VISUAL PERCEPTION AS INVARIANCE

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The railroad tracks stretch straight and far away from me over the desert and on to the horizon. I stand squarely between them, looking along them to the horizon, and I observe both that they converge as distance gets greater and also that they are at every distance equidistant. This is the perceptual paradox of converging parallels. Every one has the experience, and Blumenfeld (1, pp. 323–346) found it under the controlled conditions of his alley experiment.[1]

The convergence, when you have regard to it, is irresistible, and it is much less than the convergence of the retinal image that underlies the perception. That image might converge as much as the legs of an isosceles triangle that is almost equilateral. The base of the triangle, the tracks at your right and left, would be at the top of your inverted retinal image, and the lines of the image for the tracts would come together quickly, meeting at the fovea, on which would be the image of the vanishing point at the horizon. It is thus plain that the perceptual pattern is not the pattern of the retinal image nor any form topographically equivalent to it.

It seems improbable, furthermore, that anyone ever observes the convergence and the equidistance of the tracks simultaneously. You can see the pattern one way or the other at will, according to which question you ask yourself about the perception. There must, therefore, be two *Aufgaben,* two attitudes, one for each of these observations. Certainly it is not safe, without further inquiry, to say that one observation is more primary than the other, or more immediate (quicker), or less inferential. Presently we must relate these two attitudes to dangerous concepts like *seeing* and *knowing*, but for the moment they remain merely two landmarks in a paradigm: (1) the perceived convergence and (2) the perceived equidistance.

The phenomena of perceived size with distance variant also furnish us with another paradigm. For free binocular vision, with enough of the normal clues to the perception of distance available to the observer, the rule holds that perceived size stays constant when distance changes, that is to say, the perceived size of an object is invariant under the transformation of the object's distance from the observer, while retinal size is, of course, variant under this transformation. If the available clues are, however, reduced enough, then the perceived size comes to depend more and more on retinal size and less and less on object size when distance is varied. With complete reduction, with complete elimination of the clues to distance, retinal size (visual angle) becomes the determiner, and object size can vary with distance without af-

[1] It is to my colleague, Dr. S. S. Stevens (12), that I owe the thought that the concept of invariance can be given as much importance in psychology and biology as it has in mathematics and physics. He has criticized and improved this paper which now goes to the editor in its fourth draft. The notion that a stimulus is not something given in research but something to be discovered by research I did not get from Stevens but rather, thirty years ago, from John Dewey's famous reflex-arc discussion in 1896 (5). The motive for my paper was, of course, furnished by Gibson (3, 4).

fecting perceived size as long as retinal size stays invariant (6, 11).

These relationships hold up to about one hundred yards. What happens, we may ask, at greater distances, at 500 yards, or to the perceived six-foot man half a mile away across the valley? Common sense says that this man looks like an ant. Is it a six-foot ant that he looks like or a little one? Gibson's experiments assert that size-constancy does not fail at great distances (4, pp. 174–186). He showed that a six-foot pole half a mile away, with the intervening terrain clearly visible, is equated in perception to the six-foot pole close at hand, and I say, as Gibson did not, that the pole looks just as big *although* it looks smaller. You can judge it either way, for the paradox is comparable to the dilemma of the railroad tracks.

IMMEDIACY AND INFERENCE

The first question that arises about these paradoxes is whether there may not be two systems, two kinds of experience which occur with different points of view, that is to say, with different observational attitudes. The one system would include the converging tracks and the tiny man in the distance, the other would show size constancy. Titchener was always saying that the sciences observe the same experience but from different points of view (13, pp. 133–143, 259–266). Why may not an attitudinal difference in observation serve us here? Let us see. We shall need names for any such two kinds of experience, and a difficulty arises because every familiar term that is applied seems to prejudice the final outcome. We can, however, reduce this bias to a minimum by calling the system that includes the converging tracks and the little man the *System R* and the one that shows the size of perceived objects invariant with distance the *System O*. If I confess now that *R* seems to me to have something to do with *Reduction* and *O* something to do with *Objects*, you will see that I am begging the question, but not very much. You are still free to give other meanings to my symbols.

The first difference that suggests itself is the possible distinction between *immediate* and *inferential*, but this differentiation at once runs afoul of psychology's classical debate about the nature of experience. Wundt and Titchener would have said that sensations, contents, existential processes are immediately given, that objects, knowledge and meanings are secondary and derived from these givens. You get the givens immediately by description (*Beschreibung, cognitio rei*) and the derived entities mediately by inference (*Kundgabe, cognitio circa rem*). Titchener might have added that for the first you need *cues*, but for the second *clues*. Let us call this view the Leipzig view: Objects are made of contents.

The Gestalt psychologists, however, take exactly the opposite view. For them objects are found in immediate experience, whereas the sensations, contents and existential processes are psychologists' constructs, derived by inference and abstraction from direct experience. The immediately given are called phenomena, not contents, and phenomena are objective in their very essence. Köhler, distinguishing between value and fact, complained that the introspectionists limit themselves to the use of "concepts which have acquired a certain polish in the history of scientific thought, and," he added, "they think little of topics to which these concepts cannot be directly applied" (9, p. vii). Experience, the Gestaltists hold, is organized into objects from the first instant of its availability. Let us call this view the Berlin view: Contents are extracted from objects.

Now let us contrast the Leipzig with the Berlin view in respect of perceived size with distance variant. Leipzig says that you can see that the distant stick is smaller than the near but that you know it is just as big. Berlin says you can see that it is just as big but that you know it ought to look smaller. There is, however, an eclectic view, in which it appears that the immediate datum sometimes corresponds with the object, sometimes with the reduced sensory core of the perception, and is sometimes intermediate. Given enough clues to distance, size constancy ordinarily holds for an object placed at different distances within a couple of hundred feet of the observer; yet it may well be, as Gibson suggests, that a skilled artist can "see" or at least infer the size that he should give the object in a drawing on paper, a size that corresponds, of course, to the size of his retinal image and not to the actual constant size of the object. Conversely, an observer may see a distant man as quite small but infer that the fellow must nevertheless be a six-footer. Gibson does not say whether his stick, a half mile away, looked to his observers as small as it looks to him who observes the photograph of it in Gibson's book (4, pp. 184f.), whether his observers then inferred that, small but distant, it must match a six-foot pole nearby, or whether, on the other hand, they made their judgments immediately and with assurance. Certainly they may have done so. Even on the Leipzig view the perception of an object—the perception that Titchener called the "stimulus-error"—is often easy and quick (2, pp. 460–470).

A still better example for showing the need to compromise between the two extreme views lies in the perception of the size of the full moon's disk. The moon, 240,000 miles away, subtends a visual angle of about 0.5 degree, but the disk of light 12 feet away, the disk whose perception matches the moon's perception in size, is never, even with the moon looking small in elevation, less than 1.5 degrees (a diameter of about 4 inches). In short, two retinal images give rise to two perceptions that are equal in size when one image is three times as large as the other in diameter, or nine times as large in area. This is a deviation in the direction of object size constancy, but it does not go very far in this direction. If size constancy held, this disk 12 feet away and only 4 inches across ought to look as if it had a diameter of 2160 miles (8). It does not. The moon, a very distant object, does not look nearly so big as it would if it were close by. In other words the moon, an object, does not get itself perceived in the System *O*, the Berlin system. Is there some kind of a system *R* into which it fits? If there is, certainly that system is also not going to be one in which sizes are proportional to retinal sizes.

THE VISUAL FIELD AND THE VISUAL WORLD

Perhaps Gibson's distinction (3, 4) between the visual field and the visual world will give us the systems we are looking for. What is this field? and this world?

The visual world is the easier to understand. It is what Berlin has been calling the world of phenomena, and thus the world of perceived objects, the Gestalt world of perception, an unbounded, stable, rigid, Euclidean world, always tridimensional, with parallels always equidistant—in fact the natural world of objects duplicated in perception. Since objects do not change in size when moved, the perceptions of moved objects do not in this world change in size. Object constancy is the rule in the phenomenal perceptual world because it is the rule in the "external" natural world. In short, evolution appears to have achieved an organism in which perception duplicates or at least takes adequate account of the real external world, within small tolerances, and with only a little illusion and error. As usual, however, it is the exceptions, the alternatives, the illusions and the errors that claim our attention.

The visual field is offered us as one alternative. It is not for Gibson the visual world. It tends to be bidimensional, pictorial, and in a sense "anatomical" like the retinal image. Yet it is certainly not the retinal field, for the visual field is never doubled in binocular vision, as is the retinal field, nor is it as diplopic. The field, unlike the world, is limited in extent, changing, fluid and non-Euclidean, as you can see if you study its flow, expansion, distortion and contraction as its observer flies rapidly through it in an airplane. If the visual world is made of perceptions, perhaps the visual field is made of sensations; yet Gibson, in suggesting the appropriateness of these two classical terms, does not mean that the visual field is prior to the visual world, the basic inventory out of which the object world is made. I believe Gibson would place the converging railroad tracks and the little distant man in the visual field, because he suggests that the visual field may actually be seen by the trained artist or introspective psychologist, who can abstract from objectification and see experience as . . . as it really is? Well, at least as it really is in the visual field.

The visual field is, of course, not the brain field either. It might be isomorphic with the brain field, but that we cannot say. Here we are looking for full topographical correspondence, not mere topological identity, for a correspondence of sizes, directions and distances. The visual field must come nearer matching a monocular retinal field than the cortical field which is divided between two hemispheres.

These distinctions leave us Gibson's visual field, freely suspended *in vacuo* with full freedom to be itself. It is not the perceived visual world of objects, nor the visual projection field in the cerebral cortex, nor the retinal field, nor the pattern of optical projection on the retina, nor the pattern of the world of external objects itself. It has its own properties, rules and limitations. Certainly it is no longer possible for any of us to go along with Wundt and Titchener and to say that the visual field is immediately given. The world of objects (or of stimuli, as Titchener would have called them) can appear as promptly and as fully organized as can that specially edited experience that the trained introspectionist and the artist learn to see, perhaps at times with as much celerity as they can see the stone that Dr. Johnson kicked. Nevertheless there is a use for Gibson's visual field as well as for his visual world, although both concepts are in need of further specification. At present these two systems float freely in a parallelistic pluralism, and they can be given—it seems to me—more precise meaning and better specification by operational reduction. Let us see what operationism can do for them.

PERCEPTION AS INVARIANCE

More than fifty years ago John Dewey remarked that one problem of stimulus-response reflexology is the discovery of the stimulus (5, pp. 367–370). He was right, for the effective stimulus is not an object but a property of the stimulus-object, some crucial property that cannot be altered without changing the response, some property that remains invariant, for a given response, in the face of transformations of other characteristics. Since then scientists have been coming to realize, as Stevens points out (12, pp. 19–21), that the discovery of invariances can be regarded as the chief problem of a quantitative science that has passed beyond the stage of phenomenology. And it is in terms of invariance that perception can be specified operationally.

Again let us consider the case of perceived size. What is perceptual size constancy? It is the rule that perceived object size is invariant under the transformation of tape-measured distance and thus also under the transformation of perceived distance, since tape-measured distance and perceived distance are known to vary together. There is another rule which goes along with this one, a fact that we take for granted and do not often state in psychological context. It is the rule of physical size constancy, the rule that tape-measured object size is invariant under the transformation of tape-measured distance or other change of location. Objects do not shrink or expand as you move them around, and neither do your perceptions of them when you have those conditions of no-reduction under which size constancy occurs. We have, under these circumstances, the correlation of two similar invariances, the invariance for physical size and for perceptual size, and we are free to imagine, if we wish, that evolution aimed at this achievement, making perception adequate to reality in order to increase the organism's chance of survival.

A less dualistic way of stating this relation is as follows. You can determine the invariance of the size of objects under the transformation of location either (a) by the direct comparison of the object in one place with the object in another or (b) by indirect comparison of the object in different places through the mediation of a tape-measure. In the latter case you compare the object directly with the marking on the tape, and you can keep distance constant by always reading the tape at a fixed distance. A great deal of other evidence also contributes to the accepted theory that objects do not change size appreciably when they move around on the face of the earth with ordinary velocities. The rule of size constancy thus becomes this: Under the transformation of location, size observed by direct comparison is invariant when size observed by tape-measuring is invariant. In short, we have two invariances correlated. There can be no mistake about there being two, for one breaks down more easily than the other. Reduce the clues to distance, and the correlation no longer holds, for then receding objects are seen to shrink, although not to recede.

Size constancy, defined operationally by this correlation of two observed invariances, can be translated into the common-sense statement: A man (or a chimpanzee) can perceive correctly the physical size of an object. The perceiver can perceive in direct comparison whatever remains invariant under the transformation of distance. We may next properly ask: Can a man (or a chimpanzee) also perceive the size of his retinal images, that is to say, can he be an artist or an introspectionist? Perhaps the man can though the chimpanzee can not. We need to know exactly what observation would demonstrate that an organism is perceiving the size of its own retinal images.

For a man to perceive the size of his own retinal images his perception of size must remain invariant under all transformations that leave the size of the retinal images invariant, including the crucial transformation involving object distance. If s is the linear size of the object and d is its distance from the eye, then retinal size (visual angle) is invariant when s/d is invariant, so the question becomes: Can the artist or introspectionist acquire and use an observational attitude under which perceived size stays fixed whenever s/d remains invariant, even under the transformation of distance? Human artists can come near to maintaining this invariance, but there are conditions under which the relation breaks down.

It breaks down, for instance, in perceiving the moon. As we have already noted it is impossible to perceive the moon as big as it really is (2160 miles across) or as small as its retinal image is (0.5 degree across). You see something in between, nearer retinal size than object size (8). Certainly when celestial distances are involved neither object size nor retinal size determines perceived size. What is needed is the discovery of the size-invariant for celestial distances, the discovery of the stimulus. We might know how properly to specify the stimulus if we knew the actual sizes of many moons that, at different distances from the earth, all look the same size. How big must moons that look alike be if they are a thousand miles away and a million miles away and at many distances in between, including the 240,000 miles that our regular moon is distant? The graph of those data would disclose the law of invariance, a statement of what is perceived under the attitude for judging size at great distances. If we could find a function, ϕ, that would be invariant when perceived size is invariant—an expression in terms of actual distance, perceived distance, actual elevation of the moon, elevation of the observer's regard, observer's attitude, and any other parameters that turned out to be essential—then we could say even better what it is that is being perceived (invariant). In short, if perceived size is invariant when this function, ϕ, is invariant, then, in judging size, you are perceiving not object size, not retinal size, but ϕ. To discover the object of perception, you have to discover what function of the parameters of the stimulus is invariant when the perception is invariant. That is a good operational definition of perception in terms of stimulus invariance.

THE VISUAL FIELD AND PERCEPTUAL REDUCTION

Gibson is writing phenomenology and he tells us that we have a visual world that corresponds in general with considerable accuracy to the rigid, Euclidean, natural, tape-measured world, and with but small exceptions for illusion and error. That is good phenomenology and natural philosophy, but it is not the body of exact quantitative knowledge that we call science nowadays. Just as the scientific physics of the natural world, with its molecules, atoms and electrons, is not something that you can look at and see, so the scientific psychology of the visual world differs from phenomenology in being a collection of observed functional relations that can be approximately summarized by the hypothesization of a Euclidean model. You cannot see the visual world at any moment when you are playing scientist; you construct it out of elaborate observations that have been being collected for many years in the past.

Gibson's visual field, a concept that creates difficulty even in phenomenology, seems to me to become clear in terms of our examples—the converging tracks, the little man in the distance, the moon that is both too big and too small. I think Gibson would accept these items as belonging in the visual field, but no matter. Let us put them in our own System R, and now let us come back to what we were planning to do all along; let us say that the System R is a system of *reduced vision.*

Our examples are all instances of partially reduced perceptions of visual size with distance variant. The System *R* (and perhaps Gibson's visual field?) is the reduced visual world, the totality of those simpler sights where reduction of the total complexity of clues makes the observer dependent upon but a few parameters of the stimulus or perhaps upon only a single one, like retinal size. For this system *R* there is no obvious model, like the Euclidean model for the visual world. The System *R*, the field of reduced perceptions, is simply a congeries of observed invariances. These reductions are, moreover, not always complete. There are limits to what attitudinal abstraction in observation and to what experimental control can accomplish in the elimination of clues. If reduction were indeed complete, then the System *R* might come to resemble or even to duplicate the retinal field. In fact, it becomes clear that these cases of partial but incomplete reduction are the occasion for the present paper.

Now let us consider another case of incomplete reduction, the case of binocular vision. Can a man tell with which eye something is seen? Presumably a pigeon can (10), but for a man the answer is yes and no. His brain knows one eye from the other as it translates retinal disparity into perceived depth. His verbal mechanisms know the difference only after he has tried first shutting one eye and then the other. He can see depth based on disparity when he cannot see diplopia. Complete reduction of binocular vision would be a reduction not to *a* retinal image but to *two* retinal images. So we have here, if we are thinking of the artist's view, another instance of the partial but incomplete integration of the physiological pattern into the perceived pattern, a crucial example where perception lies intermediate between complete "reduction" to the retinal image and complete "regression" to (integration of) the real object.

In general it seems to me better not to try to create a model for the System *R* (or the visual field), but to leave these facts as they were born, in an inventory of invariances under various reductions. The invariances tell us what the organism can do under attitudinal training to perceive its own physiological bases, the data out of which it can, after much evolution, create an extremely useful apprehension of the world that it accepts as its reality.

Let me not seem to belittle phenomenology nor our debt to Gibson. Phenomenological description is a valuable *vorwissenschaftliches* undertaking. It shows what the psychological problems are. This paper of mine is concerned with indicating the nature of the next step beyond phenomenology and with demonstrating how the scientific problems of perception can be pushed forward by a study of the parametric invariances of the stimulus.

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