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COMPARISONS OF DIGITS AND DOT PATTERNS [1]

PAUL B. BUCKLEY AND CLIFFORD B. GILLMAN [2]

University of Wisconsin—Madison

The process of numeric comparison was investigated. Four groups of 10 *S*s were asked to judge which of two digits or which of two dot patterns was numerically larger. Stimuli were either digits or dot patterns in familiar, unfamiliar, or random configurations. Mean reaction time was systematically related to the difference between logarithms of the stimulus values. A single numeric comparison process gave good account of the data for all stimulus types. This process was well described by a random walk model with variable step size and fixed boundaries. Reaction time matrices were further analyzed using Kruskal's 1964 multidimensional scaling program MD-SCAL, and the recovered stimulus configurations were successfully simulated from a simple version of the model.

Moyer and Landauer (1967) presented two single digits simultaneously and asked *S*s to respond as rapidly as possible to the digit that was numerically larger. Reaction time for this judgment was reported to be a monotonic decreasing function of the difference between digits. This "Moyer–Landauer" effect has been replicated by Fairbank (1969), Parkman (1971), and Sekuler, Rubin, and Armstrong (1971). In addition, Parkman demonstrated that reaction time is a monotonic increasing function of the value of the smaller digit (the Min effect). However, after the Min effect is removed by partial correlation, reaction time was still found to be a monotonic decreasing function of the difference between digits. Moyer and Landauer (1973) reanalyzed Parkman's data and found further support for two separate effects.

Several studies indicate that reaction time to identify a single digit is uniformly distributed across the digits. North, Grant, and Fleming (1967) collected simple stimulus identification reaction times to single digits and found a significant difference among these times. However, this same pattern of differences was replicated when *S*s identified digit names, so that these differences may have been artifacts of response pronunciation and voice key transduction. To test this hypothesis, Fairbank (1969) asked *S*s to press a key when a given digit appeared, but not otherwise. Key press reaction times were not significantly different for the various digits. Theios (1973), in a similar task, found a flat function for reaction time to name single digits. These results imply that the Moyer–Landauer effect and the Min effect are properties of some higher process than simple stimulus identification. We propose that the appropriate process is the comparison of already encoded numeric values.

The present study was designed to ascertain what information is encoded and compared by *S*s in the digit comparison task. If *S*s are comparing strictly quantitative numerical information, one should observe similar results when *S*s are asked to judge two digits or two dot patterns. Alternatively, digits may be compared differently from dot patterns. As stimuli, digits and dot patterns are very different from each other. Digits are highly overlearned, familiar stimuli, as evidenced by the flat reaction time functions for digit identification. In contrast, reaction time for magnitude estimation of random dot patterns was found to be a monotonic increasing function of stimulus magnitude (Kaufmann, Lord, Reese, & Volkmann,

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[2] Requests for reprints should be sent to Clifford B. Gillman, Waisman Center on Mental Retardation and Human Development, University of Wisconsin, 2605 Marsh Lane, Madison, Wisconsin 53706.

1949). A difference in stimulus familiarity may be responsible for these results.

To test the above hypotheses, one group of *S*s was presented with all possible nonequal pairs of single digits (1–9), as was done by Moyer and Landauer (1967), Fairbank (1969) and Parkman (1971). Other groups of *S*s judged simultaneously presented pairs of dot patterns.

To vary familiarity among dot patterns, three classes of dot patterns, each containing one–nine dots, were generated. For familiar dot patterns, the configurations of symbols found on the faces of playing cards were used. These patterns tend to be bisymmetric or regular in form. Completely unfamiliar dot patterns are those which have random configuration features. For patterns of intermediate familiarity, a single irregular dot pattern was generated for each of the nine stimuli and was used consistently throughout the experiment. The three stimulus sets are referred to as *regular, random,* and *irregular,* respectively.

Method

Subjects. Forty volunteers from introductory psychology classes at the University of Wisconsin served as *S*s in this experiment. The *S*s were randomly assigned to one of four stimulus groups.

Apparatus. Stimuli were generated as combinations of points in a 5×7 grid by a PDP-8 computer, and were presented in pairs on a Tektronix RM503 oscilloscope. The *S*s were placed so that each 5×7 grid subtended $54' \times 27'$ of visual angle, and the pair was separated by 1°.

Procedure. Each *S* was seated in a semidarkened room in front of the oscilloscope. The *S* was instructed to press one of two buttons to indicate whether the left or right stimulus of each pair was numerically larger. Stimuli remained on display until *S* responded. Whenever an error was made, the stimulus pair and the word *error* were displayed until *S* acknowledged it by pressing a response button. This stimulus pair was then randomly reinserted among those pairs still to be presented. Thus each block of trials consisted of the 72 trials necessary to obtain one correct response to each of the 72 possible pairs, plus any trials on which an error was made.

For the dot pattern stimulus conditions, *S*s were given pretraining to become familiar with the stimuli. Before each block, each dot pattern and a digit corresponding to the number of dots in the pattern were displayed on the oscilloscope. Pretraining continued until *S* indicated that he was comfortable in dealing with each stimulus. For *regular* and *irregular* patterns, *S* was told that he would be seeing the same patterns throughout the study. For *random* patterns, *S*s were warned that the pattern configurations changed from trial to trial, and were instructed to make note of this during identification practice.

Each *S* received 11 blocks of trials. The response–stimulus interval was 512 msec. Each block was separated by a rest interval of at least 1 min., terminated by *S*.

Results

The first block of trials was discarded as practice, and only the last 10 blocks were analyzed. Thus each *S* made 10 correct responses to each pair of stimulus values. Since there were 10 *Ss* per group, each mean reaction time is based on 100 observations. Error rate was less than 3% and was positively correlated with correct time ($r = .68$, $p < .001$); there were too few errors for more systematic analysis. Figure 1 plots mean reaction time for correct judgments against the difference between stimulus values. The magnitude of the reaction time effect for digits was comparable to that found by Moyer and Landauer (1967) and Parkman (1971). For digits that differ by one, a correct response required approximately 100 msec. longer than for correct response to digits that differ by eight. Regular patterns of dots required more time for comparative judgment, but their reaction times also decreased monotonically with the difference between stimuli, as did *irregular* dot patterns and *random* dot patterns. The form of all functions was quite similar. Surprisingly, *irregular* dot patterns required more time for judgment than *random* dot patterns. This may reflect *S*'s attempts to learn labels for the recurring *irregular* patterns with indifferent success.

In Figure 2 is plotted Parkman's (1971) Min effect as found in these data. Again, a systematic effect was present in all four groups of *S*s. Although there were significant differences among stimulus types, F (3, 36) $= 6.19$, $p < .005$, again the form of the function was similar for each group.

The significant differences among stimulus types may be attributed to differential stimulus encoding. The similarity of the Moyer–Landauer and Min effects for all stimulus conditions is evidence for a single underlying comparison process. To test this hypothesis of a single comparison process, the differential contribution of stimulus condition to reaction time was partialed out and only the ordinal properties of the data were examined, using the nonmetric multidimensional scaling program MD-SCAL, version 5MS (Kruskal, 1964a, 1964b). It was assumed that longer latencies to correctly judge the larger of two stimuli reflect the fact that two stimuli are subjectively similar or close together. The MD-SCAL program provides the best monotone fit of recovered interpoint distances to observed reaction times. If a single comparison process was used in all stimulus conditions, one should recover the same stimulus configuration for each stimulus type. Although it may seem that a single dimension, numeric value, should be sufficient to represent these stimuli in Euclidean space, this was not the case. Two-dimensional representations of the four stimulus types gave the best account of the data. The recovered stimulus configurations are presented in Figure 3. The three types of dot patterns yielded very similar recovered configurations. Each stress curve (Kruskal's Stress Formula I) contained a marked break and acceptably small stress (.04–.07) at two dimensions. Interpoint distances between successive stimulus values decreased as stimulus value increased from one–nine dots. The stress curve for digits contained no "elbow" at any dimensionality. However, all two-dimensional configurations were very similar. In the digit representation, only

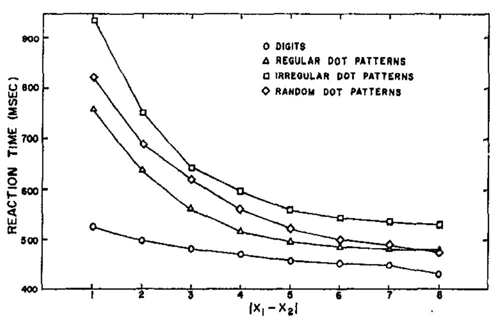

FIGURE 1. Mean response time for comparative judgment of digits and dot patterns as a function of the absolute difference in stimulus magnitudes.

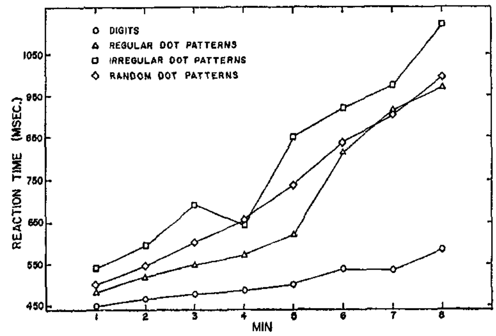

FIGURE 2. Mean response time for comparative judgment of digits and dot patterns as a function of the magnitude of the smaller stimulus.

the points corresponding to the Digits 1 and 2 deviated markedly from the corresponding scale values for dot patterns. Despite this anomaly, the recovered configuration of the Digits 4–9 corresponded well to that for the dot patterns. Thus, at least for these values, multidimensional scaling provided support for the hypothesized single underlying comparison process. (See Figure 4.)

DISCUSSION

That there should be a single comparison process for both digits and dot patterns is not intuitive. Digits are symbolic and must be translated to numeric representation for quantitative comparison. On the other hand, dot patterns may be successfully compared on the basis of stimulus properties such as display size, relative brightness, density, etc. However, reaction time data in this study furnish no evidence that any of these differential stimulus characteristics were used for comparison of dot patterns. Each *S* saw only a single type of stimulus to avoid inducing one judgmental strategy for all stimuli, yet the data indicate a similar comparison process was employed in all cases.

The simple Moyer–Landauer model and Parkman Min model may be rejected on the basis of the obtained multidimensional scaling configurations. The Moyer–Landauer model requires reaction time to be a monotonic function of the arithmetic difference between stimulus values and predicts a unidimensional solution with equal interpoint distances when reaction times are scaled. This prediction was not confirmed. A logarithmic transformation of the arithmetic differences (Moyer, 1973) does not change the predicted unidimensional solution. The Min effect predicts reaction time to

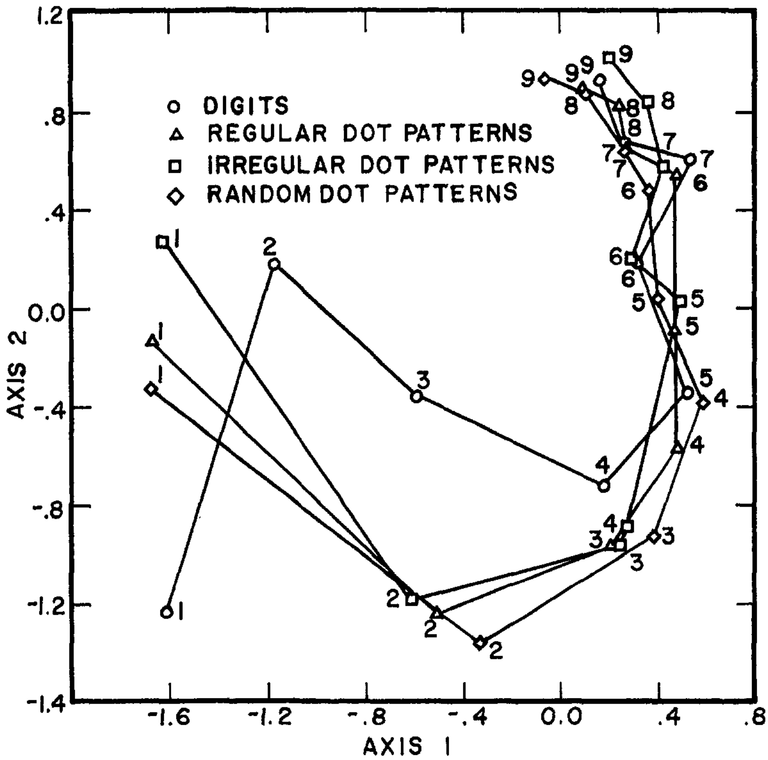

FIGURE 3. Stimulus configurations recovered by MD-SCAL analysis of response times.

be a linear function of the smaller stimulus value and consequently decreasing interpoint distances. However, simulation of this model required eight dimensions to recover nine different scale values for the stimuli. In fewer dimensions, scale values for several stimuli were indistinguishable. Thus, neither model can account for the obtained two-dimensional scaling.

Crossman (1956) asked his *S*s to sort cards according to the number of spots on each card and found that reaction time was linearly related to the reciprocal of the difference between the logarithms of the stimulus values. This relationship between reaction time and stimulus value is similar in form to the Min and Moyer-Landauer effects, but is not functionally related to them. The proportion of variance accounted for in these data by the difference between logarithms was larger than that for the Min or Moyer–Landauer effect. Furthermore, the difference between logarithms is readily interpretable and lends itself to a simple model.

Five assumptions are made. (*a*) The transformation from external stimulus to internal representation is logarithmic. Thurstone (1929) and Parducci (1963) have found evidence for the logarithmic transformation of dot patterns. (*b*) The internal representation is representable by a random variable, and the distributions of random variables representing two simultaneously presented stimuli may overlap. (*c*) The *S* computes the difference between the two random variables and adds this to a counter. The initial value of this counter reflects response bias. Since the difference between the random variables may be positive or negative, the counter increases or decreases in value. (*d*) The *S* has preestablished upper (positive) and lower (negative) boundaries, such that if the count exceeds either of these boundaries, the corresponding response is made. The values of these boundaries are fixed and independent of stimulus value. (*e*) If after an increment or decrement the value in the counter does not exceed either boundary, *S* is assumed to sample new stimulus information, compute a new difference, add this value to the counter, and again compare the counter value to the boundaries. Thus *S* resamples successively until a boundary is reached and a response can be made. This sequential process is completely described as a random walk with variable step size (Wald, 1947). The larger the difference between the logarithms of the stimulus values, the larger the expected value of the difference between internal representations, and the sooner S exceeds a boundary. The decision boundaries of the random walk model may be moved in response to experimental demand for speed and accuracy. By decreasing the absolute values of the boundaries, less information is required for a decision. Consequently, on the average, fewer samples are required for decision and mean latency is short. However, accuracy is sacrificed for speed. Similarly, larger absolute values of the boundaries increase accuracy by decreasing the probability of a false response at the expense of reaction time.

A simple version of this model assumes that internal representations have no variability and are exactly equal to the logarithm of the

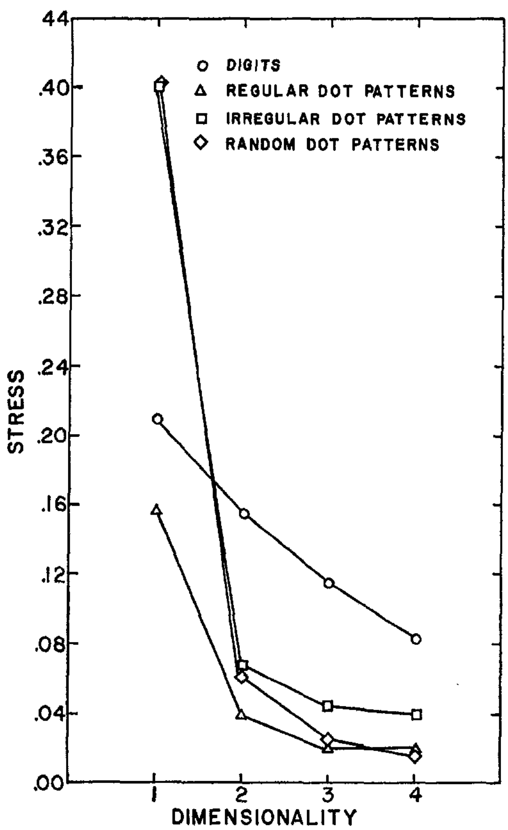

FIGURE 4. Stress for MD-SCAL solutions as a function of configuration dimensionality.

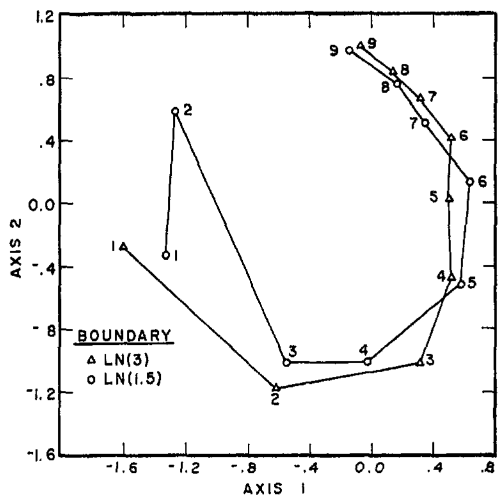

FIGURE 5. Recovered MD-SCAL stimulus configurations obtained by scaling data simulated from the random walk model.

stimulus value. This error-free version of the model was used to simulate reaction time data matrices for ideal Ss using symmetric boundary values. These simulated data matrices were subjected to MD-SCAL. For two representative boundary values, the recovered configurations are presented in Figure 5. For simulated boundary values of $\pm$log 3, the obtained configuration was very similar to that for the dot pattern data. For simulated boundary values of $\pm$log 1.5, the configuration contained anomalous scale values for Stimuli 1 and 2, comparable to those observed for digits. Boundary values above $\pm$log 3 approach a unidimensional logarithmic scale, while those boundaries smaller than $\pm$log 1.5 yield solutions of increasing dimensionality. For zero boundary (log 1), of course, the model predicts all response times to be equal. The similarity of the configurations in Figure 5 and Figure 3 is striking, and provides evidence that a single random walk comparison process can account for the data from both digits and dot patterns. Furthermore, the smaller boundary value for digits was quite reasonable and interpretable. Since digits are more discriminable from each other than are dot patterns, the corresponding internal representations for digits are less variable. For a response with equivalent confidence, S required less information for digits than for dot patterns, and a lower boundary value was sufficient.

A feature of the random walk model accounts for the Min effect as a direct consequence. According to this model, the more disparate the means of the internal representations, the greater is the probability of exceeding a boundary on a given step. This probability decreases directly as the Min value increases. Thus, reaction times are an increasing function of the Min. The Moyer–Landauer effect may be similarly explained. This random walk model accounts for all data collected to date. To the extent that it describes comparisons of both digits and dot patterns, this simple model is proposed for the process of numeric comparison.

REFERENCES

Crossman, E. R. F. W. The measurement of discriminability. *Quarterly Journal of Experimental Psychology,* 1956, **7**, 176–195.

Fairbank, B. A. Experiments on the temporal aspects of number perception. Unpublished doctoral dissertation, University of Arizona, 1969.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. The discrimination of visual number. *American Journal of Psychology,* 1949, **62**, 498–525.

Kruskal, J. B. Multidimensional scaling by optimizing goodness of fit to a non-metric hypothesis. *Psychometrika,* 1964, **29**, 1–27. (a)

Kruskal, J. B. Non-metric multidimensional scaling: A numerical method. *Psychometrika,* 1964, **29**, 115–129. (b)

Moyer, R. S. Comparing objects in memory: Evidence suggesting an internal psychophysics. *Perception & Psychophysics,* 1973, **13**, 180–184.

Moyer, R. S., & Landauer, T. K. The time required for judgments of numerical inequality. *Nature,* 1967, **215**, 1519–1520.

Moyer, R. S., & Landauer, T. K. Determinants of reaction time for digit inequality judgments. *Bulletin of the Psychonomic Society,* 1973, **1**, 167–168.

North, J. A., Grant, D. A., & Fleming, R. A. Choice reaction time to single digits, spelled numbers, "right" and "wrong" arithmetic problems and short sentences. *Quarterly Journal of Experimental Psychology,* 1967, **19**, 73–77.

Parducci, A. Range-frequency compromise in judgment. *Psychological Monographs,* 1963, **77** (2, Whole No. 565).

Parkman, J. M. Temporal aspects of digit and letter inequality judgments. *Journal of Experimental Psychology,* 1971, **91**, 191–205.

Sekuler, R., Rubin, E., & Armstrong, R. Processing numerical information: A choice time analysis. *Journal of Experimental Psychology,* 1971, **90**, 75–80.

Theios, J. Reaction time measurements in the study of memory processes: Theory and data. In G. H. Bower (Ed.), *The psychology of learning and motivation: Advances in research and theory.* Vol. 7. New York: Academic Press, 1973.

Thurstone, L. L. Fechner's Law and the method of equal-appearing intervals. *Journal of Experimental Psychology,* 1929, **12**, 214–224.

Wald, A. *Sequential Analysis.* New York: Wiley, 1947.

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