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PERCEIVED SHAPE AND ITS DEPENDENCY ON PERCEIVED SLANT[1]

PETER K. KAISER[2]
University of California, Los Angeles

Koffka's suggestions that perceived shape and perceived slant "will be coupled together so that if one changes, the other changes also" and that errors in perceived shape vary as some function of errors in perceived slant were examined. Ss described, by means of appropriate response mechanisms, the shapes and slants of trapezoids. Shape and slant responses were made both monocularly and binocularly. The changes in reported shape varied as a function of changes in reported slant. Also, shape response errors varied as a function of slant response errors under monocular viewing when Ss had no prior binocular experience with the trapezoid. The functions relating perceived shape to perceived slant were comparable to the function predicted by the Beck and Gibson shape-slant invariance hypothesis.

As one moves about, his visual surroundings appear stable and objects maintain their apparent shapes regardless of their orientation with respect to *O*. Since visual information is mediated through the retina, how is it that one tends to report the object shape of things and not the shape projected on the retina? One explanation postulates that the slant of the object is taken into account. Descartes (1638) noted that "... the shape is judged by our knowledge or opinion of the disposition of the diverse parts of the objects. . . ." More recently, William James (1890) noted, that "it is not the cross and ring pure and simple which we perceive, but the cross so held, the ring so held [p. 259]."

The first formal statements of this shape-slant relationship were presented by Koffka (1935, pp. 229–233). He proposed "invariant" relationships between perceived shape and perceived slant. From his statements, these relationships are interpreted by the present writer, as being monotonic and the precise functions are dependent on the "total sets of conditions." Beck and Gibson (1955) restated Koffka in a more restricted form, proposing the shape-slant invariance hypothesis. This hypothesis states that "a retinal projection of a given form determines a unique relation of apparent[3] slant to apparent shape [p. 126]." This unique relation is described by the family of shape-slant combinations that project identical retinal images. Koffka proposed multiple invariant relations, while Beck and Gibson proposed one.

Several studies (Beck & Gibson, 1955; Bower, 1966; Wallach & Moore, 1962) have shown that perceived shape depends on cues for slant. Limited support for a general shape-slant correspondence has been found by Stavrianos (1945), Beck and Gibson (1955),

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[2] Now at the University of Rochester.

[3] No distinction is made among the terms apparent, perceived, and reported.

Epstein, Bontrager, and Park (1962), and Winnick and Rogoff (1965). However, their data do not demonstrate the function or functions relating perceived shape to perceived slant, and cannot be considered sufficient support to verify a shape-slant invariance hypothesis. At least two investigators, Nixon (1958) and Flock (1964), were unable to find any systematic relationship between perceived shape and perceived slant.

The research reported in this paper examines Koffka's (1935) general suggestions that "the two aspects of the percept (shape and slant) will be coupled together so that if one changes the other changes also [p. 229]"; and that "it . . . is probable that the amount by which the figure appears turned from normal decreases as the constancy of shape decreases [p. 232]." More specifically these suggestions are examined in the context of the Beck and Gibson shape-slant invariance hypothesis.

Koffka's second suggestion implies that as errors in perceived slant increase (decrease), errors in perceived shape will also increase (decrease). Error refers to the discrepancy between the physical shape (slant) and *S*'s report of shape (slant) on different observations. If Beck and Gibson (1955) are correct, the function relating shape errors to slant errors and the function relating reported shape change to reported slant change should correspond to the function (unique relation) of the invariance hypothesis.

Changes in reported slant were obtained by varying the cues for depth, i.e., binocular disparity. The *S*s viewed the stimuli monocularly and binocularly. The difference between stimulation produced by monocular and binocular viewing is manifested mainly in the perception of depth (i.e., in this case, slant). Without binocular disparity as a depth cue, *S* has less information with which to make judgments of slant. Therefore, one can expect that *S* will report slant differently under monocular viewing than under binocular viewing. If this is the case, will he also report shape differently? In utilizing this technique, it is assumed that no distinction is necessary in the interpretation of the invariance hypothesis for monocular and binocular viewing. If this assumption is valid, we may conclude that obtained changes in reported shape are primarily due to the effect of changes in perceived slant.

METHOD

Apparatus.—All stimulus displays were housed in a common box (Fig. 1). The shape response apparatus (a), standard stimulus (b), and slant response apparatus (c) were placed 30° apart, tangent to an arc of radius 105 cm. They were viewed through a viewing slot (d) placed at the center of the arc. The box was illuminated by four 20-w. cool white fluorescent tubes (e) placed on the floor, ceiling, and two walls at the front of the box. Light baffles (f) were placed in front of each tube so that the displays would not receive direct illumination. The inside front wall and a 7-in. border on the walls, floor, and ceiling adjacent to the inside front wall were painted flat white. Illuminating the inside of the box in this manner produced homogeneous illumination with no visible shadows about the standard stimulus. The remainder of the box was painted flat gray. Mounted on the front of the viewing box was a combination head-holder chinrest.

The standard stimuli were three trapezoids cut from tempered aluminum, .0020 in. thick. The sizes and shapes of the trapezoids were designed so that when placed at 15°, 45°, and 65° away from the frontal-parallel plane top receding away, they projected identical retinal images. Each trapezoid at its respective slant subtended the following visual angles: top = 8°, height = 5°, and base = 10°. The formulas used to determine these shapes are reported by Kaiser (1966). The surfaces of the trapezoids were painted flat white. The base edge of each stimulus was

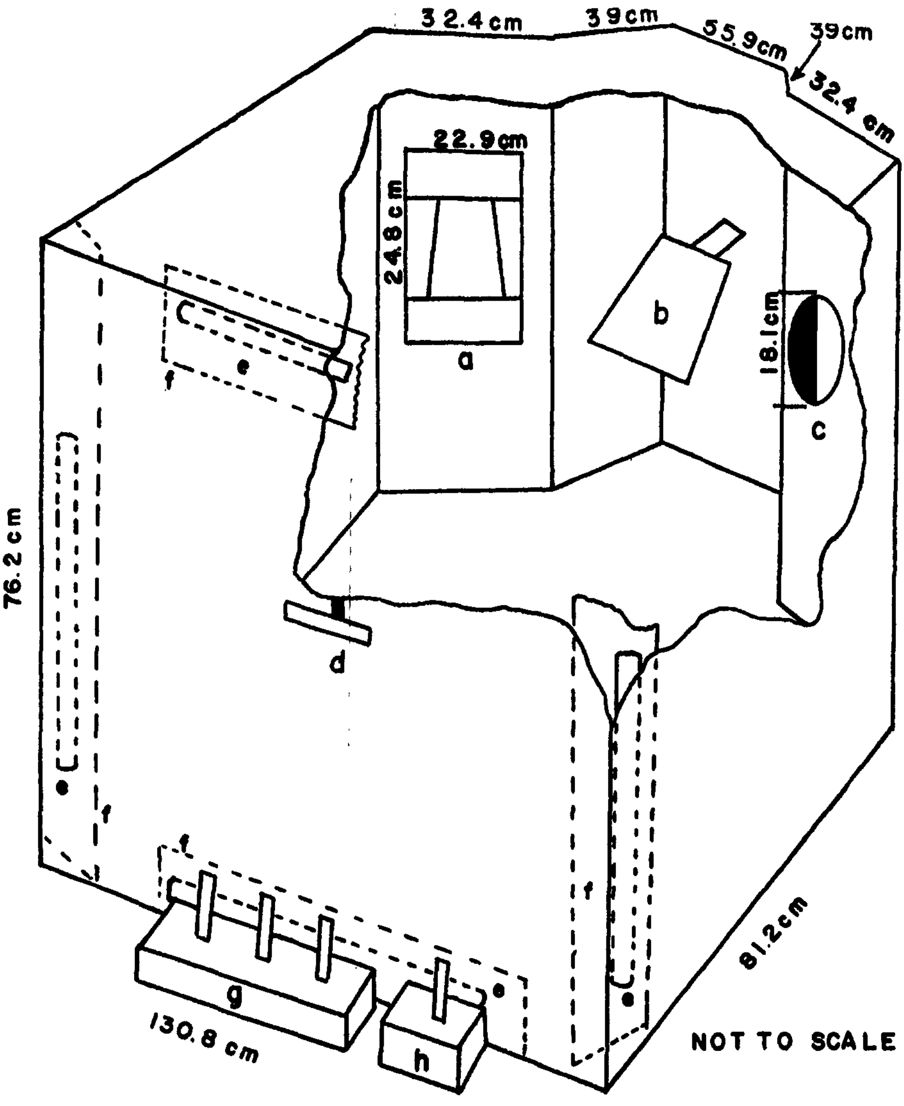

FIG. 1. Schematic drawing of viewing box, standard stimulus, and response mechanisms. (Letters identify components in text.)

filed to a knife edge so that when slanted the thickness of the metal would be minimally visible. The luminance of the stimuli were 12 ftl. for the 15° stimulus, 8.5 ftl. for the 45° stimulus, and 6.5 ftl. for the 65° stimulus. The luminance of the gray background about the stimuli was approximately 3 ftl. These measurements were made with a Spectra brightness spot meter. The stimuli were mounted on a plate whose angle of slant was controlled by turning a crank attached to a worm gear and nut arrangement. The stimuli were slanted about a horizontal axis placed at eye level, i.e., directly opposite the viewing slot at a distance of 105 cm. Access to the mounting plate was through a side door.

The shape-response apparatus permitted *S* to report shapes ranging from isosceles triangles with the apex at the top to isosceles triangles with the apex at the bottom. These shapes included triangles, trapezoids, and rectangles. The equipment imposed the restriction that the maximum size of the shape response could not be more than 24 cm. high and 22 cm. wide. This restriction is more than 2 cm. larger than the maximum height and width of any of the standard stimuli. Only a few *S*s were affected by this restriction. They attempted to report the base of the 15° stimulus as being wider than 22 cm. and were asked to scale down the size of the shape response. The shape response was made by adjusting four masks surrounding a flat white background. The top and bottom masks moved vertically; the vertical masks moved horizontally. Also, the vertical masks could be rotated to vary the slant of the sides of the shape response. A more detailed description of this apparatus is given by Kaiser (1966). The fronts of these masks were painted the same color as the inside of the box. The background behind these masks presented a homogeneous white surface regardless of the position of the masks. The *S*s were given three controls (Fig. 1g) to make the shape responses: one varied the height, a second the width, and the third the angles of the sides.

The *S*s made slant responses by rotating a half-black half-white disc (Flock, 1964) by means of a control switch (Fig. 1h). When the division between the black and white halves was vertical this indicated that the standard stimulus was vertical; when it was horizontal, the standard was parallel to the line of sight. Rotation clockwise indicated that the top of the standard was slanted toward *S* and vice versa.

Subjects.—The *S*s were 30 undergraduates fulfilling a requirement of the introductory psychology course at UCLA. They were naive with respect to the purpose of the experiment. On their last eye examination, all *S*s had normal acuity without corrective lenses.

Procedure.—Each *S* was assigned to one of three stimulus-slant conditions and one of two viewing conditions (monocular or binocular first). The assignment was made in an irregular order. There were five *S*s in each of the six conditions. Thus, each *S* viewed only one stimulus shape at its respective slant. The *S* was instructed in the use of the response mechanisms and given a short practice period with a stimulus that was not one of the standards. Practice was conducted monocularly when monocular was the first condition and binocularly when binocular viewing was the first condition. The right eye was used under the monocular condition; the left eye being occluded by an eye patch. Each *S* was given the following instructions:

> I will place various shaped objects at various slants into the box. Your task is to answer two questions. 1. What is the shape of the object placed in the box by *E*? 2. At what slant did *E* place the object in the box? When you make the shape response, the figure you make in the shape response apparatus should be the same size and shape as the object placed in the box. There might possibly be one difference between the object I placed in the box and your shape response. Your reproduction of the object should look as if the object is in a vertical position, i.e., perpendicular to your line of sight. The object in front of you, however, may be slanted to your line of sight. For example, suppose you look at the object in the box, and you wish to say that it is a circular disc and it is slanted away from you. You would make a circular disc[4] on the shape apparatus that is the same size and shape as the disc in front of you. However, the disc that you make on the response mechanism will be vertical while the disc placed in the box will be slanted. If you make a perfect response, you, theoretically, should be able to reach into the box, take the circular disc and place it directly over your response and it would fit perfectly.

At this point *S* was asked if he had any questions and *E* explained any portion of the instructions that were unclear. The *S* was told, further, that he would always make the slant response first. After completing the slant response, he would make the shape response. Then he could look iteratively among the standard stimulus, his slant response and shape response as often as he liked, making any adjustments to his responses that he felt were required. A trial was ended when *S* said that the shape and slant responses indicated exactly what he wanted to tell *E* about the shape and slant of the standard stimulus.

At the end of each trial *E* closed the viewing slot and measured the slant response from the rear of the slant response apparatus. This measure was taken from a protractor-

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[4] This is only an example. A circular disk could not be constructed with this shape-response apparatus.

pointer arrangement attached to the rear of the black-white disc. The *E* also measured the height, width of the top, and width of the base of the shape response directly from the rear of the masks. The slant responses were measured to the nearest degree and the shape responses to the nearest millimeter.

The *E* then went through the motions of changing the standard stimulus. However, the standard was not changed. The purpose of this procedure was to discourage *S*s from responding from trial to trial under the assumption that they were always looking at the same stimulus. Finally, *E* changed the setting of the shape and slant response mechanisms to new positions.

Each *S* made three sets of responses under each viewing condition, e.g., three shape and three slant responses monocularly and three shape and three slant responses binocularly.

TABLE 1

SUMMARY OF SHAPE AND SLANT RESPONSES

Stimulus Subject	Monocular Mean			Binocular Mean		
	Slant	h/b	t/b	Slant	h/b	t/b
Monocular First						
15						
YD	27	.83	.90	15	.65	.85
MF	20	.63	.86	24	.62	.91
JB	28	.60	.82	16	.52	.81
GT	11	.62	.88	13	.54	.88
KV	−1	.50	.77	15	.58	.80
M	17.0	.64	.85	16.5	.58	.85
45						
LF	59	1.24	.94	40	.83	.85
ED	11	.55	.83	26	.76	.94
AB	38	.78	.91	36	.93	.98
JR	12	.52	.82	32	.57	.83
SW	27	.76	.85	27	.76	.85
M	29.4	.77	.87	32.2	.77	.89
65						
RB	−21	.49	.81	58	1.15	.95
DT	20	.83	.88	38	1.38	.98
PT	10	.62	.86	55	1.27	.98
SC	43	.91	.98	58	1.31	1.00
ED	21	.54	.75	64	1.09	.97
M	14.6	.68	.86	54.6	1.24	.98
Binocular First						
15						
AS	2	.54	.77	15	.53	.80
FW	35	.84	.96	9	.53	.77
RP	18	.71	.86	10	.58	.87
NB	26	.51	.82	22	.51	.81
RB	17	.65	.89	3	.54	.83
M	19.6	.65	.86	11.8	.54	.82
45						
MK	25	.68	.77	25	.77	.80
DJ	14	.62	.86	37	.70	.83
ME	17	.72	.87	24	.76	.89
GB	38	.68	.87	44	.75	.86
KH	24	.62	.84	44	.72	.87
M	23.6	.66	.84	34.8	.74	.85
65						
GS	46	1.19	.98	51	1.29	.97
NS	56	1.04	.98	58	1.18	.98
JC	59	1.13	.99	68	1.23	1.01
MT	46	.55	.81	61	.99	.93
JW	60	1.21	.99	63	1.33	.99
M	53.4	1.02	.95	60.0	1.20	.98

RESULTS

The shape response measures were reduced to height to base (h/b) ratios and to top to base (t/b) ratios. Analyses of the change scores and error scores were not performed on the raw h/b and t/b measures. These measures for stimuli that project identical retinal images are not linear functions of slant, but curvilinear. Therefore, it was not meaningful to combine the data in this form for the different stimuli at their respective slants. This difficulty was overcome by transforming h/b and t/b scores so that, under the Beck and Gibson hypothesis, they are linear functions of slant. These transformations[5] and the rationale for them are reported by Kaiser (1966). The *nontransformed* shape scores will be written as h/b and t/b while the *transformed* values will be written as h′/b′ and t′/b′. In addition to combining the transformed shape response data for different slant conditions, linear regression analyses may now be used to evaluate

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[5] Briefly, the transformations were obtained as follows. The curvilinear function relating h/b to slant was derived for the stimuli subtending visual angles described above. An arbitrary straight line (h′/b′) was drawn through this function and an equal interval scale assigned to the associate ordinate. To determine the value of h′/b′ corresponding to a particular h/b, the slant value associated with that h/b is obtained. The h′/b′ associated with this same slant is the transformed value. An analytical derivation proved this graphical technique accurate. Transformation of the t/b was obtained by a similar graphical technique. The following expression describes the transformations:

$h'/b' = k[f^{-1}(h/b)] + a$. Where: k = slope of linear function; a = intercept of linear function; f = curvilinear function of slant.

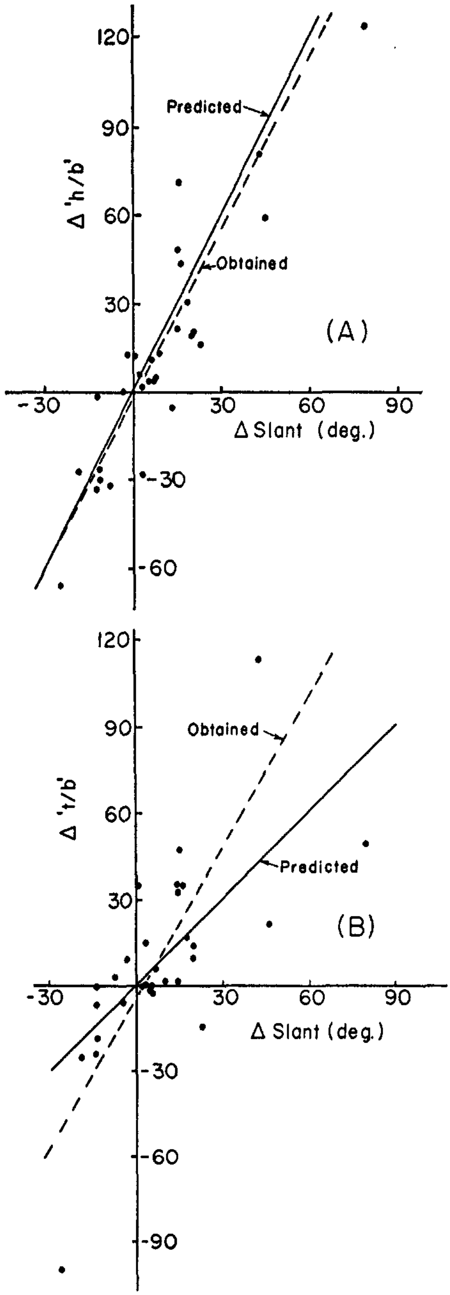

FIG. 2. Change in h'/b' and t'/b' as a function of change in slant responses when going between monocular and binocular viewing.

the agreement between obtained and predicted results.

Table 1 shows the mean slant responses, mean h/b (untransformed) and mean t/b (untransformed). Each mean is the average of three responses. In previous experiments (Kaiser, 1966, Exp. A & I) *S*s were given 10 trials. These investigations showed that the intra-*S* reliability was very high. Therefore, the use of three trials per *S*, per condition was considered justified. As expected, slant judgments were more accurate binocularly than monocularly.

Change scores were computed as follows. The slant response, h'/b' and t'/b' (transformed scores) obtained under monocular viewing were subtracted from the slant response, h'/b' and t'/b' under binocular viewing for each *S*.

Figure 2A shows the change in h'/b' as a function of the change in reported slant when going between monocular and binocular viewing. The change in reported h'/b' ($\Delta h'/b'$) is significantly correlated with the change in reported slant (Δ slant), r (30) $= .90$ $p < .01$. A regression line was computed by the method suggested by Worthing and Geffner (1943) for the case when "liability of error occurs" in both variables. The slope of this regression is 1.89. The predicted line, if *S*s responded in terms of the Beck and Gibson hypothesis, for this experiment, is 2.00. The change in t'/b' (Fig. 2B) is significantly correlated with the change in slant responses, r (30) $= .68$, $p < .01$. The slope of the obtained regression line is 1.80; however, the predicted slope for this experiment is 1.00.

Figure 3 shows the results of the error analyses for monocular viewing conditions as the first condition and as the second condition. Error was computed by subtracting *S*s' responses from the physical measures. For example, if *S* reported a slant of 60° when the physical slant was 45°, he made a slant response error of 15°. When monocular viewing is the first condition, the shape errors are significantly correlated with slant errors. The correlation between h'/b' error and slant response error is .95 ($df = 15$; $p < .01$). The obtained regression line for the h'/b' is 1.87; the predicted line is 2.00. For t'/b' it is 1.25 and the predicted line is 1.00. Under the monocular condition, when binocular viewing was first, the shape response errors were not significantly correlated with the slant response errors.

Table 2 presents a summary of the error analyses and the change analyses.

For binocular viewing, both as the first and second viewing condition the correlations between shape error and slant error did not reach significance. However, the binocular response errors fell within the range of the monocular response errors. Most of the binocular responses fell near the point of zero-slant error and zero h'/b' and t'/b' error. The failure to obtain significant results under binocular viewing is probably due to the small range of response errors relative to the inter-*S* variability; therefore, these results do not detract from the importance of the monocular results.

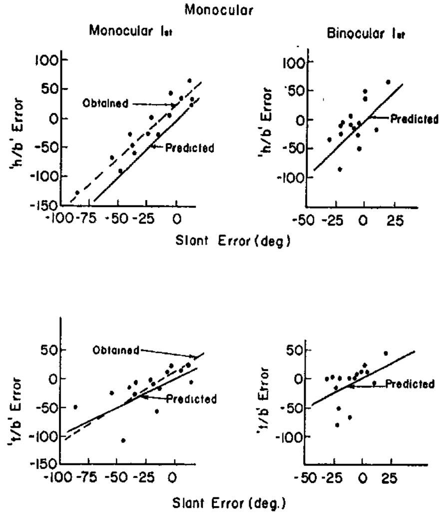

FIG. 3. h'/b' and t'/b' errors as a function of errors in slant responses under monocular viewing when used as first and second viewing conditions.

TABLE 2

SUMMARY OF THE ANALYSES

Error Analyses	*r*	*df*	Regression Coefficient	
			Predicted	Obtained
h'/b' monocular-monocular 1st	.95*	15	2.00	1.87
t'/b' monocular-monocular 1st	.65*	15	1.00	1.25
h'/b' monocular-binocular 1st	.63	15	2.00	
t'/b' monocular-binocular 1st	.46	15	1.00	
h'/b' binocular-monocular 1st	.38	15	2.00	
t'/b' binocular-monocular 1st	.13	15	1.00	
h'/b' binocular-binocular 1st	−.36	15	2.00	
t'/b' binocular-binocular 1st	.35	15	1.00	
Error Analyses[a]				
h'/b'	.90*	30	2.00	1.89
t'/b'	.68*	30	1.00	1.80

[a] In the analysis of response changes, monocular first and binocular first data are combined.

* $p < .01$.

DISCUSSION

The error analyses in this experiment are in accord with the error analyses of two previous investigations (Kaiser, 1966, Exp. A & I). In agreement with Koffka's (1935) suggestion, a close correspondence was obtained between errors in perceived shape and errors in perceived slant. The relationship between errors in h'/b' and errors in slant is a fair approximation of the function predicted by the Beck and Gibson (1955) shape-slant invariance hypothesis. While t'/b' errors as a function of slant errors show a similar relationship, the correspondence to the Beck and Gibson hypothesis is not as good as that provided by h'/b'. Agreement with the predicted function is found only under monocular viewing when *S*s had no prior binocular experience with the stimuli. The relationship was not found under binocular viewing primarily due to the small range of response errors compared with the inter-*S* variability. These results lend support to Epstein's et al. (1962) findings that under objective instructions *S*s made accurate shape responses. While the instructions in the experiment reported in the present paper can be considered objective, they differ from Epstein's et al. in one significant way. Their *S*s were instructed to report the "actual physical dimensions of the target even if the match you make doesn't look equal in shape to you." The *S*s in my experiment did not receive the latter part of this instruction. It would seem that the important question is how the actual physical dimensions appear to *S*s.

The results of the change analyses lend support to Koffka's proposal concerning the covariance of changes. Furthermore, this relationship can be described by the function proposed by the invariance hypothesis. The h′/b′ changes demonstrate this function considerably better than the t′/b′ changes do.

In the error and the change analyses, the relationship between h′/b′ and slant adheres more closely to the predicted relationship than does the relationship between t′/b′ and slant. This difference can, in some measure, be accounted for by the smaller range of t′/b′ responses. If *S*s had responded precisely as predicted by the invariance hypothesis, the range of h/b (not transformed) scores would have been .81 while the range of t/b scores would have been only .16. This difference in ranges may account for the smaller correlation coefficients obtained for t′/b′ data than for the h′/b′ data. However, one would expect that if the correlations were smaller, then the slope of the regression lines should also be smaller than predicted. The regression lines for the t′/b′ data in all analyses were greater than the predicted 1.00. No interpretation is made for this paradoxical finding.

The failure of previous investigators to find particular relationships or to provide strong support for a given shape-slant relationship probably can be attributed to the experimental methodology and the analyses to which the data were subjected. The following methods employed in the present research seem to have contributed to obtaining the "invariant" (Koffka, 1935) relationships reported in this paper. (*a*) Since both conceptions (Koffka's and Beck and Gibson's) of the invariant relations state that the shape-slant relationship should be obtained for a retinal image of a given form, all the standard stimuli projected identical retinal images. Thus, if the invariance hypothesis were tenable, the relationships between reported shape and reported slant should yield a single function. However, this function has to be evident over the inter-*S* variability. Therefore, several stimuli projecting identical retinal images, instead of one stimulus, were used in order to obtain a larger range of slant response changes and errors. Extending the range of these slant responses should not affect the relationship between reported shape and reported slant if an invariance hypothesis is valid. (*b*) For a given retinal image object shape (h/b, t/b) is a curvilinear function of object slant. If *S* responds in terms of the invariance hypothesis, the reported shape should also be a curvilinear function of reported slant. Demonstrating this function can be difficult, unless *S* variability is small. Also, when using differently shaped stimuli at different slants, untransformed error and change scores cannot be meaningfully combined in one analysis. Therefore, h/b and t/b scores were transformed to permit straight line analyses of the data from different stimuli, facilitating the comparison of obtained data with predicted functions. It was thus possible to test the Beck and Gibson hypothesis in a manner less obscured by *S* variability.

The following methodological improvements may have contributed to obtaining data in accord with the function predicted by the invariance hypothesis. (*a*) The shape and slant response mechanisms were viewed under the same conditions as the standard stimulus by housing them in a common viewing box. (*b*) Simultaneous shape and slant responses were approximated by allowing *S*s to make adjustments to their initial responses in an iterative fashion until satisfied that *both* responses represented their perception of the shape and slant of the stimulus. (*c*) The shape-response mechanism had sufficient degrees of freedom so *S*s could report all aspects of their perception of the shape of the stimulus. The *S*s could report the shape as they perceived it. They were not constrained to construct the shape response about any single fixed dimension (e.g., fixed width of response apparatus).

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