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# Perceiving the sweet spot

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**Abstract.** Many sports involve aligning a hitting implement with a ball trajectory such that contact is made at the implement's center of percussion or 'sweet spot'. This spot is not visibly distinct; its perception must be haptic. Although it is functionally defined with respect to contact—it is the point of impact that produces the least vibration in the hand holding the implement—hitting success requires appreciating the location of the sweet spot prior to contact. Two experiments verified that perceivers (novices as well as expert tennis players) distinguished perception of length from perception of the position of the sweet spot simply on the basis of wielding, both for tennis rackets and for bats contrived from wooden rods with attached masses. Results conformed to previous research on dynamic touch in showing that perceiving the lengths of wielded objects, including selectively perceiving partial lengths, is constrained by inertial properties of the object.

## 1 Introduction

The successful striking of a ball with a bat or racket entails aligning the hitting implement with the ball trajectory such that contact is made at the implement's center of percussion or 'sweet spot'. When this alignment is achieved, one has the satisfying feeling of hitting the ball just right—an impression that "the bat or racket is doing the work unaided" (French 1971, page 668). It is often the case, however, that impact of implement and ball occurs at locations other than the sweet spot. The consequence is an impression of hitting the ball with effort, accompanied by a stinging feeling in the hands. Both impressions are the dynamic consequences of a rigid object subject to a sudden impulse.

Alignment of ball and sweet spot would seem to be a cooperative achievement of visual and haptic perception. During flight, a ball is essentially visible but nontangible and, during the swing, a striking implement's sweet spot is essentially tangible but nonvisible. The ball trajectory and time-to-contact with the plane of the bat or racket is registered by eye; the center of percussion of the bat or racket relative to the grasp and to the body's main axes is registered presumably by feel. In order to bring about the controlled interception of the trajectory of the sweet spot with the trajectory of the ball, the visual and haptic perceptual systems must coordinate their respective detections of task-relevant information.

It is evident that, with a little trial and error, striking a ball with a bat or racket will lead gradually to an appreciation of where the sweet spot is localized. The felt consequences of the different impacts over trials would be the basis for such learning. At issue in the present article is the haptic perception of the sweet spot of a racket unmediated by experiences with the dynamic consequences of racket – ball contact. Can one perceive the sweet spot of a striking implement simply on the basis of wielding it? The haptic perceptual subsystem of greatest relevance to this task is dynamic touch (Gibson 1966). When an object is firmly grasped, as a racket is at the handle, and then swung or wielded, the primary deformations of the body tissues take place in the muscles and tendons. The capabilities of dynamic touch are tied to these deformations and their afferent consequences. A large body of research has shown that perception by dynamic touch is constrained by a physical characterization of objects that reflects their mass distributions—not simply how massive they are but where that mass is relative to the grasping hand (figure 1). Mass distributions can be quantified in terms of *moments*, or tendencies to produce (or resist) motion about an axis. As has been shown repeatedly, exteroperception capabilities are related by power functions to the moments of the mass distribution (see reviews by Carello and Turvey, in press; Turvey 1996; Turvey and Carello 1995). The technical details of this argument have been discussed thoroughly elsewhere (eg Fitzpatrick et al 1994; Turvey 1996; Turvey and Carello 1995) and an overview can be found in the appendix. For present purposes we note two kinds of facts. First, the nonvisible perception of the lengths of hand-held wielded objects tends to scale as the second moment of the mass distribution. In particular, perceived length is constrained by a quantification of the maximal resistance to rotation, the so-called *eigenvalue* $I_1$, raised to the ⅓ power (eg Turvey et al 1998a). Second, when an object is simply held rather than actively wielded, the first or static moment is a contributing factor (eg Burton and Turvey 1990). These facts have important implications for the sweet spot, because its distance from the rotation axis is given by a ratio of the second moment to the first moment, that is the ratio of the moment of inertia about the relevant axis to the static moment about that axis (figure 1). In principle, therefore, the sweet spot of a hand-held implement should be perceptible by dynamic touch simply on the basis of wielding the implement.

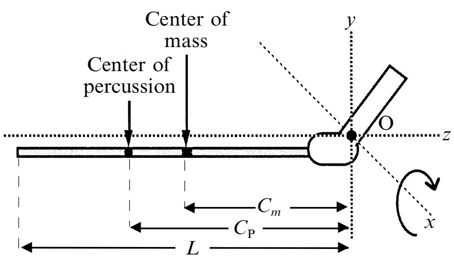


**Figure 1.** A homogeneous cylinder grasped at one end can be characterized with respect to how its mass is distributed relative to the *x*, *y*, and *z* axes with their origin at a center of rotation O in the wrist. Several physical quantities can be defined for this simple object in terms of its mass, *m*, and its length, *L*: the distance of the center of mass from the wrist, $C_m = \frac{1}{2}L$; the first or static moment, $M = mC_m$; the second moment about the *x* axis, $I_{xx} = \frac{1}{3}mL^2$; and the distance of the center of percussion from the wrist, $C_P = I_{xx}/M$. (For simplicity, this illustration assumes a negligible diameter.)

An examination of the nonvisible perception of center of percussion of a hand-held implement in conjunction with an examination of the nonvisible perception of the implement's length should provide important insights into the capabilities of dynamic touch. Experiments show that people have unambiguous but inexact impressions of the distance from the hand to the tip of a wielded object. The inexactness is not arbitrary. To the contrary, perceptions of different lengths are ordered correctly and tend to be in the range of the actual lengths; to use Bingham's (1993) argument, the perceptions are definite rather than absolute or relative. Inexactness must necessarily be the case because the quantities of relevance to dynamic touch are mechanical not geometric. Thus, as noted above, nonvisibly perceived length is a function of the magnitudes of the inertial eigenvalues. The length of a hand-held object, as measured by a ruler, is complexly encoded within the inertia tensor and for objects that are oddly shaped and materially inhomogeneous the correlation between components of the inertia tensor and the lengths of such objects will be low (Turvey and Carello 1995). It needs to be underscored, however, that although object length is an ill-posed question for dynamic touch, it is the case that, under each and every constraint on the nature of the rigid hand-held objects that have been studied thus far, length perception has proven to be specific to the tensor eigenvalues. As noted, the specificity takes the form of a power law with an approximately ⅓ scaling on $I_1$. One broad theoretical implication of the power law is that length perception by dynamic touch is a self-similar, fractal process (eg Peitgen et al 1992; Schroeder 1991). That is, it depends in the same way on the largest eigenvalue of the inertia tensor at all object sizes or scales (eg consider very small objects with little resistance to being rotationally accelerated, such as sewing needles, and very large objects with great resistance, such as mallets), within as yet unidentified largest and smallest scale sizes (Turvey 1996; Turvey et al 1996). The importance of power laws for psychophysics was well-respected by Stevens (1961, 1962), but the fullness of their implications has only become clear in recent years. A system whose functioning abides a power law, such as dynamic touch in the perception of length, does not conform to the principle of superposition. That is, it does not partition into a number of noninteracting components and it is not understandable as the sum of these separate components. To the contrary, the power law behavior indicates that the underlying processes are multiple and interdependent, operating at many time scales that are distributed lognormally (eg West and Deering 1995).

The significance of the comparison between the perceptions of the distance from the rotation point in the wrist to (a) the object's tip and (b) the object's center of percussion, lies with the fact that the latter distance is strictly determined by, and perfectly correlated with, the moment of inertia and the static moment in ratio. Unlike the distance to the tip of the object, the distance to the center of percussion from the rotation point is a physically defined, a posteriori fact of the object's mass distribution relative to that point. The distance to the sweet spot would seem to be a well-posed problem for dynamic touch. What should be expected, therefore, from this contrast between dynamic touch responding to the ill-posed problem of perceiving an implement's length and dynamic touch responding to the (apparently) well-posed problem of perceiving an implement's center of percussion? If the contrast truly matters to dynamic touch, and if the center of percussion is perceptible, then we might expect measures of perceptual success (accuracy, reliability) for the well-posed problem to be superior to those for the ill-posed problem. An additionally significant feature of the comparison is in respect to the question of whether participants can, in fact, achieve two reliably distinct nonvisible length perceptions for the same object. Is the sensitivity of dynamic touch to invariants in the play of forces such that it can support the separate perceptions of two distances in the same direction (eg along the longitudinal axis of a racket)? It has already been shown that dynamic touch can resolve perceptions of two lengths in orthogonal directions (Turvey et al 1998a) and that it can resolve both the whole longitudinal length and a fractional longitudinal length (eg the portion forward of the grasp) of an object held at an intermediate position between its ends (eg Carello et al 1996; Pagano et al 1996; Solomon and Turvey 1988; Solomon et al 1989). A successful demonstration of distinct perceptions of an implement's length and center of percussion would add to our growing understanding of the nonvisible differentiations of the properties of a hand-held object by dynamic touch.

In the present research, we applied the magnitude production methods used in previous research on the nonvisual perception of lengths and widths by dynamic touch (Turvey and Carello 1995). Participants wielded occluded hand-held implements under either the instruction to produce the felt distance to the end of an implement or the instruction to produce the felt distance to that point on the implement at which they would prefer to strike a ball. It was hoped that the latter instruction would suffice to constrain a participant to the task of perceiving the wielded implement's center of percussion. Additionally, two groups of participants were studied: one group highly skilled in tennis and the other relatively naive in respect to racket sports in general. At issue was whether the nonvisual ability to perceive the sweet spot would be dependent on experiences that seem to demand the application of such an ability.

## 2 Experiment 1: The sweet spot of a tennis racket

In addition to the center of percussion, there are two additional spots on a tennis racket that can be designated 'sweet' (Brody 1987). These correspond to the node of the first harmonic (of the racket's vibrations when struck) and the place at which the racket's coefficient of restitution is maximal. Whereas ball contact with the center of percussion (sweet spot 1) produces the least initial shock to the hand, ball contact with the node of the first harmonic (sweet spot 2) produces the least uncomfortable vibration felt by the hand and arm, and ball contact with the place of greatest elasticity (sweet spot 3) produces the maximum speed of rebound. Typically, sweet spot 3 tends to be closest to the hand, sweet spot 2 tends to be furthest from the hand, and sweet spot 1 tends to lie in between them (Brody 1987). Highly skilled tennis players may be accustomed to exploiting all three sweet spots as the situation within a game of tennis demands. One might also expect, on first blush, that skilled tennis players are more attuned haptically than non-players to the respective locations of a racket's sweet spots and to the location, relative to the hand, of a racket's distal tip (that is the racket length).

Six tennis rackets of six different sizes were used in the experiment. We presumed that, under the instructions to perceive the place on the racket at which one would want to make contact with a ball, participants would most likely be restricted to sweet spot 1, the center of percussion, insofar as that is defined by the moments of the racket mass distribution. The other two sweet spots seem to require contact (and not simply wielding) for their determination.

### 2.1 *Method*

2.1.1 *Participants.* Seven undergraduates from the Introductory Psychology class at the University of Connecticut participated in the unskilled group in partial fulfillment of a course requirement. None of this so-called novice group were recreational tennis players. The participants in the skilled or expert group were seven undergraduate varsity tennis players plus one varsity coach at the University of Connecticut (four males and four females). Each was paid \$5 for participating (the athletes had exhausted their NCAA eligibility). All participants had normal mobility in their right arms.

2.1.2 *Materials and apparatus.* Six Wilson graphite tennis rackets were used. They included three junior models and three adult models (one standard, one long, and one stretch). All were strung with 135 kg of force. Their dimensions are provided in the first four columns of table 1.

**Table 1.** Linear dimensions, inertial characteristics, and perceived extents of rackets by novices and experts in experiment 1.

| Racket | $L$/cm | $I_1$/g cm$^2$ | $C_P$/cm | Novices | | Experts | |
|---|---|---|---|---|---|---|---|
| | | | | $L$/cm | $C_P$/cm | $L$/cm | $C_P$/cm |
| 1 | 53.3 | 213 067 | 35.9 | 43.4 | 32.5 | 49.1 | 34.9 |
| 2 | 58.4 | 255 792 | 38.3 | 43.6 | 36.1 | 51.0 | 35.4 |
| 3 | 63.5 | 302 419 | 43.2 | 50.6 | 43.5 | 56.7 | 41.0 |
| 4 | 66.0 | 326 700 | 46.4 | 54.5 | 45.8 | 61.7 | 44.4 |
| 5 | 70.7 | 374 887 | 47.2 | 56.6 | 48.3 | 63.8 | 46.8 |
| 6 | 72.4 | 393 132 | 48.0 | 58.6 | 49.3 | 64.8 | 46.0 |

The report apparatus was a wooden track supported on wooden legs at a height of 75 cm. A small wooden block could be slid along the track from 0 to 150 cm by means of a string and pulleys. On sweet-spot trials, a miniature tennis ball was affixed to the block (figure 2). An opaque curtain to the right of the track occluded the

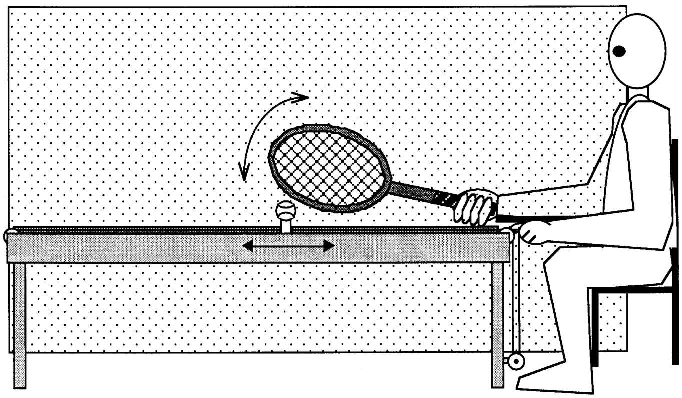

**Figure 2.** A tennis racket (occluded from the participant's view by an opaque curtain) was wielded about a rotation point in the right wrist. Participants were instructed to indicate the requested perceived extent by positioning a wooden marker (topped by a small tennis ball for sweet-spot trials) with a rope-and-pulley.

participant's view of the rackets as they were wielded. The zero point on the visible track was lined up with the wrist of the participant's occluded right arm.

2.1.3 *Procedure.* A participant was seated at a student desk with the right forearm supported out to the wrist. On a given trial, a racket was placed in the right hand with the end of the handle flush with the base of the hand. Participants were instructed to hold the racket firmly so that it did not twist in the hand. All wielding motions were about the wrist and typically occurred in all three spatial dimensions. Participants were free to wield the racket as long as needed and to adjust the report apparatus as much as needed to arrive at a confident judgment. On length trials, the position of the wooden block was adjusted so that its front face coincided with the felt position of the far end of the racket. On sweet-spot trials, the position of the tennis ball was adjusted to the location at which the participant would like to hit it with the racket held in the prescribed manner. Length and sweet-spot judgments were blocked, with their order counterbalanced over participants. Each racket was presented three times in random order within each block.

### 2.2 *Results and discussion*

Mean perceived length and perceived distance to the sweet spot are provided in table 1. A 2 (group) $\times$ 2 (property) $\times$ 6 (racket) mixed-design ANOVA was performed on the participants' mean perceived extents. The groups did not differ significantly from one another, $F < 1$ (novice, 46.9 cm; expert, 49.6 cm). The two racket properties were distinguished from one another ($F_{1,13} = 32.16$, $p < 0.0001$); in particular, perceived distance to the sweet spot (42.0 cm) was, on average, shorter than perceived length (54.5 cm). The main effect of racket ($F_{5,65} = 69.84$, $p < 0.0001$) indicates that perceived extents increased with increases in actual extents. No interactions were significant [all $F$s $\approx 1$ except Group $\times$ Property ($F_{1,13} = 3.11$, $p > 0.10$)].

Additional analyses allow a comparison of observers' reliability and accuracy in judging the two racket properties. A measure of the consistency or reliability of a participant's perceptions is provided by the average deviation, expressed as a percentage of the mean perceived extent (Norman et al 1996). A reliability measure (for each property separately) is obtained from the three judgments of each racket, with these racket measures then averaged to yield a reliability measure for each participant. The reliabilities across expert participants ranged from 3.3% to 10.6% for judgments of the distance to the sweet spot with a mean of 5.8%, and from 3.0% to 8.1% for judgments of length with a mean of 4.3%. Consider racket 3 (see table 1). For the experts, the mean perceived distance to the sweet spot for this racket was 41.0 cm and the mean perceived length was 56.7 cm. An expert's judgment on any given trial tended to vary by only 2.5 cm and 2.2 cm, respectively, from these mean perceived values. The experts were clearly very reliable in their judgments, and they were no less reliable for length than they were for the location of the sweet spot. Very much the same conclusion can be drawn with respect to the novices. Their reliabilities ranged from 3.0% to 9.8% for judgments of the distance to the sweet spot with a mean of 6.7% and from 3.0% to 19.4% for judgments of length with a mean of 7.9%. Consider racket 3 again. For the novices, the mean perceived distance to the sweet spot for this racket was 43.5 cm and the mean perceived length was 50.6 cm. A novice's judgment on any given trial tended to vary by only 2.9 cm and 4.0 cm, respectively, from these mean perceived values.

The corresponding accuracy measures are provided by the root-mean-square (RMS) errors. These reveal how much a participant's judgments of sweet spot and length varied from their actual magnitudes. The percentage RMS error for each property was calculated according to the following equation:

$$\text{RMS error}/\% = \frac{\sum\sum \dfrac{\sqrt{(\text{perceived} - \text{actual})^2}}{\text{actual}}}{\text{objects} \times \text{repetitions}} \times 100 \,,$$

where summation is over the number of rackets and the number of repetitions. The RMS errors for expert participants ranged from 3.9% to 24.8% for judgments of the distance to the sweet spot with a mean of 16.1%, and ranged from 5.0% to 38.9% for judgments of length with a mean of 15.3%. These RMS errors indicate how much reported extent varied from the correct value as a percentage of the correct value. Consider racket 3. The distance of the sweet spot from the wrist for this racket was 43.2 cm and the length was 63.5 cm. On the average, the experts varied by 6.9 cm and 9.7 cm, respectively, from these values. For the novices, the RMS error ranged from 8.4% to 29.2% for judgments of the location of the sweet spot with a mean of 13.1%, and ranged from 6.6% to 50.3% for judgments of length with a mean of 20.4%. On the average, the novices varied by 5.7 cm and 12.7 cm, respectively, from the values for the actual distance to the sweet spot and length of racket 3. Novices seem as adept as experts in perceiving the location of a racket's sweet spot and length.

It is instructive to compare reliability and accuracy. Suppose that one had to measure a well-defined length with a ruler. One source of uncertainty or error would arise from interpolating between scale markings. This error would probably be random, given that the interpolation is just as likely to result in an underestimation as it is to result in an overestimation. The reliability measure captures this random error. A contrasting systematic uncertainty or error would arise if the ruler was distorted in some way—for example, it had been stretched or shrunk or bent. A stretched ruler would always underestimate, a shrunk ruler would always overestimate, and a bent ruler might either overestimate or underestimate depending on the magnitude of the well-defined length. The RMS or accuracy measure captures this systematic error. Returning to the present length measures by dynamic touch, if a participant judged the distance to the sweet spot of a racket or the distance to its tip correctly apart from random fluctuations, then the participant's RMS measure and reliability for that racket should be equal. If, instead, the RMS measure is greater than the reliability, then it means that the judgments are systematically distorted (Norman et al 1996). A 2 (group) $\times$ 2 (property) $\times$ 2 (analysis) mixed-design ANOVA revealed that the participants were less accurate (average = 16.7%) than they were reliable (average = 6.1%) ($F_{1,13} = 25.65$, $p < 0.0002$). (For comparison, Norman et al's investigation of the visual perception of horizontal distances and distances in depth yielded average accuracy and reliability measures of 22.6% and 7.1%, respectively.) Neither the main effect of group nor the main effect of property was significant ($F$s $< 1$), and there were no significant interactions [all $F$s $\approx 1$ except Group $\times$ Property, $F_{1,13} = 1.98$, $p > 0.05$)]. In summary, there was systematic error in the perceptual measures of both sweet spot and length. This systematic error or distortion was identical for the two measures and for the two groups of participants.

The systematic error could arise from a number of sources. A possible methodological source of systematic error is the location of the zero point adopted by the individual participant for the distances at which the report marker was positioned (see figure 2). If a participant perceived his or her right hand to be forward (or backward) of the actual zero used by the experimenter, then the participant might be inclined to place the marker at distances that were systematically greater (or smaller) than the rackets' extents. A possible perceptual-system source of systematic error is the fact that the location of racket sweet spot and racket length are specified by the moments of the racket mass distribution. As noted in the introduction, length perception by dynamic touch is a power law of the eigenvalues of the inertia tensor (the units of the 'ruler' at the disposal of the participant are dimensionally $ML^2$ rather than L). A perceptually based origin of systematic distortion would have implications for the issue of racket length as an ill-posed problem and the location of racket sweet spot as a well-posed problem. The observation that the degree of systematic distortion was the same for the two properties, together with the observations that the two properties were perceived equally reliably and equally accurately, suggests that they posed the same kind of problem, not different kinds of problems, for the haptic subsystem of dynamic touch. The larger implication is that the perception of the location of a racket's sweet spot and that of a racket length share a common basis. The likely candidate is the inertia tensor.

In sum, for both the novice and expert participants it seems as if information about the location of a racket's sweet spot (its center of percussion) can be obtained simply by wielding. It also seems to be the case that wielding a racket can lead to two distinct (but equally accurate and equally reliable) perceptions of distance from the hand along the longitudinal dimension of the racket. Both groups of participants reported significantly larger values of distance to the tip of the racket than distance to the sweet spot.

## 3 Experiment 2: The sweet spot of a weighted rod

Assume a cylindrical hitting implement of length $L$ with a metal ring attached either closer to the hand or closer to the distal end of the implement, as shown in figure 3. The moment of inertia about the $x$ axis will be greater for the further ring position

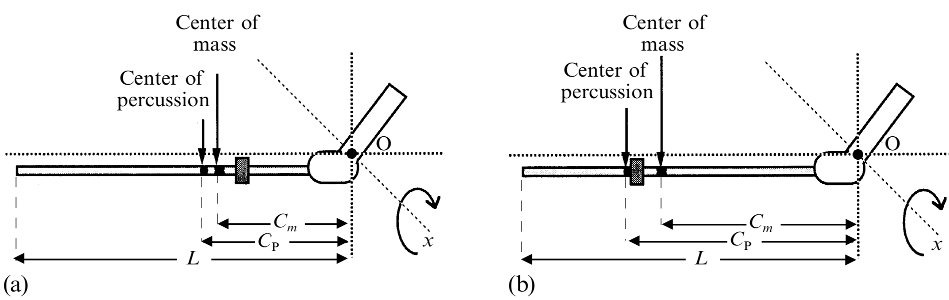


**Figure 3.** (a) A mass attached near to the hand (one-third the length of the rod measured from the rotation point O in the wrist) produces a smaller moment of inertia and a closer center of percussion than (b) a mass attached far from the hand (two-thirds the length of the rod). (Compare the locations of the center of mass and the center of percussion to their locations on the homogeneous rod of figure 1.)

as will the distance $C_P$ from the $x$ axis to the center of percussion. Given the evidence that moments of inertia constrain length perception, there should be two perceived lengths for the single $L$, with this one-to-two map converged into a one-to-one map when $I_1$ is substituted for $L$. In contrast, if the center of percussion constrains the perception of $C_P$, then the mapping from actual $C_P$ to perceived $C_P$ should be one-to-one.

The preceding expectations were evaluated in experiment 2 with participants classified into two groups in the manner of experiment 1, that is varsity tennis players made up the expert group and regular undergraduates made up the novice group. With respect to evaluating the sensitivity of participants to the two distance measures of $L$ and $C_P$, the rods plus metal rings of experiment 2 present a certain advantage over the rackets of experiment 1. Foreknowledge of the set of actual lengths in experiment 2 was minimal given that (a) the objects were unknown and not experienced visually or tactually prior to the experiment (in contrast to the frequent viewing and, perhaps, frequent handling, of objects of the kind used in experiment 1) and (b) the visible report apparatus suggested that perceived extents could range from 0 to 1.5 m.

### 3.1 *Method*

3.1.1 *Participants.* Eight undergraduates from the Introductory Psychology class at the University of Connecticut participated in the unskilled group in partial fulfillment of a course requirement. None of the novice group were recreational tennis players. The participants in the skilled group were eight undergraduate varsity tennis players at the University of Connecticut (four males and four females). Each was paid $5 for participating (the athletes had exhausted their NCAA eligibility). All participants had normal mobility in their right arm. None had participated in experiment 1.

3.1.2 *Materials and apparatus.* Rods were cut from 0.6-cm-radius pine dowels in three lengths: 44.3, 60, and 75 cm. A 60 g mass was affixed to each so that in the near condition it was ⅓$L$ from the hand and in the far condition it was ⅔$L$ from the hand. The inertial characteristics are provided in the first four columns of table 2. The same pulley apparatus was used to report perceived extents. As in experiment 1, a small tennis ball was attached to the visible marker for sweet-spot trials.

**Table 2.** Linear dimensions, inertial characteristics, and perceived extents of rods by novices and experts in experiment 2.

| Mass position | $L$/cm | $I_1$/g cm$^2$ | $C_P$/cm | Novices | | Experts | |
|---|---|---|---|---|---|---|---|
| | | | | $L$/cm | $C_P$/cm | $L$/cm | $C_P$/cm |
| Near | 44.3 | 74 182 | 20.1 | 33.6 | 22.4 | 37.0 | 22.9 |
| | 60.0 | 144 624 | 26.9 | 37.6 | 27.0 | 40.6 | 27.0 |
| | 75.0 | 230 747 | 33.3 | 49.6 | 36.2 | 48.1 | 33.1 |
| Far | 44.3 | 205 015 | 30.8 | 48.6 | 34.1 | 44.1 | 31.9 |
| | 60.0 | 384 624 | 41.0 | 60.9 | 47.6 | 53.6 | 39.8 |
| | 75.0 | 605 747 | 50.8 | 70.2 | 54.7 | 58.7 | 48.1 |

3.1.3 *Procedure.* The procedure closely followed that of experiment 1. On a given trial, a rod was placed in the right hand so that the appropriate end (attached mass near or far) was flush with the base of the hand. As in experiment 1, length and sweet-spot judgments were blocked with their order counterbalanced over participants. Each rod was presented three times in random order within each block.

### 3.2 *Results and discussion*

Mean perceived length and distance to the sweet spot for the two groups are provided in table 2. Inspection of table 2 suggests that the perception of length, $L$, for a given rod depended on the position of the attached metal ring and that, for both novices and experts, the perception of $C_P$ closely tracked the position of the sweet spot. A 2 (group) $\times$ 2 (property) $\times$ 2 (rod length) $\times$ 2 (mass location) mixed-design ANOVA was performed on the mean perceived extents. As in experiment 1, the groups did not differ significantly from one another [$F < 1$ (novice, 43.5 cm; expert, 40.4 cm)]. The two rod properties were distinguished from one another ($F_{1,14} = 28.40$, $p < 0.0001$), again revealing perceived distance to the sweet spot (35.4 cm) to be shorter than perceived length (48.5 cm). The main effect of rod length ($F_{2,28} = 109.64$, $p < 0.0001$) indicates that perceived extents increased with increases in actual extents (rods of 44.3, 60, and 75 cm averaged 34.3, 41.8, and 49.8 cm, respectively). The main effect of mass location ($F_{1,14} = 81.13$, $p < 0.0001$) reflects longer perceived extents for the farther mass attachments (near averaged 34.6 cm, far averaged 49.4 cm). The interaction of mass attachment and length ($F_{2,28} = 20.99$, $p < 0.0001$) indicates the contribution of moments of the mass distribution. Two marginal interactions involving group—that with mass location ($F_{1,14} = 4.56$, $p = 0.05$) and that with length ($F_{2,28} = 2.79$, $p < 0.08$)—suggest that novice participants were somewhat more sensitive to the inertial manipulations. No other interactions were significant [all $F$s $\approx 1$ except Group $\times$ Property $\times$ Mass location ($F_{1,14} = 1.93$, $p > 0.15$)].

Reliability and RMS error were calculated as in experiment 1 (with averages taken over the six rod configurations). For the experts, the reliabilities ranged from 3.6% to 16.4% for judgments of the distance to the sweet spot with a mean of 8.3%, and from 5.7% to 16.8% for judgments of length with a mean of 9.5%. For the novices, the reliabilities ranged from 6.6% to 17.1% for judgments of the distance to the sweet spot with a mean of 11.0%, and from 4.5% to 22.8% for judgments of length with a mean of 9.5%. With respect to RMS errors, experts ranged from 12.2% to 83.9% for judgments of the distance to the sweet spot with a mean of 28.8%, and from 15.8% to 70.1% for judgments of length with a mean of 39.0%. The highest RMS values of 83.9% for the distance to the sweet spot and 70.1% for length were from the same expert participant who overestimated dramatically but reliably (5.5% and 7.7%, respectively). The RMS errors for the novices ranged from 18.8% to 69.5% for judgments of the distance to the sweet spot with a mean of 34.1%, and from 21.3% to 43.8% for judgments of length with a mean of 31.7%. It seems that the two properties were reported with equivalent accuracy and reliability and that the novices were no different in these respects from the experts. A 2 (group) $\times$ 2 (property) $\times$ 2 (analysis) mixed-design ANOVA confirmed these impressions. Neither the main effect of property nor the main effect of group was significant ($F$s $< 1$), and there were no significant interactions [all $F$s $\approx 1$ except Group $\times$ Property ($F_{1,14} = 2.51$, $p > 0.10$)]. As in experiment 1, there was a systematic distortion: accuracy (average = 33.4%) did not match reliability (average = 9.6%) ($F_{1,14} = 30.48$, $p < 0.0001$).

Simple regressions assessed the dependence of perception on geometric and mechanical properties of the rod configurations. As expected, perception of the distance to the sweet spot was a single-valued function of the actual position of the center of percussion (figure 4a). In contrast, the perception of length yielded two values (corresponding to the near and far placements of the attached mass) for each value of actual length (figure 4b). Perceived length was, instead, dependent, as expected, on $I_1$ (figure 4c). The seemingly straightforward dependence of perceived $C_P$ on actual $C_P$ is illustrated by the common function that is obtained for that property for the two very different implements of experiments 1 and 2 (figure 5). Although the expert group shows a somewhat tighter fit, the shared range in which the two groups of participants operate is noteworthy.

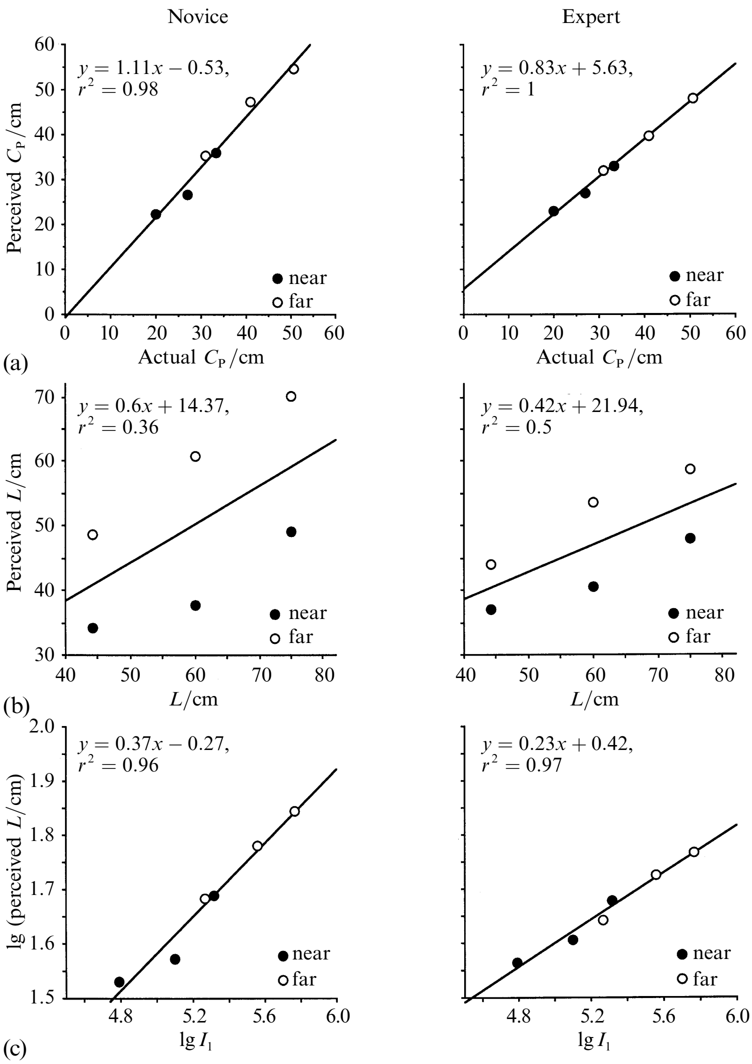


**Figure 4.** (a) Perceived distance of the sweet spot is a single-valued function of actual distance of the sweet spot for both groups of participants. (b) Perceived length is not a single-valued function of actual length of rods with attached masses. (c) Perceived length is a single-valued function of $I_1$ (in log – log coordinates).

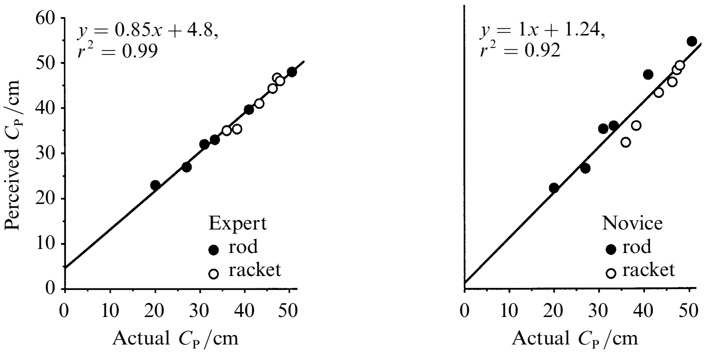


**Figure 5.** Combined data from experiments 1 and 2 for expert (left) and novice (right) participants.

## 4 General discussion

The nonvisible perception of the striking implement is an important but relatively unexamined aspect of sports involving rackets, bats, and so on. The necessity of keeping the eye on the ball means that haptic perception must coordinate with visual perception in aligning the center of percussion of the racket [or one of the other sweet spots, Brody (1987)] with the ball at the time of contact. The experiments reported in this article suggest that the center of percussion of a tennis racket is perceptible by the haptic subsystem of dynamic touch during the course of the stroke (and, therefore, prior to the moment of contact with the ball). The experiments also suggest that such perception may be a basic capability of dynamic touch that does not require specialized experiences (such as playing tennis regularly) for its realization; the ordinary everyday wielding, hefting, and swinging of objects may be sufficient to attune the haptic subsystem to the information about center of percussion. In general, experienced athletes are more accurate than novices in responding to the variables of special relevance to a given skill (Abernathy 1990a, 1990b, 1994). The results of Oudejans et al (in press) suggest, however, that for a variable of fundamental significance to basic actions (such as the vertical optical acceleration of a projectile that specifies the direction of locomotion for its interception), novices may not differ from experts in sensitivity; they differ only in the ability to gear the appropriate behavior to the variable. The present research on perceiving the location of the sweet spot was limited to the issue of sensitivity. It seems relatively apparent that experts would be superior in their ability to organize their hitting behavior in relation to the felt location of the sweet spot.

What might be the information about center of percussion? For objects held and wielded freely in three dimensions, the weight of the evidence points to the inertia tensor as the basis for the nonvisible perceptions one has of the object's spatial dimensions and of the hand's relation to them (Turvey 1996; Turvey and Carello 1995). There are indications that the first moment can also be registered, particularly under conditions of minimal wielding when the movement of the hand-held object relative to the wrist and other joints is limited to tremor (eg Burton and Turvey 1990). Because the position of the center of percussion, $C_{\mathrm{P}}$, is given by the ratio of second to first moment about the relevant axis, it is tempting to think that dynamic touch is sensitive to the muscle/tendon stress/strain patterns induced by the relation of these two moments. The implication of the latter is that dynamic touch registers a variable of even higher order, namely a ratio of second and first moments. Against this conclusion, however, stands the evidence from the reliability and accuracy measures that the perception of the position of the center of percussion, $C_{\mathrm{P}}$, in the present experiments was no different in these respects than the perception of length, $L$. Perceiving the linear dimension of a hand-held object is realized as power functions of the eigenvalues of the inertia tensor (eg Fitzpatrick et al 1994; Solomon and Turvey 1988; Turvey et al 1998a) and perceiving fractions of these linear dimensions (the fractionation defined by hand position) is similarly a matter of power functions but now involving the eigenvector directions as well as the eigenvalues (eg Pagano et al 1996; Turvey et al 1996, 1998b). The possibility must be entertained, therefore, that perceiving the location of the sweet spot is similarly a matter of a power function dependence on the eigenvalues of the inertia tensor. Figure 6 shows the power function dependences of perceived distances of the sweet spot and perceived rod length on the major eigenvalue for the expert and novice participants in experiment 2. The implication is that the difference may be only in the coefficient or measure constant (the antilog of the intercept in log – log coordinates).

In sum, the present research adds to the growing appreciation of the ability of dynamic touch to contribute to the perceptual control of manual actions. The major lesson here is that an object property of relevance to bat – ball skills, namely, the place on the implement at which contact should be made, is perceptible during movement

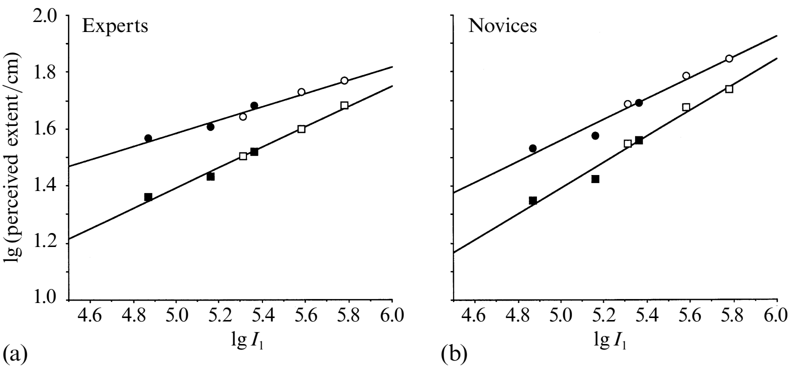


**Figure 6.** Perceived rod length (circles) and perceived sweet spot (squares) are distinct functions of moment of inertia for (a) experts and (b) novices. Filled symbols represent near mass placement; open symbols represent far mass placement.

prior to contact. The minor lesson, but one that calls out for further examination, is that the nonvisible perception of the location of an object's sweet spot and its length may be similarly based in the inertia tensor, distinguished only by the measure constants of the governing power laws.

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## APPENDIX

The first and second moments of a mass distribution quantify tendencies to produce (or resist) motion about an axis. The motion is, therefore, rotational. Consider first the relevance of rotations for the manipulation of hand-held objects. When an object is grasped firmly in the hand, the only way it can be moved is by rotations about the joints of the arm. For the case of a hammer swung just at the wrist, the rotational nature of the movement is fairly obvious. But even for a multi-joint movement such as a tennis ground stroke that serves to translate the racket head, that translation is brought about by cascaded rotations about the wrist, elbow, and shoulder [Carello and Turvey (in press)]. Each joint is a center of rotation. For simplicity, let us focus just on the wrist. Depending on how an object is constructed, it will be more or less difficult to rotate about that joint. In particular, the way in which the mass, $m$, of an object is distributed affects its resistance to rotational acceleration about a point. Imagine a horizontal rod spinning around a vertical axis through its center of mass. It spins easily about the axis, although a longer rod resists rotation more than a shorter rod, and a heavier rod resists rotation more than a lighter rod. If the axis were moved off-center, the resistance of a given rod to rotation would be greater—not because the mass changed but because the way in which the mass is distributed about the rotation point changed.

Simply holding an object still requires resisting the pull of gravity twisting the object in the hand; this torque is proportional to the first moment of the object mass distribution. In its simplest form, the *static moment* is given by the product of an object's mass and the distance $C_m$ of the object's center of mass from the wrist. Setting the object in motion requires overcoming the resistance to rotational acceleration; this is quantified by the second moment of the mass distribution. In its simplest form, the *moment of inertia* is given by the product of the object's mass times the squared distance of the object's center of mass from the wrist. Of course, objects are often asymmetric in their mass distribution—for example, both a hammer and a tennis racket have a head at one end. In addition, rotations occur about more than just one axis. The *inertia tensor*, $I_{ij}$, is a $3 \times 3$ matrix that quantifies the different resistances to being rotated about the $x$, $y$, and $z$ axes (the moments of inertia) as well as in directions orthogonal to those axes (essentially, the tugs off those axes brought about by the aforementioned asymmetries, the products of inertia). These quantifications are labeled in a way that indicates the direction of the resistance relative to the elected coordinate system: $I_{xx}$, $I_{yy}$, and $I_{zz}$ are the components on the diagonal of the matrix; $I_{xy}$, $I_{xz}$, and $I_{yz}$ are the products above the diagonal; and $I_{yx}$, $I_{zx}$, and $I_{zy}$ are the products below the diagonal (because this is a symmetric matrix, corresponding components above and below the diagonal are equivalent).

Intuitively, the $x$, $y$, and $z$ directions can be interpreted as the spatial axes of the environment (as they are depicted in figures 1 and 3). But they could also legitimately be designated as the spatial axes of the forearm. Indeed, the orientation of the axes is arbitrary—an infinite number of sets could be anchored in the wrist and they would be as mathematically legitimate as the intuitive sets. There is one set of axes that is nonarbitrary, however, and that is the set that comprises the symmetry axes of the object relative to the rotation point. All other axis choices can be related to the symmetry axes by a 3-D rotation that finds the orientation about which the mass is evenly distributed. Because this transformation eliminates the off-diagonal components of the matrix (ie the products of inertia), it is referred to as diagonalization. The orientation of the new tensor is given by its principal directions, or *eigenvectors*, and the magnitudes of the resistances are given by the principal moments, or *eigenvalues*. The resulting *diagonalized* tensor, $I_k$, is characterized by the maximum eigenvalue $I_1$, the minimum eigenvalue $I_3$, and the intermediate eigenvalue $I_2$, along with the orientations of the three eigenvectors, $\boldsymbol{e}_k$. For the most part, given the typical orientation of an object in the hand relative to a common convention for labeling axes, $I_{xx}$ in the coordinate-system-dependent tensor is equivalent to $I_1$ in the diagonalized form.