3	−12	−7	0.01	0.01	0.01	0.04	0.02
	L	6	5	6	15	8*	0.02	0.01	0.04	0.06	0.02
	RL	1	−1	−4	−7	−3	0.00	0.00	0.01	0.01	0.00
NK	R	−9	−10	−4	−5	−7*	0.05	0.06	0.02	0.01	0.03
	L	8	10	8*	8	8*	0.07	0.05	0.17	0.04	0.06
	RL	5	0	2	−1	2	0.01	0.00	0.00	0.00	0.00
RH	R	−15	−14	−17*	−20*	−17**	0.09	0.06	0.12	0.11	0.09
	L	3	10	9*	5	7*	0.01	0.04	0.10	0.01	0.02
	RL	−17	4	−4	−11	−7	0.09	0.00	0.00	0.02	0.01

(for ML, $F_{2,477} = 1.9$, $p > 0.1$; for NK, $F_{2,477} = 2.0$, $p > 0.1$; for RH, $F_{2,477} = 7.5$, $p \leqslant 0.0006$). In the case of subject RH, a paired t-test reveals that all three conditions are significantly different from each other ($p < 0.05$). The influence of condition on standard deviation is significant for subject NK only (for ML, $F_{2,477} = 0.3$, $p > 0.7$; for NK, $F_{2,477} = 22.0$, $p < 0.0001$; for RH, $F_{2,477} = 1.0$, $p > 0.3$). It appears that for her the standard deviations are smaller if she performs the task with her left hand (paired t-test, $p < 0.05$).

A more detailed comparison of the differences between conditions is possible by means of a multiple-regression analysis on the signed deviations. With such an analysis it is possible to determine whether the slopes belonging to the various conditions as given in table 2 and shown in figure 3 are significantly different from each other (since the slopes of the lines fitted through the deviations as a function of the vertical distance are mostly not significantly different from zero, it does not make sense to analyse them in the same way). For subject ML the slope of the bimanual condition is significantly larger than those of two unimanual conditions ($p < 0.0001$). For subject NK, the slope belonging to the left-hand condition is significantly smaller than those of the other two conditions ($p < 0.05$). Last, for subject RH, all three slopes are significantly different from one another ($p < 0.05$).

4 Discussion

The results clearly show that what subjects feel as parallel deviates systematically from what is actually physically parallel. Several analyses showed that these deviations are highly significant. The deviations can be described with a horizontal gradient in the settings of the subject with respect to veridical. What this means is that the deviations depend only on the horizontal distance between the reference and test bars and not on the vertical distance or the exact positions. This is in agreement with our previous findings (Kappers and Koenderink 1999) with measurements on the right side of the median plane. Thus we have established the validity of a description of the deviations in terms of a horizontal gradient over a large part of haptic space (table plane), over large distances, and in both unimanual and bimanual conditions. The subject-dependent horizontal gradients lie within the range of gradients reported previously (Kappers and Koenderink 1999).

That a description of the deviations by means of a horizontal gradient is suitable for the data of both the two unimanual and the bimanual conditions is not at all trivial if one considers that the movements involved for a certain setting are completely different in the three cases. Moving between two locations with the right hand or with the left hand involves the activation of different muscles, and in the bimanual case arm movements are not necessary at all. This provides a strong indication that the cause of the patterns of deviation cannot solely be based on limitations on possible movements. Apparently, the perception of what is parallel is based on some internal representation, which is more or less independent of the hand(s) used, which is similar for all subjects, and which is non-Euclidean.

Although the patterns of signed deviations of the three conditions are similar, this does not mean that there are no differences between the results of these conditions. These differences are, however, quantitative rather than qualitative. Remarkably, for all subjects the gradient belonging to the bimanual condition was always the largest and the one belonging to the left-hand condition always the smallest. Most of these differences are indeed significant (see section 3). Interestingly enough, this disadvantage for the bimanual condition (if a larger gradient can be called a disadvantage) is in agreement with the results of Appelle and Countryman (1986) from their matching task. They also report larger mean errors in the bimanual case. Furthermore, this finding is also what one would expect if the results from the shape-discrimination and curvature-discrimination experiments (eg Kappers and Koenderink 1996; Kappers et al 1994) can be extrapolated into the domain of spatial relations.

One can only speculate at the cause of these quantitative differences. One obvious difference between the unimanual and the bimanual conditions is that in the latter case the reference and test bars are touched simultaneously and not successively as in the former two cases. Thus, in the two unimanual conditions a memory component is involved which does not necessarily play a role in the bimanual condition. Although it cannot be excluded that this memory component causes the differences, for a number of reasons this explanation seems unlikely. First, the larger the delay between touching two bars, the larger the involvement of the memory component and thus the more difficult the task becomes. If 'more difficult' corresponds with 'larger deviations' then the results are opposite from what one would expect if memory plays a determining role. Second, there are also significant differences between the two unimanual conditions where the influence of a memory component must be more or less identical. Last, in the experiments of Appelle and Countryman (1986) both bimanual and unimanual stimuli were touched successively, but they also found a disadvantage for the bimanual condition.

In the haptic literature there is some evidence for the existence of haptic 'oblique effects'. What is usually meant is that performance for a certain task is worse for stimuli presented under oblique orientations (eg Appelle and Countryman 1986; Appelle and Gravetter 1985; Gentaz and Hatwell 1995; Lechelt et al 1976; Lechelt and Verenka 1980). Since our reference stimuli were presented under the orientations 0°, 45°, 90°, and 135°, our data also permitted an analysis with respect to a possible oblique effect. There was no oblique effect for the standard deviations in the settings. However, the absolute deviations were significantly larger for the two oblique reference orientations of 45° and 135° than for horizontal (0°) or vertical (90°) reference orientations. We previously reported similar findings (Kappers and Koenderink 1999) for a smaller section of haptic space and a unimanual (right-hand) condition.

It is of course tempting to compare the present results with those reported by Blumenfeld (1937) although one should bear in mind that his stimuli and experimental task were quite different from mine. In his experiment, two stimuli (consisting of threads fixed to a needle) were always located at the same vertical distance from the subject and at the same horizontal distance from the median plane. Subjects were asked to pull the threads in such a direction that they became parallel to each other and to the median plane. The so-constructed alley curves diverge towards the subject as long as the horizontal distance between the lines is less than the distance between the shoulder joints; above this distance the lines gradually become parallel and for some subjects eventually converge (maximum distance was 90 cm). In the present setup the horizontal distance between two symmetrically positioned bars was 60 or 120 cm. In Blumenfeld's setup the settings belonging to 60 cm would be approximately parallel; for those belonging to 120 cm he does not have a comparison. In the present experiment, bars symmetrically positioned 60 cm from each other were never parallel and certainly do not diverge, not even if the reference orientation was 90°. Thus, apparently, the precise task influences the patterns of deformation.

Which difference between the two experiments is the major factor is hard to determine from the data currently available. One obvious candidate is that in his case both stimuli have to be adjusted, whereas in the present experiments one stimulus is fixed as reference. That this can indeed be of influence can be seen from the existence of the oblique effect. If, for example, the reference orientation is oblique, the deviation can be so large that the test bar is oriented horizontally. If now the positions of the reference and test bar are interchanged and the reference bar is oriented horizontally (just like the test bar in the other case), the deviation of the test bar is smaller than in the previous case. Since the two bars are identical, this can only mean that somehow the fact that only a specific bar may be rotated influences the results. Other differences, such as the fact that subjects had to pull threads in his experiment and thus could not really touch the stimuli, might also play a role. Clearly further experiments are needed before conclusions can be draw on the cause of the different outcomes.

Summarising it is possible to say that the results presented in this paper provide strong proof that haptic space is not Euclidean. The deformations in the haptic table space can be adequately described with a horizontal gradient in the orientation settings with respect to veridical. In future studies we will try to find and understand the cause of the deformations of haptic space.

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References

Appelle S, Countryman M, 1986 "Eliminating the haptic oblique effect: influence of scanning incongruity and prior knowledge of the standards" *Perception* **15** 325–329

Appelle S, Gravetter F, 1985 "Effect of modality-specific experience on visual and haptic judgment of orientation" *Perception* **14** 763–773

Battro A M, Pierro Netto S di, Rozestraten R J A, 1976 "Riemannian geometries of variable curvature in visual space: visual alleys, horopters, and triangles in big open fields" *Perception* **5** 9–23

Blumenfeld W, 1937 "The relationship between the optical and haptic construction of space" *Acta Psychologica* **2** 125–174

Brambring M, 1976 "The structure of haptic space in the blind and sighted" *Psychological Research* **38** 283–302

Coren S, 1993 *The Left-hander Syndrome* (New York: Vintage Books)

Gentaz E, Hatwell Y, 1995 "The haptic 'oblique effect' in children's and adults' perception of orientation" *Perception* **24** 631 – 646

Indow T, Watanabe T, 1984 "Parallel-alleys and distance-alleys on horopter plane in the dark" *Perception* **13** 165 – 182

Kappers A M L, Koenderink J J, 1996 "Haptic unilateral and bilateral discrimination of curved surfaces" *Perception* **25** 739 – 749

Kappers A M L, Koenderink J J, 1999 "Haptic perception of spatial relations" *Perception* **28** 781 – 795

Kappers A M L, Koenderink J J, Pas S F te, 1994 "Haptic discrimination of doubly curved surfaces" *Perception* **23** 1483 – 1490

Lechelt E C, Eliuk J, Tanne G, 1976 "Perceptual orientational asymmetries: A comparison of visual and haptic space" *Perception & Psychophysics* **20** 463 – 469

Lechelt E C, Verenka A, 1980 "Spatial anisotropy in intramodal and cross-modal judgments of stimulus orientation: the stability of the oblique effect" *Perception* **9** 581 – 589

Lederman S J, Klatzky R L, Barber P O, 1985 "Spatial and movement-based heuristics for encoding pattern information through touch" *Journal of Experimental Psychology: General* **114** 33 – 49

Luneburg R K, 1950 "The metric of binocular visual space" *Journal of the Optical Society of America* **40** 627 – 642

Yamazaki T, 1987 "Non-Riemannian approach to geometry of visual space: an application of affinely connected geometry to visual alleys and horopter" *Journal of Mathematical Psychology* **31** 270 – 298