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The generic viewpoint assumption and Bayesian inference

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Abstract. The task of human vision is to reliably infer useful information about the external environment from images formed on the retinae. In general, the inference of scene properties from retinal images is not deductive; it requires knowledge about the external environment. Further, it has been suggested that the environment must be *regular* in some way in order for *any* scene properties to be reliably inferred. In particular, Knill and Kersten [1991, in *Pattern Recognition by Man and Machine* Ed. R J Watt (London: Macmillan)] and Jepson et al [1996, in *Bayesian Approaches to Perception* Eds D Knill, W Richards (Cambridge: Cambridge University Press)] claim that, given an 'unbiased' prior probability distribution for the scenes being observed, the generic viewpoint assumption is not probabilistically valid. However, this claim depends upon the use of representation spaces that may not be appropriate for the problems they consider. In fact, it is problematic to define a rigorous criterion for a probability distribution to be considered 'random' or 'regularity-free' in many natural domains of interest. This problem is closely related to Bertrand's paradox. I propose that, in the case of 'unbiased' priors, the reliability of inferences based on the generic viewpoint assumption depends partly on whether or not an observed coincidence in the image involves features known to be on the same object. This proposal is based on important differences between the distributions associated with: (i) a 'random' placement of *features* in 3-D, and (ii) the positions of features on a 'randomly shaped' and 'randomly posed' 3-D *object*. Similar considerations arise in the case of inferring 3-D motion from image motion.

1 Introduction

The task of human vision is to reliably infer useful information about the external environment from the images formed on the retinae. Since these images are 2-D projections of a 3-D scene, the depth of features in the scene—the spatial component along the line of sight—is not specified.

In general, the inference of scene properties (eg depth, surface reflectance, illumination) from retinal images is not deductive. It has been suggested that the reliability of perception depends on the *regularities* of the external environment, and that biological organisms have evolved or learned to exploit these regularities. If the observed scenes are simply 'random' 3-D arrangements of features, then, so the story goes, retinal images cannot provide a reliable basis for perceptual inferences. These environmental regularities are generally of a statistical nature and are expressed as *probability distributions* over the space of possible scene interpretations.

The importance of environmental regularities is a time-honored principle of much of the literature on visual perception, dating back to Helmholtz (1909/1962). A recent collection of essays on applications of Bayesian probability theory to visual perception (Knill and Richards 1996) amply demonstrates how various environmental regularities may be used to probabilistically justify perceptual inference. In a Bayesian framework, biases in the *prior probability distributions* appearing in Bayes's rule embody a mathematical representation of these environmental regularities.

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One way to formulate the application of Bayes's rule to vision is:

$$P(S|I) = \frac{P(I|S) \times P(S)}{P(I)} ,$$

where I is the image formed on the retina, S is a possible scene interpretation, and $P(S)$ is the prior probability distribution for scenes in the environment. $P(S|I)$ (read "the probability of the scene given the image") is the posterior probability distribution for scenes in the environment. It represents the 'refinement' of the prior probability distribution made possible by the information present in the image I. On the other hand, $P(I|S)$ is the probability of the image given the scene. This probability distribution is deter-mined by prior assumptions about the relative probabilities of the different *viewpoints* from which the scene might be observed. One such assumption is the generic viewpoint assumption.

2 The generic viewpoint assumption

The generic viewpoint assumption (Binford 1981; Witkin and Tennenbaum 1983; Lowe 1985; Koenderink and van Doorn 1986; Malik 1987; Nakayama and Shimojo 1990, 1992; Albert 1992; Freeman 1992; Albert and Hoffman 1995; Yuille 1996) states that human vision assumes that salient image features are not the result of 'accidental' coincidences between the viewpoint of the observer and the internal geometry of the scene. Jepson et al (1996), Jepson and Richards (1993), and Knill and Kersten (1991) present arguments purporting to show that various instances of the generic viewpoint assumption are not probabilistically valid without assuming strong 'biases' in the prior probability distribution [ie in the distribution $P(S)$ appearing in Bayes's rule]. In particular, they suggest that such distributions should be linear combinations of: (i) smooth functions over the entire space of possible scene descriptions, which represent the 'unbiased' component of the prior, and (ii) localized, highly biased Dirac delta functions centered on certain 'favored' interpretations. Jepson et al (1996) claim that these delta functions, or 'modes', comprise a precise, formal representation of the environmental regularities that make reliable perceptual inference possible. However, their claim presupposes that there exist well-defined criteria for classifying distributions as 'regularity-free', 'unbiased', or 'random'.

To illustrate the difficulty in formulating such criteria, consider the following perceptual problem: Suppose we are given a 3-D scene containing a single line segment whose length is between zero and some maximum length L (inclusive). Further, suppose that the image we observe consists of just a single 'point'. We want to know whether the observed 3-D scene is more likely to be a 'point' in 3-D (ie a 'line' of length zero) or a 3-D line segment of nonzero length that happens to lie along our line of sight.

Now, consider two possible definitions for a 'regularity-free' or 'unbiased' prior probability distribution over such scenes: (i) With the first endpoint of the line segment fixed at the origin of our 3-D coordinate system, assume that the second endpoint is selected by using a flat distribution on distance from the origin (within the sphere of radius L). Call this prior probability distribution P_1. This distribution over the *position* of the endpoint induces a distribution over the *length* of the line segment that is proportional to the square of the length (since the surface area of a sphere is proportional to the square of its radius). In other words, P_1 is 'flat' as a function of the position of the endpoint, but it is a parabola as a function of the length of the line segment, so it is 'biased' towards *longer* line segments. It can be shown that P_1 does not reliably support the inference that a line segment has length zero in 3-D just because it has length zero in the image. However, another plausible definition for an 'unbiased' prior probability distribution in this context does, namely (ii) a flat prior for the *lengths* of the 3-D line segments. Call this distribution P_2. I will show that P_2 *does* reliably support the desired inference (see figure 1).

(a) (b)

Figure 1. (a) A flat prior probability for the length of a line segment between zero and L, inclusive. (b) A posterior probability for the length of a line segment, given a flat prior on length, and given an image of the line segment in which the endpoints coincide.

In general, given the task of inferring the 3-D *shape* of a single object, it seems natural to define an 'unbiased' prior probability distribution in this context to be a smooth distribution over a space of *viewpoint-independent* descriptions of the objects (eg the *lengths* of line segments).[(1)] The viewpoint of the observer is then chosen by using some smooth prior probability distribution on the space of possible viewpoints on the object. In other words, I suggest that the generic viewpoint assumption is implicitly based on a conceptual framework in which a prior probability distribution is placed on the *internal* geometry of the object being observed, and a uniform (or smooth) prior is placed on the space of possible viewpoints of the observer relative to the object. This contrasts with placing a smooth distribution over a space of *viewpoint-dependent* descriptions of an object, for example, the *positions of the endpoints* of a line segment (which implicitly includes a description of the shape of the object and its 3-D orientation relative to the observer). One might expect that the consequences of the choice between viewpoint-independent and viewpoint-dependent representations for defining what is meant by a 'random' or 'regularity-free' context would be qualitatively unimportant; that either representation could be used to define a 'random' prior and determine the reliability of a given perceptual inference without affecting the conclusions. However, because of the greater symmetry of certain perceptual interpretations, the choice between these representations turns out to be critical.

3 Seeing 'at rest' but perceiving motion?

Jepson et al (1996) argued that the inference: "If a particle is stationary in the image, then it is stationary in 3-D space" is not reliable unless the prior probability distribution for the motion of the particle is biased in favor of the particle being at rest (see figure 2). They showed that, if the prior probability distribution in the space of 3-D velocity vectors is given by the Maxwell distribution for the velocity of an ideal gas molecule, then, despite the fact that this distribution is at its maximum when the velocity vector is zero, this inference is unreliable.

[(1)] For example, a viewpoint-independent description of the set of all possible 3-D triangles might be to give the lengths of two sides and the included angle between them. On the other hand, a viewpoint-dependent description of the set of all possible 3-D triangles might be to give the 3-D coordinates of each of its three vertices relative to the observer, for a total of nine parameters.

In the case of static images a viewpoint-independent description is essentially the same in spirit as an object-centered description. An object-centered description is also a viewpoint-independent description; however, the reverse is not always true. For the case of moving images, a description can be viewpoint-independent, ie independent of the position of the observer, but not independent of the velocity of the observer. An object-centered description is always independent of the motion of the observer. Thus, as will be discussed below, the speed of a particle is a viewpoint-independent descriptor, in that it is the same for all observers that are at rest relative to one another, but it is not an object-centered descriptor (since its velocity is always zero relative to itself).

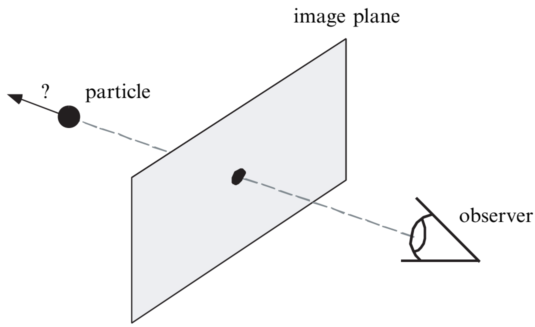

Figure 2. Is the inference "At rest in the image implies at rest in 3-D" probabilistically valid given an 'unbiased' prior probability distribution for the motions of particles in 3-D space? See text.

The Maxwell distribution is:

$$P_{\mathrm{M}}(\boldsymbol{v}) = \frac{1}{(2\pi cT)^{1/2}} \exp\left(-\frac{\|\boldsymbol{v}\|^2}{2cT}\right),$$

where T is the temperature of the gas, and c is a positive constant.

However, the velocity vector of a particle is a viewpoint-dependent description of the motion of a particle. A viewpoint-independent description of the motion of a particle would be its *speed*—the magnitude of its velocity vector. In general, the probability of the particle having speed approximately equal to s, given any probability measure M on the space of velocity vectors, is equal to the 'mass' of M inside a thin spherical shell of radius s centered at the origin of velocity space. Therefore, to convert the prior probability distribution for velocity given by the Maxwell distribution into a prior probability distribution for speed we simply multiply the Maxwell distribution by the surface area of a sphere of radius equal to the speed, ie $4\pi s^2$. The resulting prior probability distribution is zero for $s = 0$ and nonzero for all $s > 0$, suggesting that the Maxwell distribution is strongly biased *against* the particle being at rest. In other words, it is a very unlikely event to find a particular ideal gas molecule at rest.

On the other hand, consider a flat prior for *speed* (up to some maximum speed S), combined with a flat prior for viewing angle.(2) Notice that the velocity vector of length zero is a single point in velocity space, whereas in the speed/viewing-angle representation, where for $s = 0$ the viewing angles can still vary freely, they generate a 2-D subspace of the 3-D speed/viewing-angle representation space. This suggests that an unbiased prior for speed may be qualitatively very different from an unbiased prior for velocity.(3) I claim that a flat prior for speed *does* support the inference that 3-D velocity of a particle is close to zero if its image velocity is close to zero.

A rigorous proof of this claim can be constructed by calculating the limit of the ratio: (probability of the resting interpretation)/(probability of the moving interpretations) as the visual resolution becomes infinitely fine, and conditioned on the particle being at rest in the image. The complete proof is given in the Appendix. It shows that this limit is nonzero and finite, ie that the *cumulative* posterior probability of *all* the 'moving' interpretations is comparable to the posterior probability of the *single* 'at rest' interpretation (also see Yuille 1996). Furthermore, the inference: "If the particle is at rest in the image, then its 3-D speed is less than δ", where δ is any arbitrarily small positive number, is reliable (ie it is valid with probability 1). In fact, the ratio (probability of the *single* stationary interpretation)/(probability of the set of all moving interpretations with speeds greater than δ) is infinite for any arbitrarily small $\delta > 0$. This shows

(2) Here I assume orthographic projection, as in Jepson et al (1996).

(3) Similarly, Knill and Kersten (1991) argued that the generic viewpoint assumption does not reliably support the inference that a straight 'wire' in an image corresponds to a straight wire in 3-D. The ideas represented here may also be applied to their example.

that the ratio: (probability of the 'at rest' interpretation)/(probability of the set of all 'moving' interpretations) is finite only because of the cumulative probability of the moving interpretations having speeds *less* than δ—the cumulative probability of the moving interpretations with speeds greater than δ is negligible, no matter how small we choose δ to be. An analogous proof shows that the inference that a line segment has length zero in 3-D if it has length zero in the image is reliable, given a flat prior for the length of the line segment (the velocity vector is analogous to length and orientation of the line segment).

It is important to note that if we transform a flat prior for speed into velocity space, then the limiting value of the integral of the resulting distribution over a sphere centered at the origin in velocity space approaches zero as the radius of the sphere approaches zero. This follows from the fact that the integral of the original flat prior for speed between zero and some nonzero speed s approaches zero as s approaches zero. In fact, a flat prior for speed transformed into velocity space is $\sim s^{-2}$. On the other hand, the integral of a Dirac delta function over a sphere of radius s remains nonzero as s approaches zero. So, although a flat prior on speed is clearly more 'peaked' at zero in velocity space than the Maxwell distribution discussed by Jepson et al (1996), it is also clearly a much weaker regularity than the 'modes' they claim are necessary for reliable inference based on the generic viewpoint assumption.

Jepson et al (1996, page 73, line 8) claim that "The existence of this mode [for being at rest] was shown to be *critical* [emphasis added] in order to obtain a reliable inference that an object being observed is in fact at rest". We have shown that a mode for zero velocity is not necessary for this inference to be reliable.

The arguments presented here apply to inferences about the 3-D structure of a single 'object', or the motion of a single feature. It can certainly be argued that the assumption that a group of feature points belongs to a single object (such as the two endpoints of a line segment) is *itself* a bias. However, this kind of bias is very weak in comparison to delta functions placed on *single* interpretations: This kind of 'object bias' does not place a nonzero probability mass on any single object shape in either an object-centered or a viewer-centered sense (see above), whereas the 'modes' advocated by Knill and Kersten (1991), and Jepson et al (1996) do. It might further be argued that, in order for a flat prior on shape to be considered a 'random' environment, we would have to already know (by some other means) that the group of feature points of interest belongs to a single object. Fair enough. (However, note that in the motion example discussed above a similar argument *cannot* be made; a flat prior for speed implies no object bias.)

I conclude that:

Even in an environment of 'random' 3-D shapes (or motions) it is possible to make certain reliable inferences about 3-D shape (or motion) from 2-D image data.

4 Summary and discussion

I have suggested that the generic viewpoint assumption supports reliable inference given a flat prior over a viewpoint-independent representation of the environment. I argued that the meaning of statements such as: "The states of the environment are random" depends on the choice of the parameters used in the representation of that environment. If there is no 'canonical' choice for these parameters, then there can be no principled way to define what 'random' means. I demonstrated that this ambiguity can be significant enough that, given a uniform prior probability over one representation space, a certain inference can have negligible posterior probability; whereas, given a uniform prior probability over a different representation space, the same inference can have posterior probability one.

A related point was made by the French mathematician Joseph Bertrand (Bertrand 1889). One variant of the famous 'Bertrand's paradox' is the following problem in elementary geometry: Suppose we want to calculate the probability that a 'randomly selected' chord to a circle has length greater than the length of one side of an inscribed equilateral triangle of the circle. It turns out that different ways of parameterizing the set of chords to a circle leads to different answers to this question, and there is no principled reason to prefer one parameterization over the others (see figure 3). Although some have suggested that this particular paradox can be resolved by selecting a 'model' for the problem and considering the symmetry constraints it suggests (Jaynes 1973), it is known that this solution is not general. That is, the symmetry constraints entailed by 'reasonable' models do not generally lead to a unique unbiased probability distribution for other similar paradoxes (van Fraassen 1989). In particular, these techniques do not appear to be useful for the problems considered in this article.

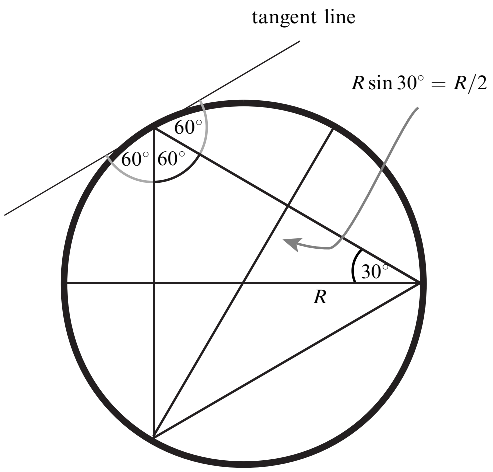

Figure 3. Bertrand's paradox. This figure illustrates two of the many possible methods that might be used to 'measure' the percentage of chords to a circle whose length is greater than the length of a side of an equilateral triangle inscribed in the circle. One method is to consider the range of angles that the chord could make relative to the tangent line to the circle at one of the chord's endpoints. Another method is to consider the range of possible perpendicular distances that the chord could have from the center of the circle. However, these two methods give different answers to the question: "What is the probability that a 'randomly selected' chord to a circle has length greater than the length of one side of an inscribed equilateral triangle of the circle (after Bertrand 1889)?"

It is possible that the ambiguity of the concept of a 'regularity' could be reduced by defining it in a more qualitative way, say by considering the properties of certain *equivalence classes* of prior probability distributions, rather than exact prior probability distributions themselves. For example, a mode for zero velocity would become a mode for zero speed when transformed into the 3-D speed/viewing-angle representation, so it seems this 'regularity' would be preserved under most reasonable re-parameterizations.

I conclude that although environmental regularities are certainly likely to be necessary to obtain the kind of rich perceptual knowledge we generally possess about our environment, my analysis shows that they are not necessary *in principle* for reliable perceptual inference, and that it is possible to reliably infer certain things about a 3-D environment from an image even when the statistics of that environment can reasonably be considered to be 'random'.

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References

Albert M, 1992 "Genericity and the perception of illusory contours", in *Proceedings of the 14th Annual Conference of the Cognitive Science Society* (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 289 – 294

Albert M K, Hoffman D D, 1995 "Genericity in spatial vision", in *Geometric Representations of Perceptual Phenomena: Papers in Honor of Tarow Indow's 70th Birthday* Ed. D Luce (Hillsdale, NJ: Lawrence Erlbaum Associates) pp 95 – 112

Bertrand J, 1889 *Calcul des Probabilités* 3rd edition (Bronx, NY: Chelsea Publishing Co)

Binford T O, 1981 "Inferring surfaces from images" *Artificial Intelligence* **17** 205 – 244

Fraassen B C van, 1989 *Laws and Symmetry* (Oxford: Clarendon Press)
Freeman W T, 1992 "The generic viewpoint assumption in a framework for visual perception" *Nature (London)* **368** 542 – 545
Helmholtz H von, 1909/1962 *Physiological Optics* volume 3 (New York: Dover, 1962); English translation by J P C Southall for the Optical Society of America (1925) from the 3rd German edition of *Handbuch der physiologischen Optik* (Hamburg: Voss 1909; first published in 1876, Leipzig: Voss)
Jaynes E T, 1973 "The well-posed problem" *Foundations of Physics* **3** 477 – 492
Jepson A, Richards W, 1993 "What is a percept?" Report RBCV-TR-93-43, Department of Computer Science, University of Toronto, Toronto, Canada
Jepson A, Richards W, Knill D, 1996 "Modal structure and reliable inference", in Knill and Richards (1996) pp 93 – 122
Knill D, Kersten D, 1991 "Ideal perceptual observers for computation, psychophysics, and neural networks", in *Pattern Recognition by Man and Machine* Ed. R J Watt (London: Macmillan)
Knill D, Richards W (Eds), 1996 *Bayesian Approaches to Perception* (Cambridge: Cambridge University Press)
Koenderink J, Doorn A J van, 1986 "The internal representation of solid shape with respect to vision" *Biological Cybernetics* **32** 211 – 216
Lowe D, 1985 *Perceptual Organization and Visual Recognition* (Boston, MA: Kluwer)
Malik J, 1987 "Interpreting line drawings of curved objects" *International Journal of Computer Vision* **1** 73 – 103
Nakayama K, Shimojo S, 1990 "Toward a neural understanding of visual surface representation" *Cold Spring Harbor Symposia on Quantitative Biology* **40** 911 – 924
Nakayama K, Shimojo S, 1992 "Experiencing and perceiving visual surfaces" *Science* **257** 1357 – 1363
Witkin A P, Tennenbaum J M, 1983 "On the role of structure in vision", in *Human and Machine Vision* Eds J Beck, B Hope, A Rosenfeld (New York: Academic Press)
Yuille A, 1996 "Commentary on 'Priors, preferences and categorical percepts'", in Knill and Richards (1996) pp 221 – 223

Appendix

In the text I considered the perceptual inference "If a particle is stationary in the image, then it is stationary in 3-D space", and gave qualitative arguments for its reliability, given a 'random' prior to the 3-D speed of the particle. Here I prove that, if a particle is stationary in an image, then the limit of the ratio: (probability that the particle 3-D speed is less than ε)/(probability that the particle 3-D speed is greater than ε) is finite as ε approaches zero, given a uniform prior probability distribution on the 3-D speed/viewing-angle representation space. The parameter ε represents the precision of the image measurements and also the precision of the 3-D perceptual category of a particle being 'at rest'.

For a given 3-D speed s, we need to calculate the range of viewing directions or the solid angle Θ within which the image speed will be less than ε. Suppose that our uniform prior probability distribution on 3-D speed has constant value equal to c up to a maximum speed S. Recall that just as an ordinary angle θ in a 2-D plane is defined by the amount of the circumference of the unit circle that the angle subtends, a solid angle Θ in 3-D space is defined by the amount of surface area of the unit sphere that the solid angle subtends. Now, the surface area of a sphere of radius R is equal to $4\pi R^2$. In the 2-D case, $\sin\theta < \theta$ for all $0 < \theta < \pi/2$, and analogously in 3-D:

$$2\pi(\sin\theta)^2 < \Theta, \text{ for all } 0 < \theta < \pi/2, \qquad \text{(inequality 1)}$$

where $\Theta/2$ is the size of the solid angle defined by a cone with its vertex at the center of the unit sphere, and with angle θ relative to its axis of symmetry. To see why, note that $\pi(\sin\theta)^2$ is the area of the base of the cone of the solid angle if we just took the base to be a flat disk. This area must always be less than the surface area of the sphere subtended by the cone, which is $\Theta/2$. The full solid angle of interest actually includes two oppositely pointing cones, and has total size Θ, since the particle could be moving either towards or away from the observer.

On the other hand, the ratio: (area of the solid angle for one cone, ie $\Theta/2$)/(area of the flat base of the cone) is maximal when the solid angle is the full sphere (ie $\Theta/2 = 2\pi$), in which case this ratio is $2\pi(1)^2/\pi(1)^2 = 2$. In other words, the solid angle for one cone has no more than twice the area of the flat base of the cone. So we also have:

$$4\pi(\sin\theta)^2 > \Theta, \quad \text{for all } 0 < \theta < \pi/2. \qquad \text{(inequality 2)}$$

We now calculate the posterior probability that the 3-D speed of the particle is greater than ε, given a flat prior on its 3-D speed P_{FS}, and an image I of the particle in which it has image speed less than ε. Note that the constraint that the image speed of the particle is less than ε implies that $\sin\theta < \varepsilon/s$, where θ is the angle between the 3-D velocity vector of the particle and the observer's line of sight, and s is the 3-D speed of the particle. Let $\Theta(s)$ represent the solid angle of possible viewing directions from which the particle, moving at 3-D speed s, would project an image speed of less than ε into I. Now, the posterior probability that the 3-D speed of the particle is greater than ε is equal to

$$c\int_{\varepsilon}^{S} \Theta(s)\mathrm{d}s\,.$$

So, substituting inequality 1 into this formula we find that the posterior probability

$$P_{\mathrm{FS}}(s \geqslant \varepsilon|I) \geqslant 2\pi c\varepsilon^2 \int_{\varepsilon}^{S} \frac{1}{s^2}\mathrm{d}s = 2\pi c\left(\varepsilon - \frac{\varepsilon^2}{S}\right),$$

and substituting inequality 2 shows that this probability can be at most twice the value of the expression on the right.

The posterior probability that the 3-D speed of the particle is less than ε is simply

$$P_{\mathrm{FS}}(s < \varepsilon|I) = 4\pi\int_0^{\varepsilon} c\mathrm{d}s = 4\pi c\varepsilon\,,$$

since the solid angle of possible viewpoints is equal to the complete viewing sphere, ie 4π. So, if the particle is moving slower than ε in the image I, then

$$\frac{P_{\mathrm{FS}}(s < \varepsilon|I)}{P_{\mathrm{FS}}(s \geqslant \varepsilon|I)} = 4\pi c\varepsilon/2\pi c\left(\varepsilon - \frac{\varepsilon^2}{M}\right) = 1\bigg/\frac{1}{2} - \frac{\varepsilon}{2M} \xrightarrow{\varepsilon\to 0} 2$$

is finite, as claimed.

Furthermore, note that, if the lower limit of the integral given above for the posterior probability that the 3-D speed of the particle is greater than ε is taken to be a fixed, but arbitrarily small, parameter δ, then the above limit 'blows-up' as ε goes to zero. In other words, the 'at rest' interpretation for the particle is *infinitely* more probable than the set of 'moving faster than δ' interpretations, for any arbitrarily small value of δ. So, we see that it is only the cumulative probability of the moving interpretations having speeds less than δ that has probability comparable to the probability of the *single* 'at rest' interpretation. The cumulative probability of the moving interpretations having speeds greater than δ is negligible.